Solid-State Electronics Vol. 34, No. 1, pp. 33–42, 1991
Printed in Great Britain. All rights reserved

0038-1101/91 $3.00 + 0.00

AVALANCHE MULTIPLICATION IN SEMICONDUCTORS: A MODIFICATION OF CHYNOWETH's LAW

THOMAS LACKNER
Siemens AG, Research Laboratories, Otto-Hahn-Ring 6, D-8000 Munich 83, F.R.G.

(Received 1 May 1990; in revised form 16 June 1990)

Abstract—A new theory is presented for calculating the avalanche multiplication factor $M(x)$ in semiconductors. The integral equation, which determines $M(x)$, takes into account that a particle has to gain a critical energy U_c (threshold energy) from the electric field $E(x)$ in order to be able to ionize lattice and impurity atoms. Therefore, our theory is also applicable to narrow junctions. The input to our theory are the mean free paths and the threshold energies of electrons (L_n, U_c^n) and holes (L_p, U_c^p), respectively.

Approximations to the integral equation for $M(x)$ lead to ionization rates $\alpha_n(E)$, $\alpha_p(E)$ for electrons and holes, respectively. The dependence of these ionization rates on the electric field is determined analytically and a correction to Chynoweth's law is found. In this paper we merely want to show that the field dependence of the ionization rates given by Chynoweth's law ($\alpha_s^c = \alpha_s^c \exp(-b_s^c/E)$ with $s = \{n, p\}$) is, in general, a crude approximation within our more general theory.

Rather we shall derive an approximation of the following form:

$$\alpha_s = \frac{a_s}{z} \exp\left(-\frac{b_s}{E(x)}\right)$$

with

$$z(x) = 1 + \frac{b_n}{E(x)} \exp\left(-\frac{b_n}{E(x)}\right) + \frac{b_p}{E(x)} \exp\left(-\frac{b_p}{E(x)}\right)$$

and

$$a_s = \frac{1}{L_s}, b_s = \frac{U_c^s}{qL_s}, s = \{n, p\}$$

We want to stress that the purpose of our paper is not to determine the physical parameters a_s, b_s from multiplication factor measurements. Rather we want to convince the reader that our expression of the ionization rate is both a simple and, for narrow junctions, a necessary correction to Chynoweth's law. Considering abrupt p–n junctions with different doping concentrations it is shown that our dependence of ionization rate on electric field $E(x)$ is valid in a range $2 \times 10^5 < E < 6 \times 10^5$ V/cm. In contrast to the results by Van Overstraeten and De Man, no splitting of the constants a_s, b_s in a high field and a low field range is required in order to obtain a correct multiplication factor.

The arguments used are self contained.

1. INTRODUCTION

The determination of ionization rates is of great importance in device physics[1–5]. In particular, impact ionization and subsequent avalanche multiplication are the leading mechanisms which limit the blocking capabilities of diodes or thyristors[6,7].

Computer simulation is often a very useful tool for getting more insight into what is going on within a device. Of course, these computer programs can only solve the device equations numerically for one[8] or two[9–12] dimensions. The form of the device equations was first presented by Van Roosbroeck[13]. However, the parameters entering the device equations (e.g. mobility, lifetime, avalanche generation rates, etc.) have been a topic of permanent discussions.

In order to get explicit expressions for these properties (e.g. dependence of the mobility on the electric field) one has to start from a more rigorous theory such as Boltzmann's equation[14–16] or even more sophisticated equations and try to derive the device equations. In doing this, one can express the physical parameters (or transport coefficients) in terms of integrals over time correlation functions[16–18] or as solutions of rather involved integral equations[14,15]. An alternate approach consists in developing plausible models directly for the transport coefficients[19]. The starting point in this approach is a model which as a result only gives the transport coefficients and not the phenomenological equations (device equation) themselves. In this paper we shall proceed the latter way.

The field dependence of the ionization rates $\alpha_n(E)$ and $\alpha_p(E)$ for electrons and holes, respectively has been a subject of numerous investigations leading to non unique results[20–26]. The most accepted form for the ionization rates is Chynoweth's law[20] ($\alpha_s^c = a_s^c \exp(-b_s^c/E)$ with fit parameters $a_s, b_s, s = \{n, p\}$ to be determined by experiments). We want to stress that ionization rates can only be extracted from multiplication factor measurements (OBIC[27], EBIC[28]) and are not quantities which can be measured as terminal properties.

In their rigorous treatment Van Overstraeten and De Man[21] use narrow p–n junctions for multiplication factor measurements. They assume the validity of Chynoweth's law. With the field dependence given in Chynoweth's law they are able to determine the coefficients a_s^c, b_s^c by multiplication factor measurements. In this context Chynoweth's law can be viewed as an excellent physical guess for the field dependence of ionization rates in order to fit experimental data of multiplication factors. However, the measurements of Van Overstraeten and De Man also show, that the original assumption of the simple exponential law does not cover the entire E-field range and has to be sacrificed in order to fit experiments well. Constants a_p^c, b_p^c different in the low field ($E < 4 \times 10^5$ V/cm), and high field ($E > 4 \times 10^5$ V/cm) region respectively were found for holes. We strongly believe, that it is very difficult to decide experimentally whether Chynoweth's law is true or not, since measurable quantities (avalanche multiplication factor) involve integrals over ionization rates only. Therefore we decided to develop a simple theory which, as an approximative result, yields the field correction to Chynoweth's law.

The paper is organized as follows. In Section 2 we derive an integral equation for the multiplication factor $M(x)$ [see eqns (3, 4a, b)]. Within the reduced description space and within the theory of Markov chains[29–31] we claim that our integral equation is exact. In Section 3 we assume a certain ionization probability [eqns (12a, b)] in order to derive a simple equation for $M(x)$. A numerical solution for this equation [eqn (15)] is termed exact, since no approximation has been made to derive it from eqns (4a, b). We only assumed a particular ionization probability. With certain constraints on the electric field $E(x)$ [see eqns (16a, b)] and on the space dependence of $M(x)$ [see eqns (18a, b)] we derive our field correction to Chynoweth's law [see eqns (20a–c)]. Section 4 only serves as a check for the validity of the approximations made in Section 3. In particular, for an abrupt p–n junction and 'realistic' parameters (mean free paths and ionization energies) we calculate the space and voltage dependent avalanche multiplication factor $M(x, V)$. For a wide range of doping concentrations we find an excellent agreement between the exact solution [eqn (15)] and our field approximation [eqn (20a–c)]. However, the simple form of Chynoweth's law shows rather large deviations from our exact equation. This deviation increases with increasing donor concentration. In order to compare Chynoweth's law with our field approximation no reference is made to experiments or other theories. In this sense we call our paper self-contained.

