Характеристики на типови динамични звена и САУ

Цели на лекцията

Запознаване с

- Типови динамични звена
- Времеви характеристики
  - Преходна функция
  - Тегловна функция
  - Времеви характеристики на типови динамични звена
  - Преходни функции на САУ
- Честотни характеристики
  - Честотни характеристики на типови звена

Нули и полюси

$$W(p)=\frac{b_0p^m+b_1p^{m-1}+\cdots+b_{m-1}p+b_m}{a_0p^n+a_1p^{n-1}+\cdots+a_{n-1}p+a_n}$$

$$W(p)=\frac{b_0(p-\mu_1)(p-\mu_2)\cdots(p-\mu_m)}{a_0(p-\lambda_1)(p-\lambda_2)\cdots(p-\lambda_n)}$$

$$\lambda_i=-\alpha_i$$

$$p-\lambda_i=p+\alpha_i=\frac{1}{T_i}(T_ip+1)$$

$$T_i=\frac{1}{\alpha_i}$$

$$\lambda_{i,i+1}=-\alpha_i\pm j\beta_i$$

$$(p-\lambda_i)(p-\lambda_{i+1})=p^2+2\alpha_ip+\alpha_i^2+\beta_i^2=\frac{1}{T_i^2}(T_i^2p^2+2\xi_iT_ip+1)$$

$$T_i=\frac{1}{\sqrt{\alpha_i^2+\beta_i^2}},\ \xi_i=\frac{\alpha_i}{\sqrt{\alpha_i^2+\beta_i^2}}$$

$$W(p)=\frac{kp^{\gamma}\prod_{i=1}^{\chi}(T_ip+1)\prod_{i=1}^{\eta}(T_i^2p^2+2\xi_iT_ip+1)}{p^{\upsilon}\prod_{i=1}^{\rho}(T_ip+1)\prod_{i=1}^{\sigma}(T_i^2p^2+2\xi_iT_ip+1)}$$

$$\gamma+\chi+2\eta=m$$

$$\upsilon+\rho+2\sigma=n$$

Типови динамични звена

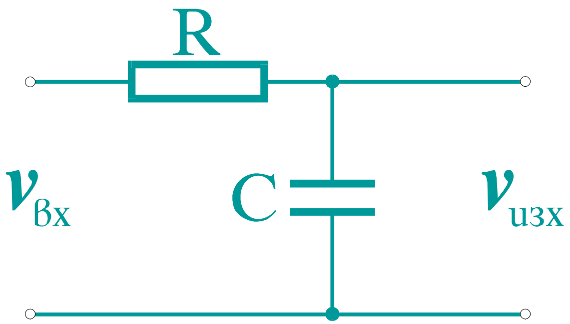
- Пропорционално звено $W(p) = k$
- Интегриращо звено $W(p) = \dfrac{1}{p}$
- Апериодично звено $W(p) = \dfrac{1}{Tp+1}$
- Колебателно звено $W(p) = \dfrac{1}{T^2p^2 + 2\xi Tp + 1}$
- Идеално диференциращо звено $W(p) = p$
- Идеално форсиращо звено $W(p) = Tp + 1$
- Идеално форсиращо звено от втори ред $W(p) = T^2p^2 + 2\xi Tp + 1$

Примери

- Апериодично звено

$$W(p) = \frac{1}{Tp+1}$$

$$T\frac{dy}{dt} + y = u$$

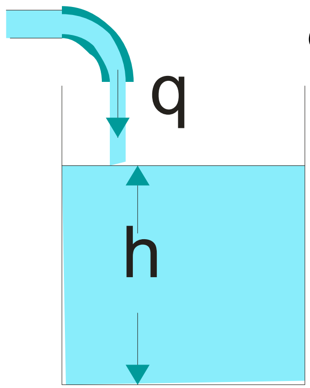


$$RC\frac{dv_{изх}}{dt} + v_{изх} = v_{вх}$$

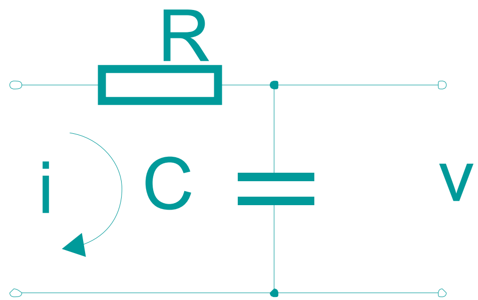
- Интегриращо звено

$$W(p) = \frac{1}{p}$$

$$\frac{dy}{dt} = k\,u$$



$$\frac{dh}{dt} = \frac{1}{S}q$$



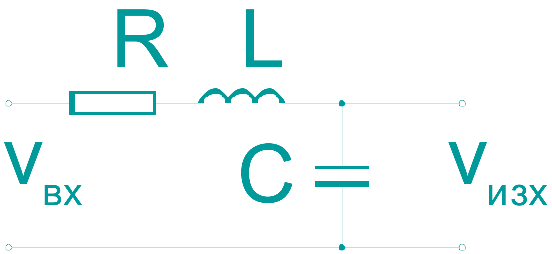
$$\frac{dv_{изх}}{dt} = \frac{1}{C}i$$

Примери

- Колебателно звено

$$W(p) = \frac{1}{T^2 p^2 + 2\xi T p + 1}$$

$$T^2 \frac{d^2 y}{dt^2} + 2\xi T \frac{dy}{dt} + y = u$$



$$LC \frac{d^2 v_{изх}}{dt^2} + RC \frac{dv_{изх}}{dt} + v_{изх} = v_{вх}$$

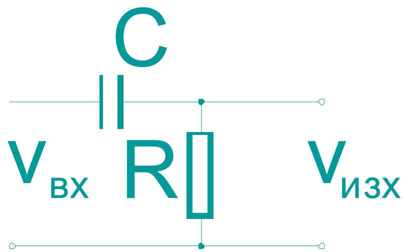
$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} < 1$$

- Реално диференциращо звено

$$W(p) = \frac{Tp}{Tp + 1}$$

$$T \frac{dy}{dt} + y = T \frac{du}{dt}$$

$$T = RC$$

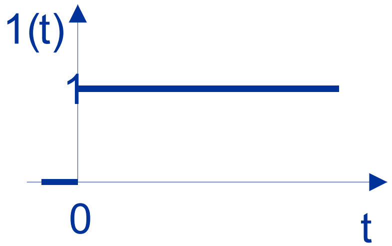


$$RC \frac{dv_{изх}}{dt} + v_{изх} = RC \frac{dv_{вх}}{dt}$$

Времеви характеристики

- Преходна функция
  - Единична функция

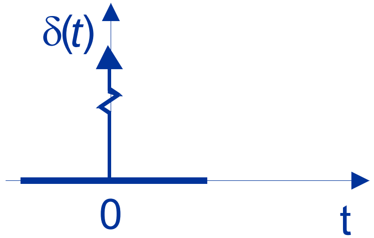
$$1(t)=\begin{cases}0,\ при\ t<0\\ 1,\ при\ t\ge 0\end{cases}$$