Remark. It is not our intention to determine the mean free path constants L_s, U_c^s from multiplication factor measurements. Rather we want to point out that the field dependence of Chynoweth's law is incorrect. The objective of this paper is to predict a field-dependence of the ionization rates which covers the entire range of interest ($2 \times 10^5 < E < 6 \times 10^5$ V/cm).

The four constants L_s, U_c^s with $\{s = n, p\}$, however, remain undetermined within this theory. Nevertheless, we have chosen a particular parameter set in order to get a feeling of the impact this new theory has on both, ionization rates and on the multiplication factor. The mean free paths L_s, L_n were taken from phonon-scattering, and the threshold energies U_c^n, U_c^p were set equal to the ionization energy in silicon.

Comparing our theory with data from Van Overstraeten and De Man already shows a surprisingly good agreement for a wide range of doping concentrations chosen for an abrupt p–n junction. If, however, Chynoweth's law is used, the above constants give rise to completely wrong results. It should be noted, that the particular parameter set chosen in this paper differs considerably from the experimentally obtained parameter set by Van Overstraeten and De Man.

The reason for this discrepancy is simple. Van Overstraeten and De Man based the evaluation of their multiplication factor measurements on the validity of Chynoweth's law. In order to get agreement with experiments, however, they had to switch the constants for holes at a field value of $E = 4 \times 10^5$ V/cm.

In contrast to Chynoweth's law, the special form of our field correction allows a unique determination of the four physical parameters L_n, L_p, U_c^n and U_c^p over the entire range of field values.

This remark should emphasize, that it is worthwhile to measure the mean free path constants and the threshold energies from multiplication factor measurements. We also view our paper as a suggestion for performing this kind of experiment.

2. MEAN FREE PATH THEORY

In this section we want to present a new theory for calculating the avalanche multiplication factor $M(x)$. Let us consider an electron–hole pair created at x, and moving under the influence of the electric field $E(x)$ [see Figs 1(a–c)]. We want to know the total number $M(x)$ of electron–hole pairs in the interval $[0, L]$ which are generated due to collisions and subsequent ionizations with lattice and impurity atoms. We introduce the following definitions.

$M(x)\cdots$ Mean number of electron–hole pairs (including the original pair) generated in $[0, L]$ given an initial electron–hole pair at x. In counting $M(x)$ we consider the stationary case.

$M_p(x)\cdots$ Mean number of electron–hole pairs generated in $[0, L]$ due to initially 1 hole at position x.

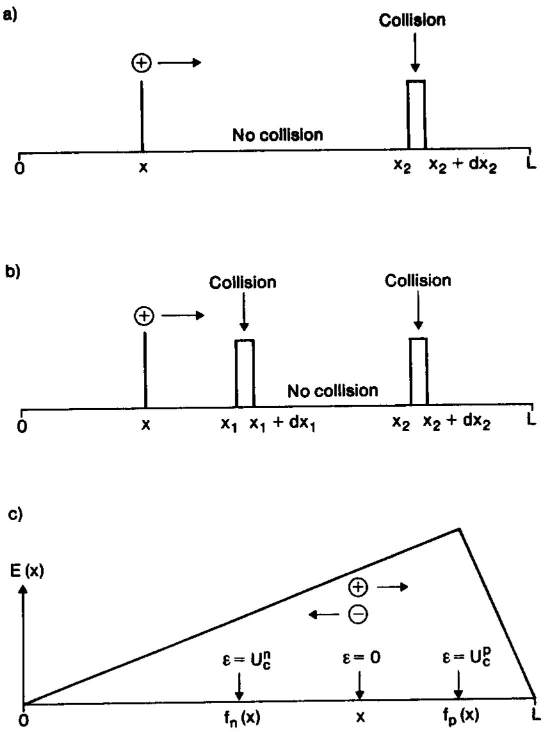

Fig. 1. Details of a collision process: (a) no collision takes place, (b) at least one collision takes place, (c) the electric field.

$M_n(x)\cdots$ Mean number of electron–hole pairs generated in $[0, L]$ due to initially 1 electron at position x.

$L_n(x)\cdots$ Mean free path for electrons.

$L_p(x)\cdots$ Mean free path for holes.

$G_n(x_1, x_2)\cdots$ Ionization probability for electrons, i.e. probability that an electron, which started at x_1 (with energy $\epsilon = 0$) and travelled without collision up to x_2 then ionizes its collision partner (thus generating secondary electron–hole pairs).

$G_p(x_1, x_2)\cdots$ Ionization probability for holes.

$\gamma_n(x)\,\mathrm{d}x = \dfrac{1}{L_n(x)}\mathrm{d}x\cdots$ Collision probability for electrons.

$\gamma_p(x)\,\mathrm{d}x = \dfrac{1}{L_p(x)}\mathrm{d}x\cdots$ Collision probability for holes.

$W_n(x_1, x_2)\cdots$ Probability for an electron not to collide in $[x_1, x_2]$.

$W_p(x_1, x_2)\cdots$ Probability for a hole not to collide in $[x_1, x_2]$.

Now that we have introduced some definitions we can describe our model in detail.

Let us assume an electron-hole pair at position x [see Figs 1(a)–(c)]. Due to the influence of the electric field ($E(x) \geqslant 0$ in $[0, L]$) the electron is moving to the left and the hole to the right. Both electrons and holes can collide with lattice and impurity atoms thus generating secondary electron–hole pairs. We assume, that after each collision, the electrons and holes respectively have lost their energy thus starting after each collision with energy $\epsilon = 0$. This assumption becomes questionable for high electric fields, see Section 4. Furthermore we make use of a simple Markov model, which roughly states that the combined probability for subsequent collision events is simply a product of the probabilities for the respective collision events.

With the aid of the ionization probabilities G_n and G_p we explicitly formulate, that a particle has to gain energy from the electric field and has to travel a distance $(x_2 - x_1) \neq 0$ in order to ionize its collision partner.

For deriving an equation for $M(x)$ we first need an explicit expression for the probability $W_n(x_1, x_2)$, that a particle undergoes no collison in the interval $[x_1, x_2]$.

Using our Markov assumption we can write

$$W_n(x_1, x_2 + \mathrm{d}x_2) = W_n(x_1, x_2)[1 - \gamma_n(x_2)\,\mathrm{d}x_2] \quad (1)$$

which merely states that the probability for a particle not to collide between $[x_1, x_2 + \mathrm{d}x_2]$ is the product of the probabilities not to collide between $[x_1, x_2]$ times the probability not to collide in $[x_2, x_2 + \mathrm{d}x_2]$. Expanding eqn (1) in a Taylor series and using the obvious initial condition $W_n(x, x) = 1$ one obtains the well known expression[29]

$$W_n(x_1, x_2) = \exp\left\{-\int_{x_1}^{x_2} \gamma_n(x)\,\mathrm{d}x\right\} \quad (2)$$

The analogous expression is valid for holes.

Next we split up to $M(x)$ according to

$$M(x) = 1 + M_n(x) + M_p(x) \quad (3)$$

where $M_n(x)$ is the number of secondary electron–hole pairs created (directly or indirectly) by the original electron. The respective definition holds for $M_p(x)$. In order to derive an equation for $M_p(x)$ in terms of $M(x)$ we assume, that starting at position x, a hole has a collision at x_2 [see Figs 1(a) and (b)]. We separate two events:

(i) There is no collision between x and x_2.

(ii) There is at least one collision between x and x_2.

It should be noted that (i) and (ii) are mutually exclusive events and cover all possible events.

A collision at x_2 will lead to an ionization if the hole has gained the threshold energy. The starting point for calculating the ionization probability $G_p(x_s, x_2)$ is for case (i) $x_s = x$ and for case (ii) $x_s = x_1$ [see Figs 1(a) and (b)]. The hole has created an electron–hole pair at x_2. By definition, this electron–hole pair gives rise to $M(x_2)$ secondary electron–hole pairs in $[0, L]$.

From the above arguments the following equation can be deduced

$$M_p(x) = \int_x^L W_p(x, x_2)\gamma_p(x_2)\,\mathrm{d}x_2 G_p(x, x_2)M(x_2) + \int_x^L \gamma_p(x_1)\,\mathrm{d}x_1 \int_{x_1}^L W_p(x_1, x_2)\gamma_p(x_2) \times \mathrm{d}x_2 G_p(x_1, x_2)M(x_2) \quad (4a)$$

where an integration with respect to the collision points x_1 and x_2 has been performed already. Similar to eqn (4a) one obtains for an electron (which in our model is moving to the left)

$$M(x) = \int_0^x W_n(x_2, x)\gamma_n(x_2)\,\mathrm{d}x_2 G_n(x, x_2)M(x_2) + \int_0^x \gamma_n(x_1)\,\mathrm{d}x_1 \int_0^{x_1} W_n(x_2, x_1)\gamma_n(x_2) \times \mathrm{d}x_2 G_n(x_1, x_2)M(x_2) \quad (4b)$$

(Note: here and in the sequel we use the convention that any integral has to be put to zero if its lower bound exceeds the upper bound.)

Once that both, the collision probabilities γ_n and γ_p (or equivalently the mean free paths L_n and L_p) and the ionization probabilities G_n and G_p are given, eqns (4a, b) together with eqn (3) form a closed integral equation for the determination of the space-dependent avalanche multiplication factor $M(x)$. This integral equation is a central result of our paper, and it will serve as a starting point for subsequent approximations.

3. IONIZATION PROBABILITY MODELS

For solving the integral equation [eqn (3)] for $M(x)$ we have to know the ionization probabilities G_n and G_p respectively. In what follows we shall propose two models, the local ionization probability (IP) and the non-local IP model. As we shall see below, the local IP model uses ionization rates $\alpha_{n,p}(E)$ as input quantities.

In contrast, the non-local IP model yields ionization rates $\alpha_{n,p}(E)$. They are determined by the model itself, and a significant improvement over Chynoweth's law is obtained.

(A) Local IP model

It is assumed, that the probability for a particle to ionize its collision partner depends only on the position of the collision partner (x_2) and not on the particle's starting position (x_1)

$$G_s(x_1, x_2) = g_s(x_2) \quad \text{with} \quad s = \{n, p\} \quad (5)$$

Equation (5) violates our previously made assumption, namely, that electrons and holes have to gain energy from the electric field $E(x)$ in order to ionize their collision partners. In other words, eqn (5) violates the following equation

$$G_s(x_1, x_1) = 0 \quad \text{with} \quad s = \{n, p\} \quad (6)$$

Nevertheless we may look at the consequences of eqn (5) in connection with eqn (4a, b). Differentiating eqn (4a, b) we immediately find

$$\frac{\mathrm{d}M_p}{\mathrm{d}x} = -\alpha_p(x)M(x) \quad \text{with} \quad M_p(L) = 0 \quad (7a)$$

$$\frac{\mathrm{d}M_n}{\mathrm{d}x} = +\alpha_n(x)M(x) \quad \text{with} \quad M_p(L) = 0 \quad (7b)$$

We introduced the ionization rates α_n and α_p respectively by the relation

$$\alpha_s(x) = \gamma_s(x)g_s(x) \quad (8)$$

However, these ionization rates remain undetermined within the local IP model. They have to be given in order to be able to solve the following integral equation for $M(x)$, which is obtained by integrating eqns (7a, b) and substituting back into eqn (3).

$$M(x) = 1 + \int_0^x \alpha_n(x_1)M(x_1)\,\mathrm{d}x_1 + \int_x^L \alpha_p(x_1)M(x_1)\,\mathrm{d}x_1 \quad (9)$$

The above integral equation and its solution are well established in literature[32]. Both, theoretical and experimental investigations suggests ionization rates of the following form (Chynoweth's law[20])

$$\alpha_s(x) = \alpha_s \exp - \left\{\frac{b_s}{|E(x)|}\right\} \quad (10)$$

where $E(x)$ is the electric field and a_s, b_s with $s = \{n, p\}$ are parameters to be fitted to the experiment. We want to stress at this point, that the ionization rates, entering eqn (9) are consistent with those determining the avalanche generation in device equations[26]. Any improvement over Chynoweth's law [eqn (10)] should be based on the form of the integral equation for $M(x)$ given in eqn (9), in order to be consistent with avalanche generation in device equations.

(B) Non-local IP model

Here we drop the rather crude assumption of eqn (5). Rather we postulate, that electrons and holes can ionize lattice and impurity atoms once that they have reached a critical energy U_c (threshold energy). Let us consider a hole starting at position x with energy $\epsilon = 0$ and moving in the electric field $E(x)$ (see Fig. 1c). In order to get the critical energy U_c^p the hole has to fly a critical distance $f_p(x) - x$, which is given by the following equation

$$\int_x^{f_p(x)} E(x_1)\,\mathrm{d}x_1 - \frac{U_c^p}{q} = 0 \quad (11a)$$

where q is the elementary charge.