- Тегловна функция
  - Делта функция

$$\delta(t)=\begin{cases}0,\ при\ t\ne 0\\ \infty,\ при\ t=0\end{cases}\qquad \int_{-\varepsilon}^{\varepsilon}\delta(t)dt=1$$

$$при\ \varepsilon>0$$



$$Y(p)=W(p)U(p)$$

$$U(p)\rightarrow\ 1(t)\ \fallingdotseq\ \frac{1}{p}$$

$$h(t)=\int_0^t w(\tau)d\tau$$

$$y(t)=\int_0^t w(t-\tau)u(\tau)d\tau$$

$$U(p)\rightarrow\ \delta(t)\ \fallingdotseq\ 1$$

$$h(t)=L^{-1}\left\{W(p)\frac{1}{p}\right\}$$

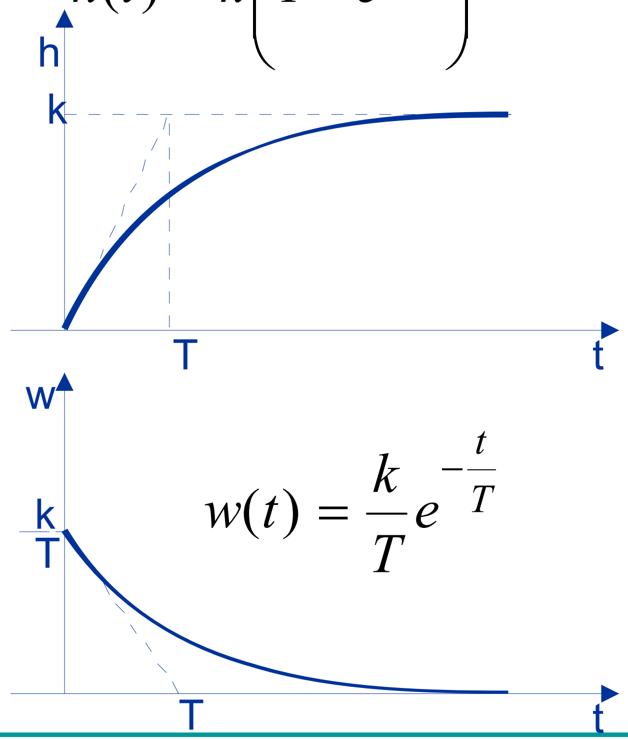
$$w(t)=\frac{dh(t)}{dt}$$

$$w(t)=L^{-1}\{W(p)\}$$

Времеви характеристики на типови динамични звена

- Апериодично звено

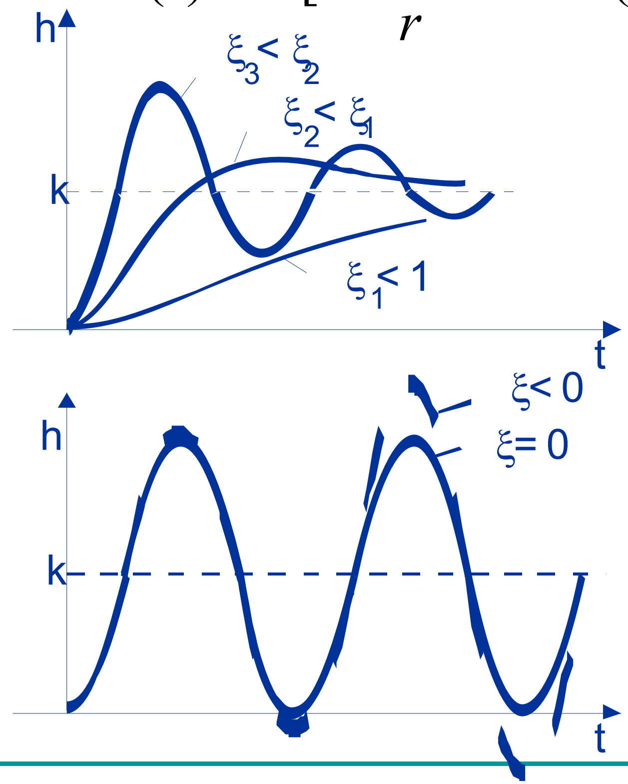
$$h(t) = k\left(1 - e^{-\frac{t}{T}}\right)$$