For electrons, which are moving to the left, one has

$$\int_{f_n(x)}^{x} E(x_1)\,\mathrm{d}x_1 - \frac{U_c^n}{q} = 0 \qquad (11b)$$

With the aid of the step function $\Theta(x)$ we can write for the ionization probability for holes

$$G_p(x, x_1) = \Theta[x_1 - f_p(x)] \qquad (12a)$$

and for electrons

$$G_n(x, x_1) = \Theta[f_n(x) - x_1] \qquad (12b)$$

The hole starts with energy $\epsilon = 0$ at position x and has to travel up to the point $f_p(x)$ in order to acquire the critical energy U_c^p. If it travels a distance $x_1 > f_p(x)$ its probability to ionize the collision partner is one, for distances $x_1 < f_p(x)$ this probability is zero.

In view of eqn (12a, b) we can simplify our general expressions for $M_p(x)$ and $M_n(x)$ respectively [see eqn (4a, b)]. Differentiating eqn (4a) and using eqn (6) yields

$$\frac{\mathrm{d}M_p}{\mathrm{d}x} = \int_x^L h_p(x, x_1)\frac{\partial}{\partial x}G_p(x, x_1)\,\mathrm{d}x_1 \qquad (13a)$$

with

$$h_p(x, x_1) = W_p(x, x_1)\gamma_p(x_1)M(x_1) \qquad (13b)$$

and

$$M_p[f_p^{-1}(L)] = 0 \qquad (13c)$$

Equation (13c) merely states, that a hole, starting within the interval $[f_p^{-1}(L), L]$ does not gain enough energy to ionize its collision partner.

Evaluating the differentiation of the step function in eqn (13a) (or integrating by parts) immediately leads to

$$\frac{\mathrm{d}M_p}{\mathrm{d}x} = -\frac{\mathrm{d}f_p}{\mathrm{d}x}h_p[x, f_p(x)] \qquad (14a)$$

With the initial condition eqn (13c) one finds

$$M_p(x) = \int_{f_p(x)}^{L} h_p[f_p^{-1}(x_1), x_1]\,\mathrm{d}x_1 \qquad (14b)$$

where $f_p^{-1}(x)$ is the inverse function of f_p. No difficulty arises when performing the same steps for electrons which, in the case of holes, led to eqn (14b). We finally end up with the following integral equation

$$\begin{aligned} M(x) = 1 &+ \int_0^{f_n(x)} \exp\left\{-\int_{x_1}^{f_n^{-1}(x_1)} \gamma_n(x')\,\mathrm{d}x'\right\}\gamma_n(x_1)M(x_1)\,\mathrm{d}x_1 \\ &+ \int_{f_p(x)}^{L} \exp\left\{-\int_{f_p^{-1}(x_1)}^{x_1} \gamma_p(x')\,\mathrm{d}x'\right\}\gamma_p(x_1)M(x_1)\,\mathrm{d}x_1 \end{aligned} \qquad (15)$$

In general, a solution of the above integral equation has to be done numerically, by first determining f_n and f_p via eqn (11a, b) and then solving eqn (15) (we assume that the electric field $E(x)$, the mean free paths $L_p(x)$ and $L_n(x)$, and the critical energies U_c^p and U_c^n are given).

In order to get local ionization rates $\alpha_n(E)$ and $\alpha_p(E)$ respectively we make the following approximations.

(i) We assume a rather weak variation of the electric field $E(x)$ within $[x, f_p(x)]$ and $[f_n(x), x]$ respectively

$$\int_x^{f_p(x)} E(x_1)\,\mathrm{d}x_1 \approx E(x)[f_p(x) - x] = E(x)d_p(x) \qquad (16a)$$

$$\int_{f_n(x)}^{x} E(x_1)\,\mathrm{d}x_1 \approx E(x)[x - f_n(x)] = E(x)d_n(x) \qquad (16b)$$

From eqns (11a, b) the distances d_n and d_p are determined by

$$d_p(x) = \frac{U_c^p}{qE(x)} \quad \text{and} \quad d_n(x) = \frac{U_c^n}{qE(x)} \qquad (17)$$

(ii) We assume a weak variation of $M(x)$ with respect to x

$$M(f_p) = M(x + d_p) \approx M(x) + d_p\frac{\mathrm{d}M(x)}{\mathrm{d}x} \qquad (18a)$$

$$M(f_n) = M(x - d_n) \approx M(x) - d_n\frac{\mathrm{d}M(x)}{\mathrm{d}x} \qquad (18b)$$

For constant mean free paths L_n and L_p respectively the derivative of eqns (15) is given by

$$\begin{aligned} \frac{\mathrm{d}M(x)}{\mathrm{d}x} = &- \exp(-d_p\gamma_p)\gamma_p M(f_p)\frac{\mathrm{d}f_p}{\mathrm{d}x} \\ &+ \exp(-d_n\gamma_n)\gamma_n M(f_n)\frac{\mathrm{d}f_n}{\mathrm{d}x} \end{aligned} \qquad (19a)$$

Inserting eqns (16) and (17) into eqn (19a) leads to

$$\begin{aligned} &[1 + d_n\gamma_n\exp(-d_n\gamma_n) + d_p\gamma_p\exp(-d_p\gamma_p)]\frac{\mathrm{d}M}{\mathrm{d}x} \\ &\approx [\gamma_n\exp(-d_n\gamma_n) - \gamma_p\exp(-d_p\gamma_p)]M(x) \end{aligned} \qquad (19b)$$

If we now compare eqn (19b) with the derivative of eqn (9) we find for the ionization rates the following expressions

$$\alpha_s = \frac{a_s}{z}\exp\left(-\frac{b_s}{E(x)}\right) \quad \text{with} \quad s = \{n, p\} \qquad (20a)$$

and

$$z(x) = 1 + \frac{b_n}{E(x)}\exp\left(-\frac{b_n}{E(x)}\right) + \frac{b_p}{E(x)}\exp\left(-\frac{b_p}{E(x)}\right) \qquad (20b)$$

and

$$a_s = \gamma_s = \frac{1}{L_s} \quad \text{and} \quad b_s = \frac{U_c^s\gamma_s}{q} \qquad (20c)$$

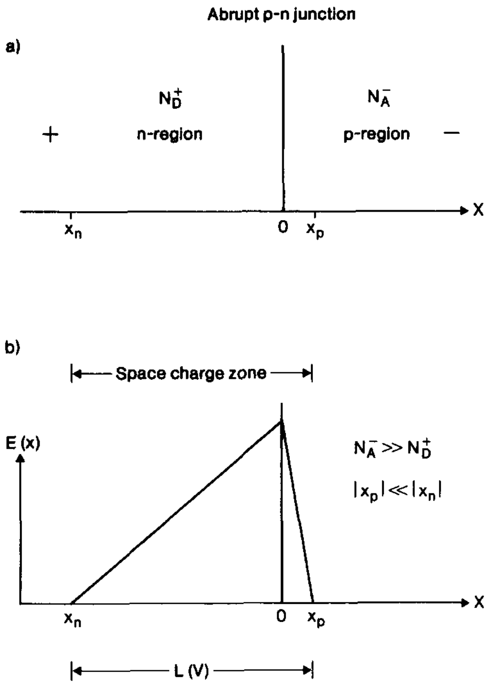

Fig. 2. The abrupt step p–n junction: (a) doping, (b) electric field. The donor concentration N_D^+ is assumed to be small compared to the acceptor concentration N_A^-.

We want to stress that we have derived the field dependence of the ionization coefficient [see eqns (20a–c)] as an approximation to eqn (15). Besides the field correction in the denominator our expression is identical to Chynoweth's law. Note that the constants appearing in eqns (20a, b) were introduced *a priori* as physical constants and not as fit parameters.

In Figs 3(a) and (b) we compare the ionization rates (ionization coefficients) for different constants a_s, b_s. We basically use two sets of constants, namely:

(a) Van Overstraeten and De Man constants (which are used in device simulators[10])

$$a_n = 0.703 \times 10^6\,\text{cm}^{-1}; \quad b_n = 1.231 \times 10^6\,\text{V/cm} \tag{21a}$$

$$a_p = 1.582 \times 10^6\,\text{cm}^{-1} \quad b_p = 2.036 \times 10^6\,\text{V/cm} \tag{21b}$$

These constants are used in curves 2a and 3a of Figs 3(a) and (b). The above constants are valid only in the range $1.75 \times 10^5 < E < 4 \times 10^5$ V/cm. In Figs 3(a) and (b), however, we used the above constants for the entire E-field range. In Figs 3(c) and (d), which were taken from Ref. [2], modified constants were used in the range $4 \times 10^5 < E < 6 \times 10^5$ V/cm. For details concerning these constants see Ref. [21].

(b) Mean free path constants (the constants for the mean free paths L_n and L_p and the ionization energy U_c were chosen from Ref. [4])

$$L_p = 0.55 \times 10^{-6}\,\text{cm}$$

$$L_n = 0.76 \times 10^{-6}\,\text{cm} \quad \frac{U_c^s}{q} = 1.12\,\text{V} \tag{22a}$$

with these constants we get using eqn (20c)

$$a_n = 1.316 \times 10^6\,\text{cm}^{-1}; \quad b_n = 1.474 \times 10^6\,\text{V/cm} \tag{22b}$$

$$a_p = 1.818 \times 10^6\,\text{cm}^{-1}; \quad b_p = 2.036 \times 10^6\,\text{V/cm} \tag{22c}$$

These constants were used in curves 2b and 3b of Figs 3(a) and (b).

In labelling the curves of Figs 3–5 we used the following notation: 1—exact model [eqn (26)]; 2—Chynoweth model [eqn (9) and eqn (20a, c) with $z = 1$]; 3—field correction to Chynoweth model [eqns (9) and (20a–c)]; a—Van Overstraeten and De Man constants [eqn (21a, b)]; (b)—mean free path constants [eqns (22a–c)]. For example, 'curve 2b' refers to Chynoweth model with mean free path constants.

For calculating curves 2a and 2b respectively we simply used eqn (20a) with $z = 1$, which is basically Chynoweth's law. Furthermore, curves 3a and 3b respectively contain the corresponding field corrections of curves 2a and 2b respectively, which are given in eqns (20a–c).

We want to point out once more that it is not our objective to determine the parameters of eqns (20a–c) from experimental data. Rather we want to show the impact our field correction [eqn (20b)] has on the ionization rate, comparing it with Chynoweth's law [eqns (20a) with $z = 1$]. For the two parameters sets chosen one observes a reduction of the ionization rates in the high field range $E > 4 \times 10^5$ V/cm and a good agreement with Chynoweth's law in the low field range $E < 4 \times 10^5$ V/cm.

In the next section, however, the accuracy of our field approximation is examined by comparing it with the exact results for an abrupt p–n junction.

4. THE ABRUPT *p–n* JUNCTION

We consider an abrupt p–n junction where the p-region (N_A^-) is highly doped compared to the n-region (N_D^+) (see Figs 2a and b). Assuming that the space charge zone is free of mobile carriers $n(x)$ and $p(x)$ respectively, we get a linear increase in the electric field $E(x)$

$$E(x) = ax \quad \text{with} \quad a = \frac{qN_D^+}{\epsilon} \tag{23a}$$

where $\epsilon = \epsilon_r \epsilon_0$ is the permittivity of silicon ($\epsilon_r \approx 11.7$, and $\epsilon_0 \approx 8.854 \times 10^{-12}$ As/V/m).

The length of the space charge zone can be expressed in terms of the voltage V by[3]