$$w(t) = \frac{k}{T} e^{-\frac{t}{T}}$$

- Колебателно звено

$$h(t) = k[1 - \frac{1}{r} e^{-\frac{\xi}{T}t} \sin(\frac{r}{T}t + \theta)]$$

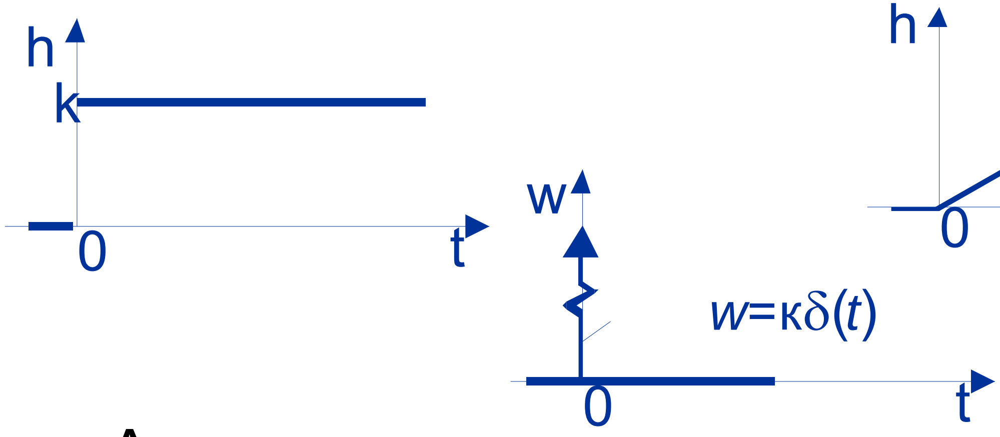


$$r = \sqrt{1 - \xi^2}$$

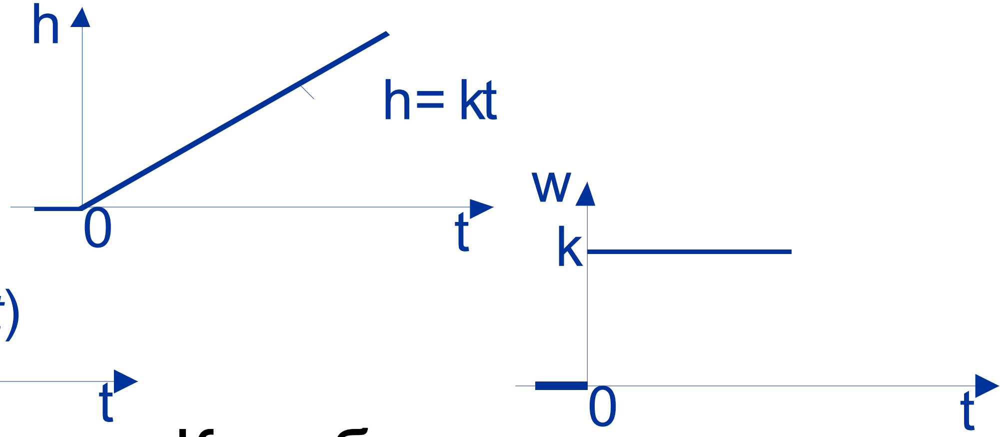
$$\theta = arctg\frac{r}{\xi}$$

Времеви характеристики на типови динамични звена

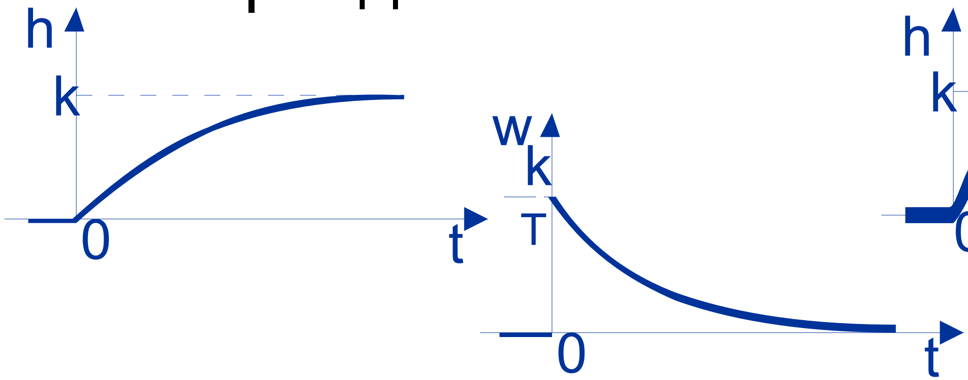
- Пропорционално звено



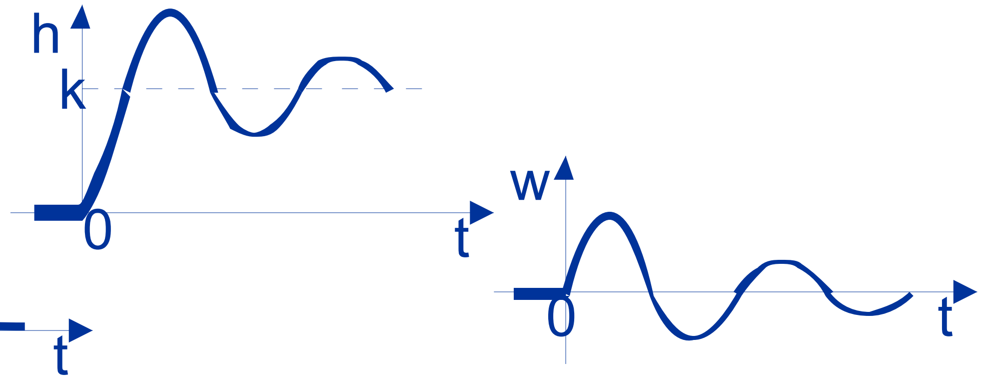
- Интегриращо звено



- Апериодично звено



- Колебателно звено



Преходна функция на САУ (теорема на разлагането)

$$W(p) = \frac{p+4}{(p+1)(p+2)^2}$$

$$h(t) = 1 - 3e^{-t} + (2+t)e^{-2t}$$

$$h(p) = \frac{W(p)}{p} = \frac{p+4}{p(p+1)(p+2)^2}$$

$$B(p) = p+4$$

$$A(p) = p^4 + 5p^3 + 8p^2 + 4p$$

$$A'(p) = 4p^3 + 15p^2 + 16p + 4$$

$\lambda_1 = 0$ $$\frac{B(\lambda_1)}{A'(\lambda_1)} e^{\lambda_1 t} = \frac{4}{4} e^0 = 1$$

$\lambda_2 = -1$ $$\frac{B(\lambda_2)}{A'(\lambda_2)} e^{\lambda_2 t} = \frac{-1+4}{-4+15-16+4} e^{-t} = -3e^{-t}$$

$\lambda_{3,4} = -2$ $$\frac{1}{(r_3-1)!} \lim_{p\to\lambda_3} \frac{d^{r_3-1}}{dp^{r_3-1}} [h(p)(p-\lambda_3)^{r_3} e^{pt}] = \lim_{p\to -2} \frac{d}{dp} [\frac{p+4}{p(p+1)} e^{pt}] = (2+t)e^{-2t}.$$

Многомерни системи

$$\mathbf{H}(t)=\begin{bmatrix} h_{11}(t) & \cdots & h_{1r}(t) \\ \vdots & \vdots & \vdots \\ h_{l1}(t) & \cdots & h_{lr}(t) \end{bmatrix} \qquad \mathbf{W}(t)=\begin{bmatrix} w_{11}(t) & \cdots & w_{1r}(t) \\ \vdots & \vdots & \vdots \\ w_{l1}(t) & \cdots & w_{lr}(t) \end{bmatrix}$$

$$\mathbf{H}(p)=\frac{1}{p}\mathbf{W}(p)=\begin{bmatrix} \dfrac{3p+1}{A(p)} & \dfrac{0.5}{A(p)} \\ \dfrac{0.5}{A(p)} & \dfrac{8p+1}{A(p)} \end{bmatrix}, \quad A(p)=p(24p^2+11p+0.75)$$