$$L = \sqrt{2/a}\,\sqrt{V} \tag{23b}$$

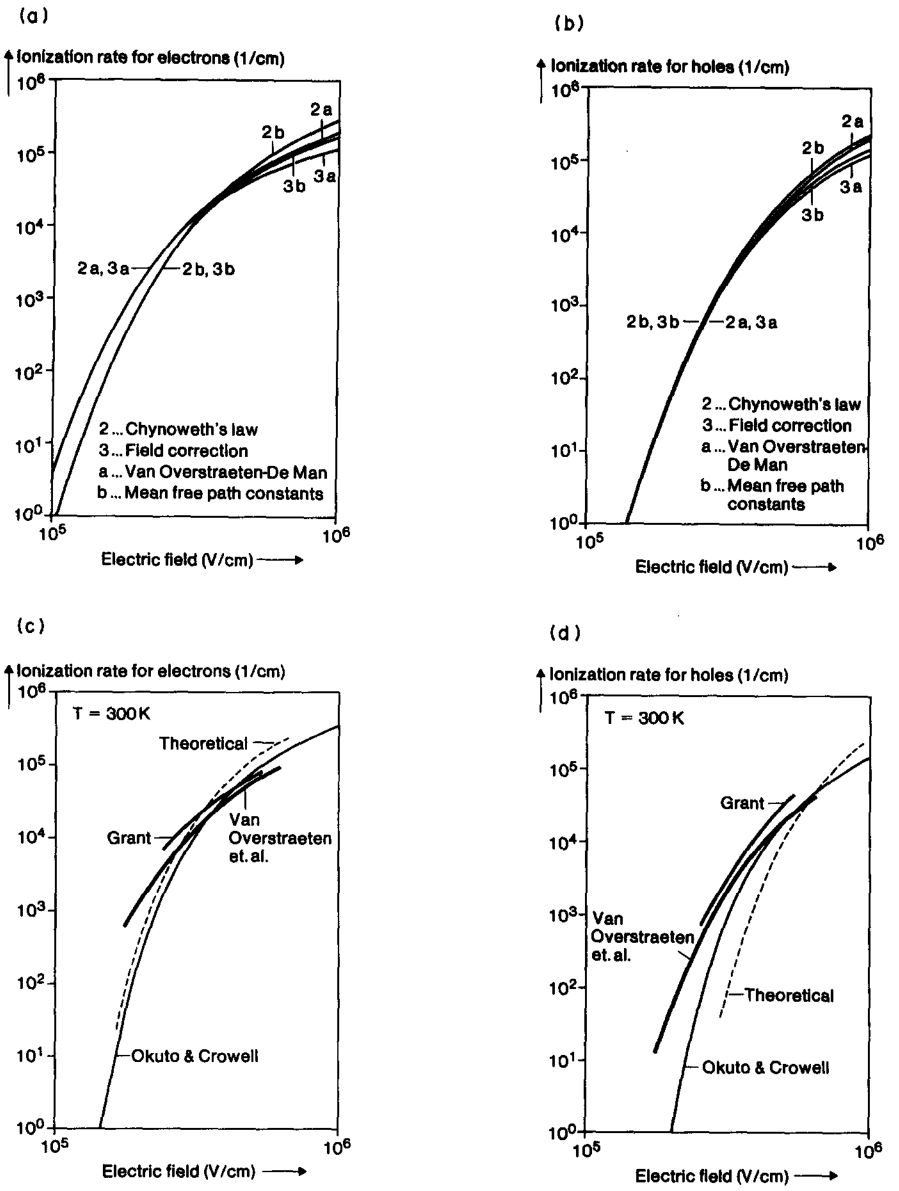

Fig. 3. The impact of the field correction [see eqns (20a–c)] on the ionization rates for electrons (a, c) and holes (b, d) respectively. For comparison, (c, d) were taken from Ref. [2].

Assuming equal threshold energies for electrons and holes ($U_c^n = U_c^p = U_c$) we find for f_n and f_p respectively [see eqns (11a, b)]

$$f_n = \sqrt{(x^2 - d^2)}, \quad f_p = \sqrt{(x^2 + d^2)}$$

with

$$d = \sqrt{2U_c/qa} \tag{24a}$$

Furthermore, one has

$$f_p^{-1}(x) = f_n(x) \tag{24b}$$

which immediately follows from eqns (11a, b) when putting $U_c^n = U_c^p = U_c$.

For computational reasons, it is advantageous to perform the following scaling

$$y = x/d \tag{25a}$$

$$M(x) = M^*(x/d) = M^*(y) \tag{25b}$$

$$l(V) = L/d = \sqrt{q/U_c}\sqrt{V} \tag{25c}$$

$$\beta_s = d\gamma_s = d/L_s \quad \text{with} \quad s = \{n, p\} \tag{25d}$$

Substituting the above equations into eqn (15) yields the following integral equation for $M^*(y)$

$$\begin{aligned} M^*(y) &= 1 \\ &+ \beta_n \int_0^{\sqrt{y^2-1}} \exp[-\beta_n/(y_1 + \sqrt{y_1^2+1})] M^*(y_1)\,\mathrm{d}y_1 \\ &+ \beta_p \int_{\sqrt{y^2+1}}^{l(V)} \exp[-\beta_p/(y_1 + \sqrt{y_1^2-1})] M^*(y_1)\,\mathrm{d}y_1 \end{aligned} \tag{26}$$

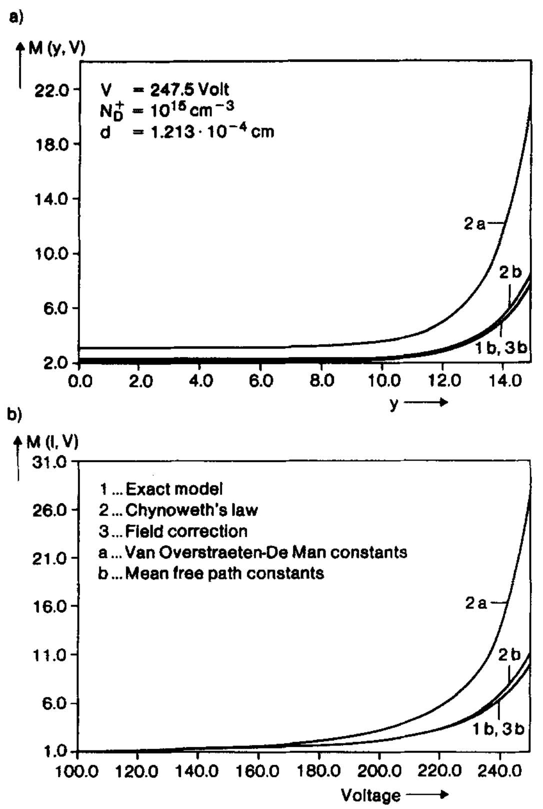

Fig. 4. (a) Space dependent multiplication factor in the direction of the space charge zone ($N_D^+ = 10^{15}\,\mathrm{cm}^{-3}$, $V = 247.5$ V). (b) Voltage dependent multiplication factor directly at the p–n junction. Curves "1b–3b" are in good agreement.

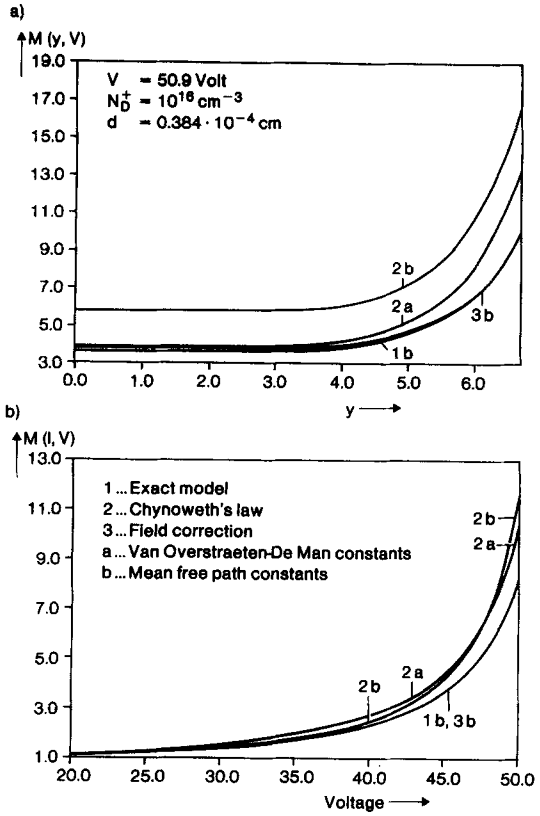

Fig. 5. (a) Space dependent multiplication factor in the direction of the space charge zone ($N_D^+ = 10^{16}\,\mathrm{cm}^{-3}$, $V = 50.9$ V). (b) Voltage dependent multiplication factor directly at the p–n junction. Chynoweth's law starts to deviate from the exact result.

In what follows any numerical solution of eqn (26) will be termed "exact".

Let us first have a glance at eqn (26). It is an integral-equation, with non-constant boundaries, for the multiplication factor $M(y)$. We still assume the convention following eqn (4b). Given the mean free paths L_n and L_p respectively, the threshold energy U_c and the donor concentration N_D^+ one can solve eqn (26) for a given voltage V [respectively space charge zone $l(V)$]. This space dependent solution will be denoted by $M(y, V)$. Furthermore, for different values of the potential V one can determine the space dependent maximum of the multiplication factor, which in our case of an abrupt p–n junction is given by $M[l(V), V]$. In Figs (4–6) these solutions are denoted by "1b" [see the notation following eqn (22c)].

For obtaining an approximation to the rather involved integral equation [eqn (26)] one can solve the more simple eqn (9), but still using the same parameter set [parameter set "b" of eqns (22a–c)]. We shall consider two approximations, namely Chynoweth's law [eqn (9) and eqn (20a) with $z = 1$] which is denoted by "2b" and our field correction to Chynowth's law [eqns (9) and (20a–c)] which is denoted by "3b". Either of the approximations made has to be compared with our exact result in order to check its accuracy. The starting point of both, the exact eqn (26) and the commonly used eqn (9) are eqns (3) and (4). Therefore, one can view both Chynoweth's law and our field correction as different approximations to an exact integral-equation [eqn (26)] which can be solved numerically only. Regardless of the specific parameter set chosen one thus can check the validity (or accuracy) of the field dependence given in Chynoweth's law and compare it with our field correction. For both, the space dependent multiplication factor $M(y, V)$ and the voltage dependent multiplication factor $M(l(V), V)$ this is done throughout Figs (4–6).

For a donor concentration $N_D^+ = 10^{15}\,\mathrm{cm}^{-3}$ we find from Figs 4(a) and (b) that both Chynoweth's law (curve "2b") and our field correction (curve "3b") are close to the exact result (curve "1b"). However, with increasing donor concentration N_D^+ Chynoweth's law deviates from the exact result significantly. Our field correction, however, remains close to the exact result (see Figs 5 and 6).

There are some quantities which facilitate the understanding of Figs 4–6, namely:

—the critical distance $d_l = f_n^{-1}(0)$ for an electron at the left end of the space charge zone. An electron, starting at position $x < d_l$ will not be able to ionize its collision partner;

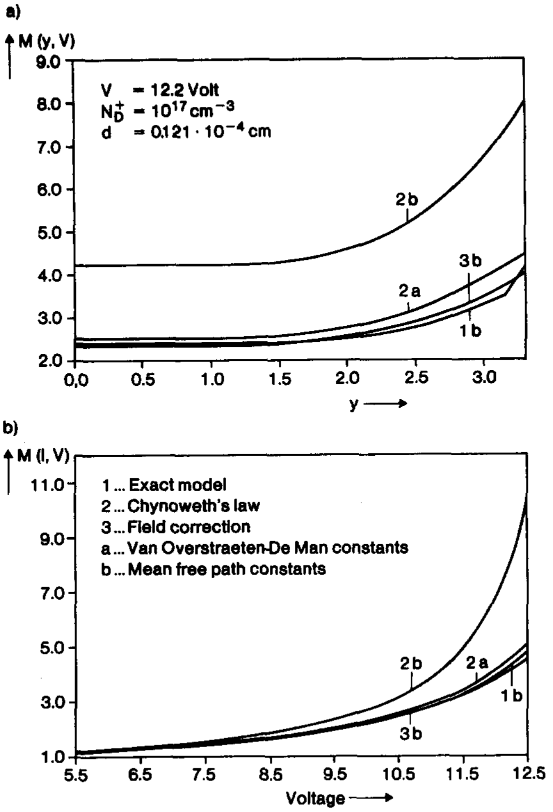

Fig. 6. (a) Space dependent multiplication factor in the direction of the space charge zone ($N_D^+ = 10^{17}\,\text{cm}^{-3}$, $V = 12.2$ V). (b) Voltage dependent multiplication factor directly at the p–n junction. Chynoweth's law deviates significantly. Field correction is still in good agreement with exact result.

—the critical distance $d_r = L - f_p^{-1}(L)$ for a hole at the right end of the space charge zone. A hole, starting at position $x > L - d_r$ cannot ionize its collision partner. The hole has to fly at least a distance d_r in order to gain enough energy to ionize its collision partner, since the maximum of the electric field is on the r.h.s. of the space charge zone (see Fig. 2);

—the maximum field $E_{max} = aL$.