$$\lambda_1=0 \quad \lambda_2=-\frac{1}{12} \quad \lambda_3=-\frac{3}{8}$$

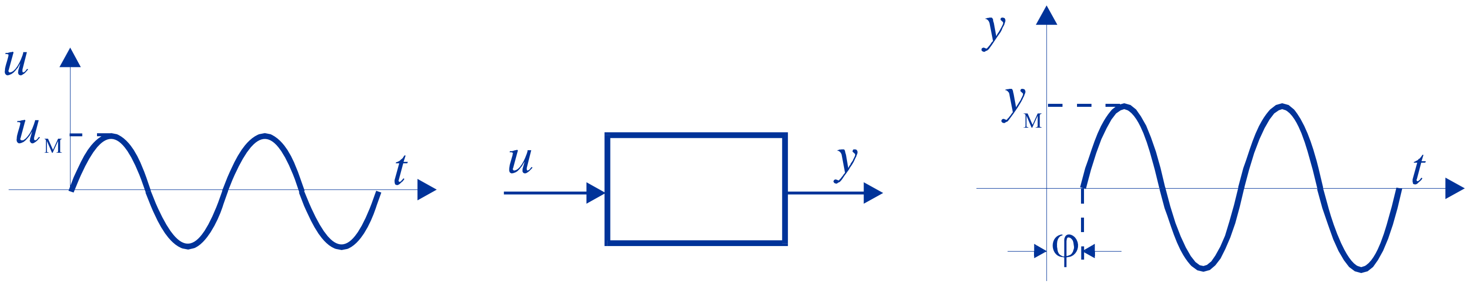
$$\mathbf{H}(t)=\frac{1}{21}\begin{bmatrix} 28-27e^{-\frac{1}{12}t}-e^{-\frac{3}{8}t} & 14-18e^{-\frac{1}{12}t}+4e^{-\frac{3}{8}t} \\ 14-18e^{-\frac{1}{12}t}+4e^{-\frac{3}{8}t} & 28-12e^{-\frac{1}{12}t}-16e^{-\frac{3}{8}t} \end{bmatrix}$$

$$\mathbf{W}(t)=\begin{bmatrix} \frac{1}{56}(6e^{-\frac{1}{12}t}+e^{-\frac{3}{8}t}) & \frac{1}{14}(e^{-\frac{1}{12}t}-e^{-\frac{3}{8}t}) \\ \frac{1}{14}(e^{-\frac{1}{12}t}-e^{-\frac{3}{8}t}) & \frac{1}{21}(e^{-\frac{1}{12}t}+6e^{-\frac{3}{8}t}) \end{bmatrix}$$

Честотни характеристики

$u(t) = u_M \sin \omega t$ $\qquad$ $y = y_M \sin(\omega t + \varphi)$

$$a_0 \frac{d^n y}{dt^n} + \cdots + a_{n-1}\frac{dy}{dt} + a_n y = b_0 \frac{d^m u}{dt^m} + \cdots + b_{m-1}\frac{du}{dt} + b_m u$$



- Амплитудно-честотна характеристика (АЧХ)

$$A(\omega) = \frac{y_M}{u_M}$$

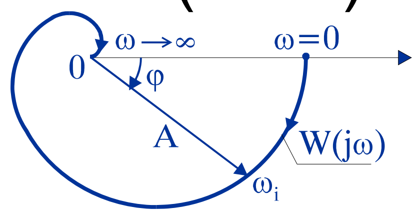
- Фазово-честотна характеристика (ФЧХ)

φ(ω)

- Амплитудно-фазова характеристика (АФХ)

$$u = u_M e^{j\omega t}$$

$$y = y_M e^{j(\omega t + \varphi)}$$



$$W(j\omega) = \frac{y_M e^{j(\omega t+\varphi)}}{u_M e^{j\omega t}} = \frac{y_M}{u_M} e^{j\varphi} = A(\omega) e^{j\varphi(\omega)}$$

Връзка между АФХ и W(p)

$$\frac{du}{dt} = u_{\text{м}} e^{j\omega t} j\omega \qquad , \qquad \frac{dy}{dt} = y_{\text{м}} e^{j(\omega t+\varphi)} j\omega$$

$$\frac{d^2u}{dt^2} = u_{\text{м}} e^{j\omega t} (j\omega)^2 \qquad , \qquad \frac{d^2y}{dt^2} = y_{\text{м}} e^{j(\omega t+\varphi)} (j\omega)^2$$

$$\vdots \qquad\qquad\qquad \vdots$$

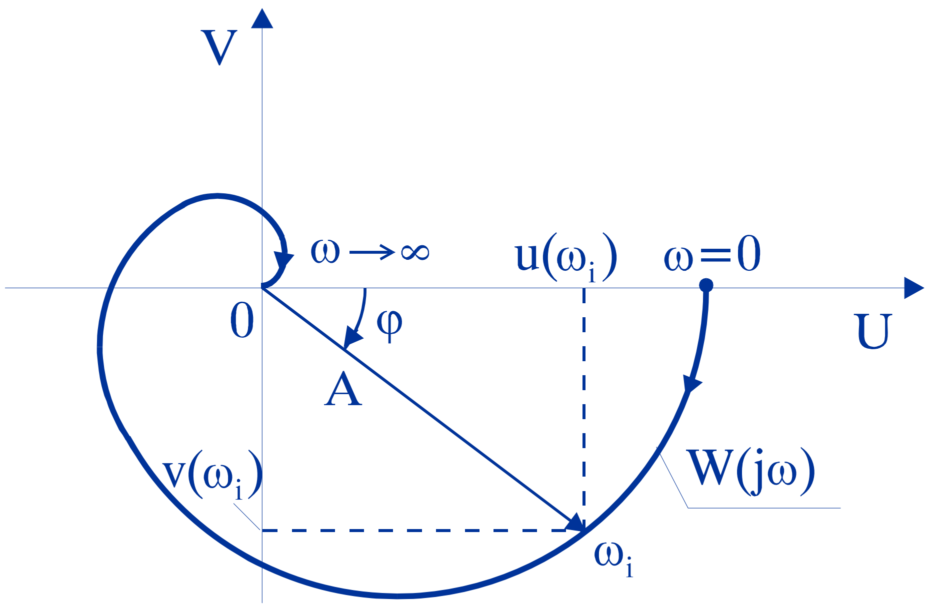
$$\frac{d^m u}{dt^m} = u_{\text{м}} e^{j\omega t} (j\omega)^m \qquad , \qquad \frac{d^n y}{dt^n} = y_{\text{м}} e^{j(\omega t+\varphi)} (j\omega)^n \quad .$$

$$[a_0 (j\omega)^n + \cdots + a_{n-1} j\omega + a_n] y_{\text{м}} e^{j(\omega t+\varphi)} = [b_0 (j\omega)^m + \cdots + b_{m-1} j\omega + b_m] u_{\text{м}} e^{j\omega t}$$

$$W(j\omega) = \frac{b_0(j\omega)^m + \cdots + b_{m-1}(j\omega) + b_m}{a_0(j\omega)^n + \cdots + a_{n-1}(j\omega) + a_n}$$