In particular we obtain the following values (see Figs 4–6):

Figures 4(a) and (b)

$$E_{max} = 2.76 \times 10^5\,\text{V/cm}, \quad N_D^+ = 10^{15}\,\text{cm}^{-3}$$

$$d_l = 1.21 \times 10^0\,\mu, \quad d_r = 4.04 \times 10^{-2}\,\mu \tag{27a}$$

Figures 5(a) and (b)

$$E_{max} = 5.07 \times 10^5\,\text{V/cm}, \quad N_D^+ = 10^{16}\,\text{cm}^{-3}$$

$$d_l = 3.84 \times 10^{-1}\,\mu, \quad d_r = 2.84 \times 10^{-2}\,\mu \tag{27b}$$

Figures 6(a) and (b)

$$E_{max} = 6.09 \times 10^5\,\text{V/cm}, \quad N_D^+ = 10^{17}\,\text{cm}^{-3}$$

$$d_l = 1.21 \times 10^{-1}\,\mu, \quad d_r = 1.88 \times 10^{-2}\,\mu \tag{27c}$$

As already anticipated in the Introduction, electrons and holes should, on average, travel at least a few mean free paths before ionizing their collision partners. Otherwise, electrons and holes can acquire high energies (compared to the ionization energy) before each collision. Therefore, our assumption, that after each collision the particle starts with energy $\epsilon = 0$ can easily become unphysical. In our case the distances d_r are always at least a few distances larger than the mean free paths ($L_n = 7.6 \times 10^{-3}\,\mu$, $L_p = 5.5 \times 10^{-3}\,\mu$). Notice the discontinuous change in the slope of curve "1b" in Fig. 6! It shows most clearly the region where holes cannot gain enough energy to create secondary electron–hole pairs, thus demonstrating the non-local nature of our theory. However, the commonly used eqn (9) cannot account for this effect (see curves "2b" and "3b" respectively).

Remark. In curves "2a" of Figs 4–6 the Van Overstraeten and De Man constants together with Chynoweth's law are used. These figures are only shown for completeness, since this parameter set is commonly used in device simulators[8, 10].

5. CONCLUSION

In this paper we presented a new theory for determining avalanche multiplication in semiconductors. We suggested a non-local ionization probability model which, under certain approximations, leads to a field correction and a simple improvement over Chynoweth's law.

The "physical need" for introducing a non-local concept has been already pointed ouy by Y. Okuta and C. R. Crowell[26]. Since their treatment is conceptionally different from ours (their starting point is not an equation for the avalanche multiplication factor) they don't arrive at a simple, solely field dependent, improvement over Chynoweth's law.

A conceptionally different approach has been presented by G. A. Baraff[24] and P. A. Wolff[23]. Their starting point is the time dependent Boltzmann-equation in position and momentum space. A certain average of the distribution function leads to ionization rates depending on the electric field. Although the Boltzmann-equation is a more rigorous starting point than our eqns (3) and (4a, b), the works of Baraff and Wolff lack the correct treatment of boundary conditions in position space. The formulation of boundary conditions is a general difficulty when using the Boltzmann-equation and can lead to problems e.g. for narrow p–n junctions.

We did not yet compare our theory with experimental data. Rather we suggest to repeat step by step the work of R. Van Overstraeten and De Man but replacing Chynoweth's law with our field correction. A glance at Figs 3(a) and (b) gives a first idea of the impact our correction will have on the ionization rates calculated from multiplication factor measurements. Finally we want to stress that any determination of our coefficients [eqn (20c)] should take into account the possible concentration dependence of the mean free paths.

Acknowledgements—The author is very grateful to R. Stengl for many fruitful and stimulating discussions on this and related topics. He would like to thank R. Stengl for permanently encouraging the author to continue this work. Furthermore, the author would like to acknowledge F. Pfirsch for critically commenting this paper, and H. Mitlehner for supporting an environment and providing an atmosphere which advances this kind of work. Finally, the author would like to thank O. J. Eder (Austrian Research Center Seibersdorf) for numerous discussions concerning concepts of collision models.

REFERENCES

1. A. Anselm, *Introduction to Semiconductor Theory*. Prentice-Hall, Englewood Cliffs, NJ (1981).
2. S. Selberherr, *Analysis and Simulation of Semiconductor Devices*. Springer, Wien (1984).
3. E. Spenke, *P–N Uebergaenge*. Springer, Berlin (1979).
4. S. M. Sze, *Physics of Semiconductor Devices*. Wiley, New York (1969).
5. W. Gerlach, *Thyristoren*. Springer, Berlin (1981).
6. A. Herlet, *Solid-St. Electron.* **8,** 655 (1965).
7. W. Moench, *Physical Status Solidi* **36,** 9 (1969).
8. W. L. Engl, R. Laur and K. Dirks, *IEEE* **CAD-1,** 85 (1982).
9. J. W. Slotboom, *IEEE* **ED-20,** 669 (1973).
10. R. E. Bank, W. M. Coughran, W. Fichtner, E. H. Grosse, D. J. Rose and R. K. Smith, *IEEE* **ED-32,** 1992 (1985).
11. S. E. Laux and B. M. Grossman, *IEEE* **ED-32,** 2076 (1985).
12. A. Franz and G. A. Franz, *IEEE* **CAD-4,** 178 (1985).
13. W. Van Roosbroeck, *Bell Syst. tech. J.* **29,** 560 (1950).
14. S. Chapman and T. G. Cowling, *Mathematical Theory of Non-Uniform Gases*. Cambridge University Press, London (1939).
15. J. H. Ferziger and H. G. Kaper, *Mathematical Theory of Transport Processes in Gases*. North-Holland, Amsterdam (1972).
16. J. P. Hansen and I. R. McDonald, *Theory of Simple Liquid*. Academic, London (1976).
17. R. Kubo, *J. Phys. Soc. Jap.* **12,** 570 (1957).
18. N. S. Green, *J. chem. Phys.* **22,** 398 (1954).
19. O. J. Eder and T. Lackner, *J. chem. Phys.* **74,** 3065 (1981).
20. A. G. Chynoweth, *Phys. Rev.* **109,** 1537 (1958).
21. R. Van Overstraeten and H. De Man, *Solid-St. Electron.* **13,** 553 (1969).
22. K. G. McKay and K. B. McAfee, *Phys. Rev.* **91,** 1097 (1953).
23. P. A. Wolff, *Phys. Rev.* **95,** 1415 (1954).
24. G. A. Baraff, *Phys. Rev.* **128,** 2507 (1962).
25. W. Shockley, *Solid-St. Electron.* **2,** 35 (1961).
26. Y. Okuta and C. R. Crowell, *Phys. Rev.* **B10,** 4284 (1974).
27. R. Stengl, *IEEE* **ED-34,** 911 (1987); see also his thesis Physikalische Untersuchungen au Hochsperrenden Planaren *p–n* Uebergaengen, Erlangen (1986).
28. J. Sandoe and J. Hughes, *Acta electron.* **25,** 201 (1983).
29. N. G. Van Kampen, *Stochastic Processes in Physics and Chemistry*. North-Holland, Amsterdam (1981).
30. O. J. Eder and T. Lackner, *Phys. Rev.* **A28,** 952 (1983).
31. T. Lackner and S. Yip, *Phys. Rev.* **A31,** 451 (1985).
32. R. J. McIntyre, *IEEE* **ED-13,** 164 (1966).