Реални и имагинерна честотни характеристики

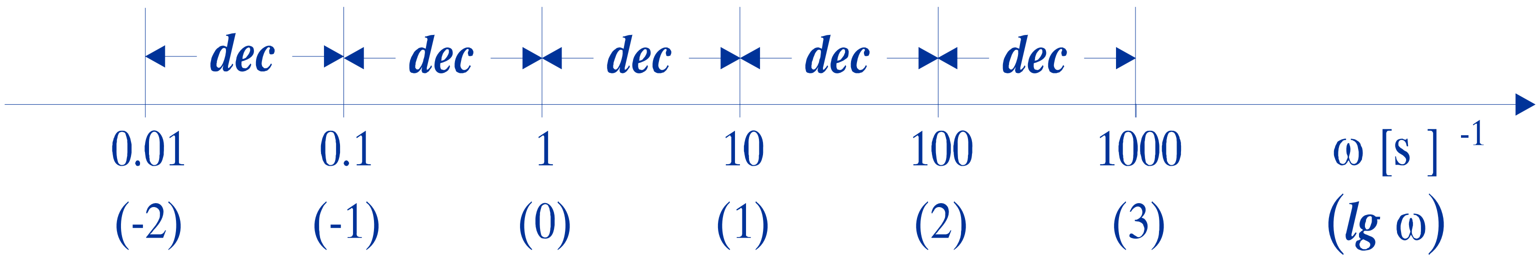
$$W(j\omega) = U(\omega) + jV(\omega)$$



$$A(\omega) = \sqrt{U^2(\omega) + V^2(\omega)} \quad ;$$

$$\varphi(\omega) = arctg\frac{V(\omega)}{U(\omega)} \quad .$$

Логаритмични честотни характеристики



- Логаритмична амплитудно-честотна характеристика (ЛАЧХ)

$$L(\omega) = 20\lg A(\omega)$$

"децибел" $[dB]$

- Логаритмична фазо-честотна характеристика (ЛФЧХ)

$$\varphi(\omega)$$

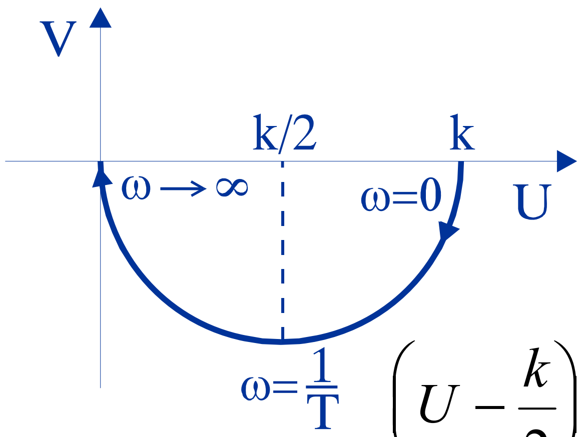
Апериодично звено

$$W(p) = \frac{k}{Tp+1}$$

$$W(j\omega) = \frac{k}{1 + j\omega T}$$

$$W(j\omega) = \frac{k}{1 + j\omega T}\,\frac{1 - j\omega T}{1 - j\omega T} = \frac{k}{1+\omega^2 T^2} - j\frac{k\omega T}{1+\omega^2 T^2}$$

$$U(\omega) = \frac{k}{1+\omega^2 T^2} \quad ; \quad V(\omega) = -\frac{k\omega T}{1+\omega^2 T^2} \quad .$$



$$\left(U - \frac{k}{2}\right)^2 + V^2 = \left(\frac{k}{2}\right)^2$$

- АЧХ

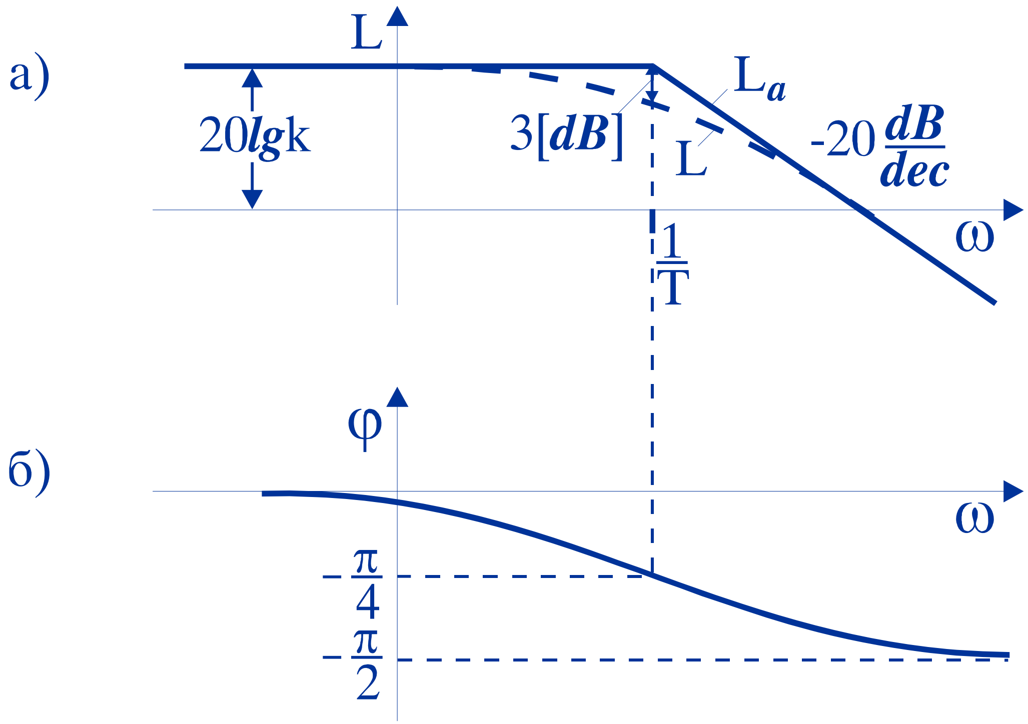
$$A(\omega) = \frac{k}{\sqrt{1+\omega^2 T^2}}$$

- ФЧХ

$$\varphi(\omega) = -arctg(\omega T)$$

Апериодично звено - ЛАЧХ, ЛФЧХ

$$L(\omega) = 20\lg k - 20\lg\sqrt{1+\omega^2 T^2}$$



$$\omega \ll \frac{1}{T}$$

$$L(\omega) = 20\lg k$$

$$\omega \gg \frac{1}{T}$$

$$L(\omega) =$$

$$= 20\lg k - 20\lg\sqrt{1+\omega^2 T^2} =$$

$$= 20\lg k - 20\lg \omega T$$

Идеално форсиращо звено от първи ред

$$W(p) = k(Tp + 1)$$

$$W(j\omega) = k(1 + j\omega T)$$

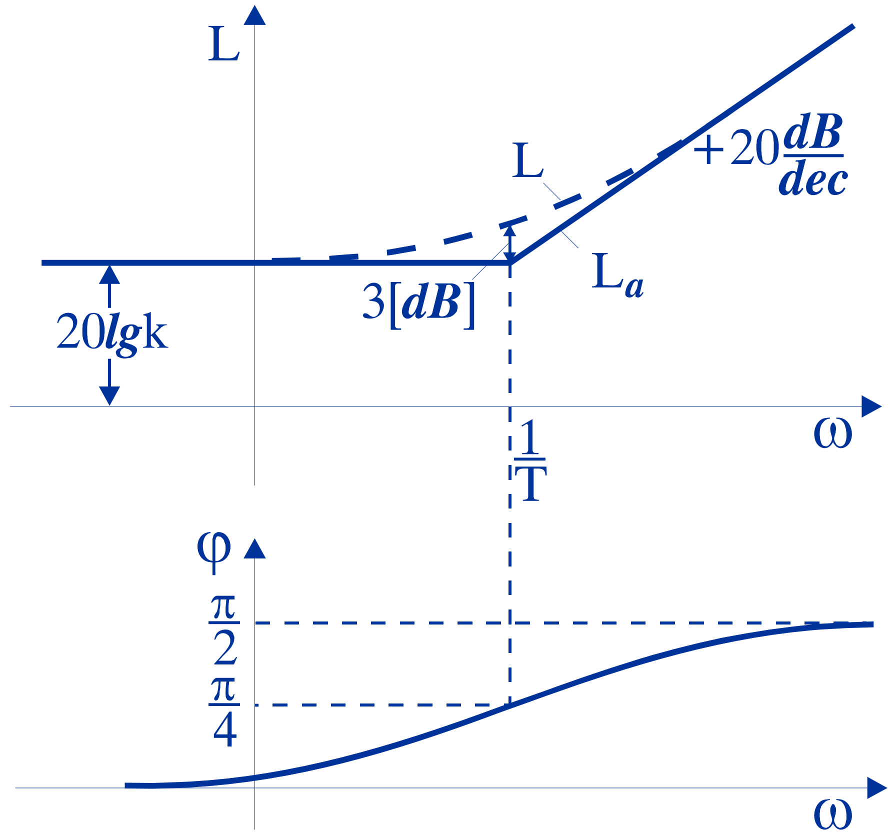
$$U(\omega) = k$$

$$V(\omega) = k\omega T$$

$$A(\omega) = k\sqrt{1 + \omega^2 T^2}$$

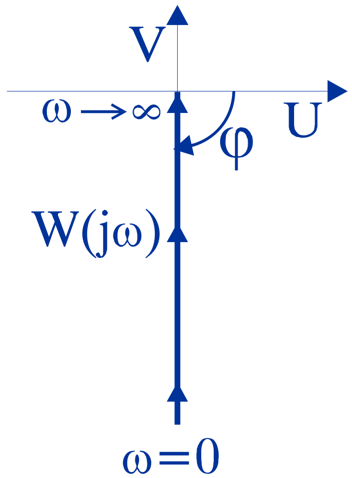
$$\varphi(\omega) = arctg(\omega T)$$

$$L(\omega) = 20\lg k + 20\lg\sqrt{1 + \omega^2 T^2}$$

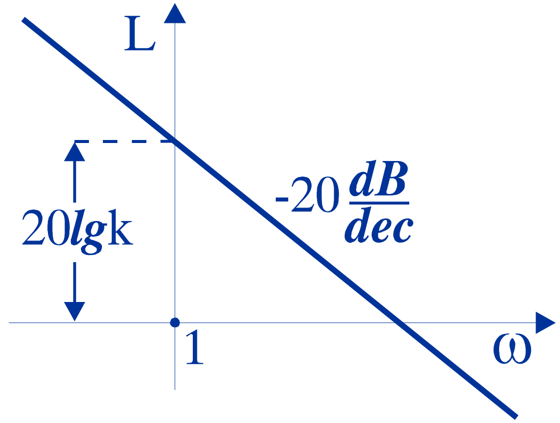


Интегриращо и идеално диференциращо звена

а)



б)



$$W(j\omega) = \frac{k}{j\omega} = -j\frac{k}{\omega}$$

$$U(\omega) = 0 \qquad V(\omega) = -\frac{k}{\omega}$$

$$A(\omega) = \frac{k}{\omega} \qquad \varphi(\omega) = -\frac{\pi}{2}$$

$$L(\omega) = 20\lg k - 20\lg\omega$$

$$W(j\omega) = j\omega k$$

$$U = 0 \qquad V(\omega) = \omega k$$

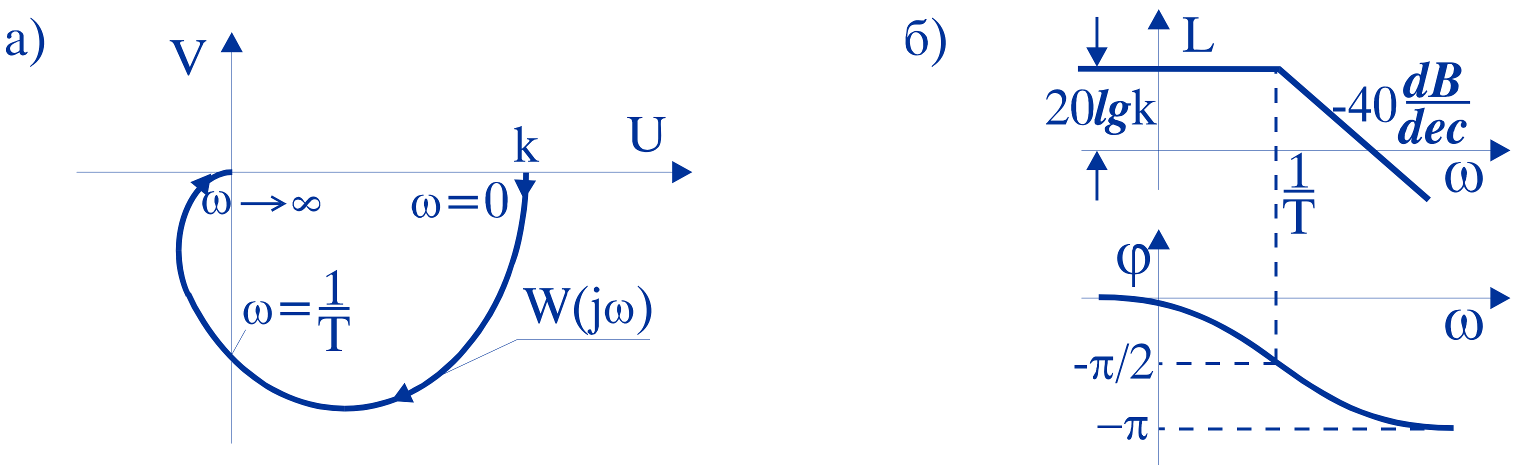
$$A(\omega) = \omega k \qquad \varphi = \frac{\pi}{2}$$

$$L(\omega) = 20\lg k + 20\lg\omega$$

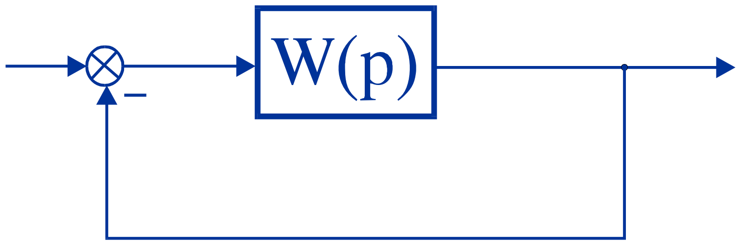
Колебателно звено (идеално форсиращо от втори ред)

$$W(j\omega)=\frac{k}{1-\omega^2T^2+j2\xi\omega T}=\frac{k(1-\omega^2T^2)}{(1-\omega^2T^2)^2+(2\xi\omega T)^2}-j\frac{k2\xi\omega T}{(1-\omega^2T^2)^2+(2\xi\omega T)^2}$$

$$A(\omega)=\frac{k}{\sqrt{(1-\omega^2T^2)^2+(2\xi\omega T)^2}} \qquad L(\omega)=20\lg k-20\lg\sqrt{(1-\omega^2T^2)^2+(2\xi\omega T)^2}$$



Честотни характеристики на отворени САУ



$$W(p) = \prod_{i=1}^{n} W_i(p)$$

$$W(j\omega) = A_1(\omega)e^{j\varphi_1(\omega)} A_2(\omega)e^{j\varphi_2(\omega)} \cdots A_n(\omega)e^{j\varphi_n(\omega)} =$$

$$= A_1(\omega)A_2(\omega)\cdots A_n(\omega)e^{j[\varphi_1(\omega)+\varphi_2(\omega)+\cdots+\varphi_n(\omega)]}$$

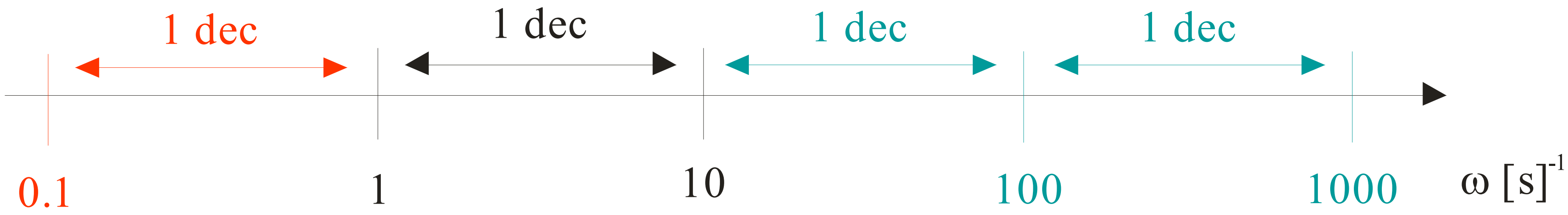
$$A(\omega) = \prod_{i=1}^{n} A_i(\omega)$$

$$\varphi(\omega) = \sum_{i=1}^{n} \varphi_i(\omega)$$

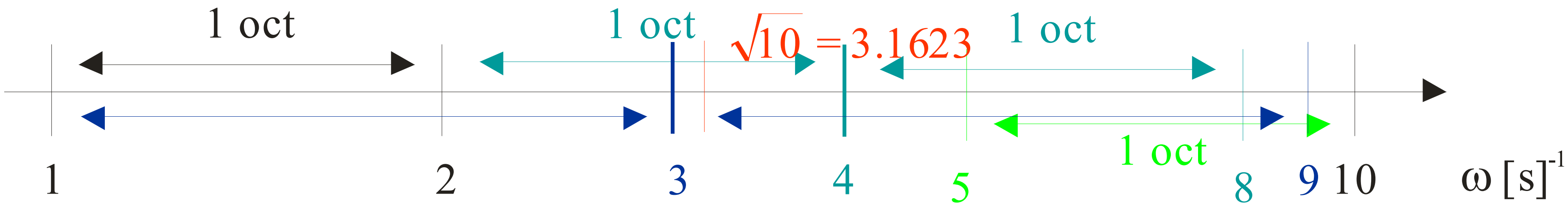
$$L(\omega) = 20\lg A(\omega) = \sum_{i=1}^{n} L_i(\omega)$$

Абсцисна ос

- ***Декада*** е всеки интервал, който съответства на <u>десетократно</u> изменение на честотата



- ***Октава*** е всеки интервал, който съответства на <u>двукратно</u> изменение на честотата

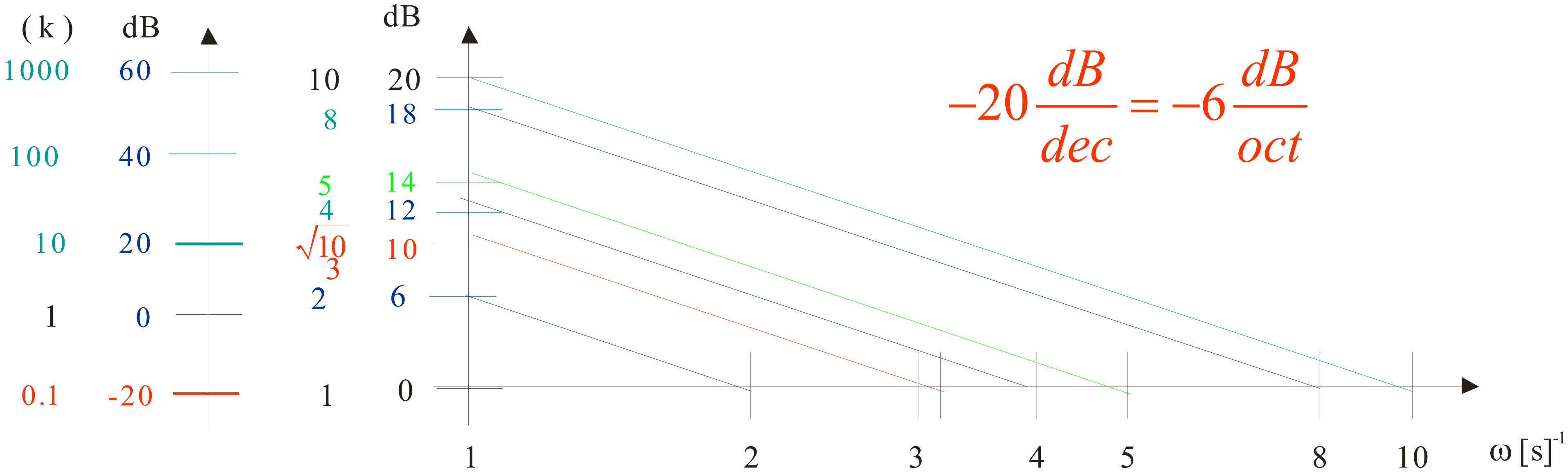


Ординатна ос

$$20\log(0) = 20\log(10^0) = 0$$

$$20\log(10) = 20\log(10^1) = 20$$

$$20\log(0.1) = 20\log(10^{-1}) = 20*(-1) = -20$$



$$-20\frac{dB}{dec} = -6\frac{dB}{oct}$$

$$20\log(4) = 20\log(2*2) = 20\log(2) + 20\log(2) = 12$$

$$20\log(5) = 20\log(10/2) = 20\log(10) - 20\log(2) = 20 - 6 = 14$$

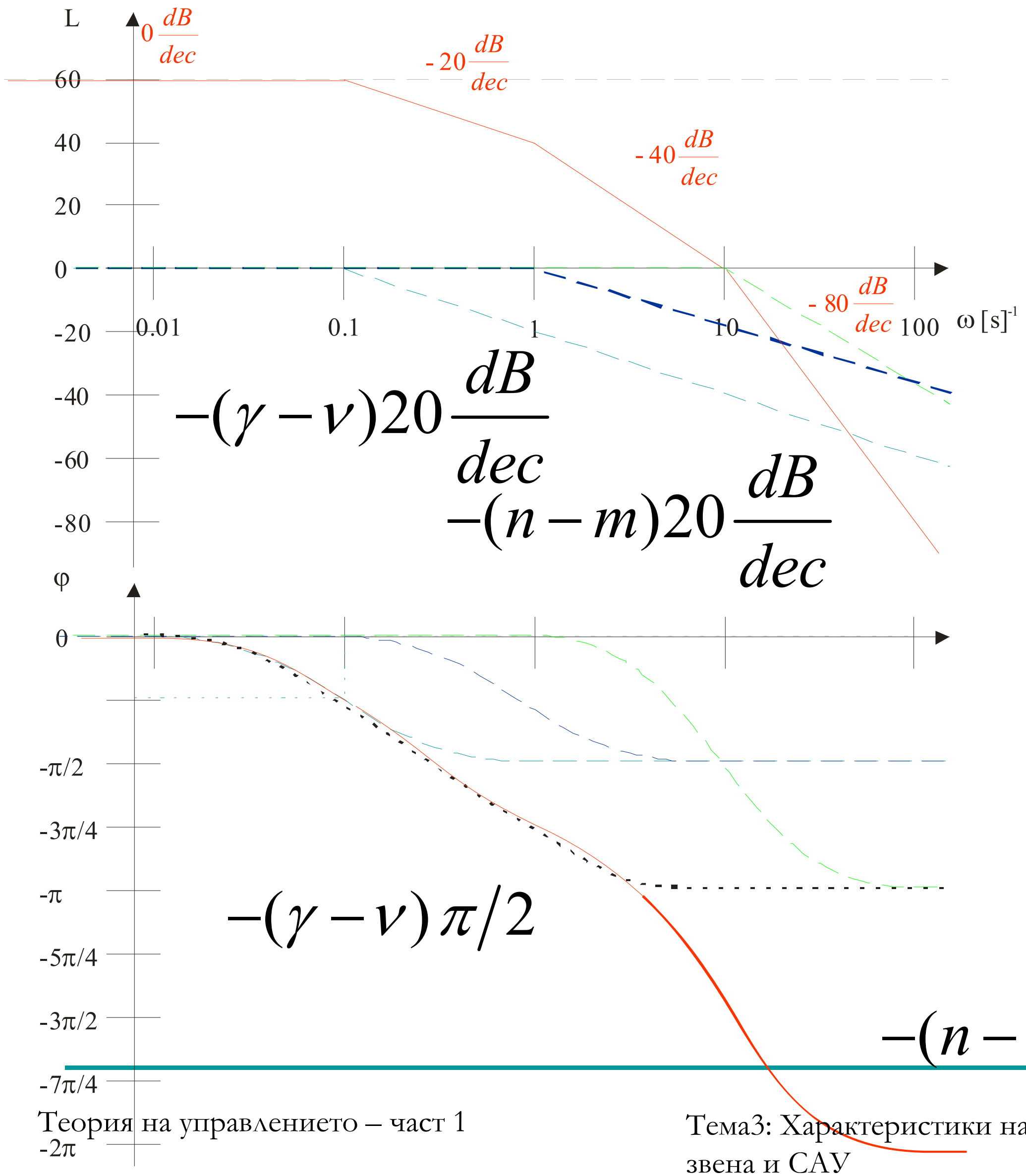
Пример 1

$$W(p) = \frac{1000}{(10p+1)(p+1)(0.01p^2+0.1p+1)}$$

1. Пропорционално $W_1(p) = k = 1000$ $\quad 20\lg 1000 = 60[dB]$
2. Апериодично $W_2(p) = \frac{1}{T_2 p + 1}$ $\quad T_2 = 10s$, $\omega_2 = \frac{1}{T_2} = 0,1s^{-1}$
3. Апериодично $W_3(p) = \frac{1}{T_3 p + 1}$ $\quad T_3 = 1s$, $\omega_3 = \frac{1}{T_3} = 1s^{-1}$
4. Колебателно $W_4(p) = \frac{1}{T_4^2 p^2 + 2\xi T_4 p + 1}$ $\quad T_4{}^2 = 0.01s$, $T_4 = 0.1s$, $\xi = 0.5$, $\omega_4 = \frac{1}{T_4} = 10s^{-1}$

ЛАЧХ и ЛФЧХ

$$W(p) = \frac{1000}{(10p+1)(p+1)(0.01p^2+0.1p+1)}$$



$$W_1(p) = k = 1000$$

$$W_2(p) = \frac{1}{T_2 p + 1}$$

$$\omega_2 = 0{,}1 s^{-1}$$

$$W_3(p) = \frac{1}{T_3 p + 1}$$

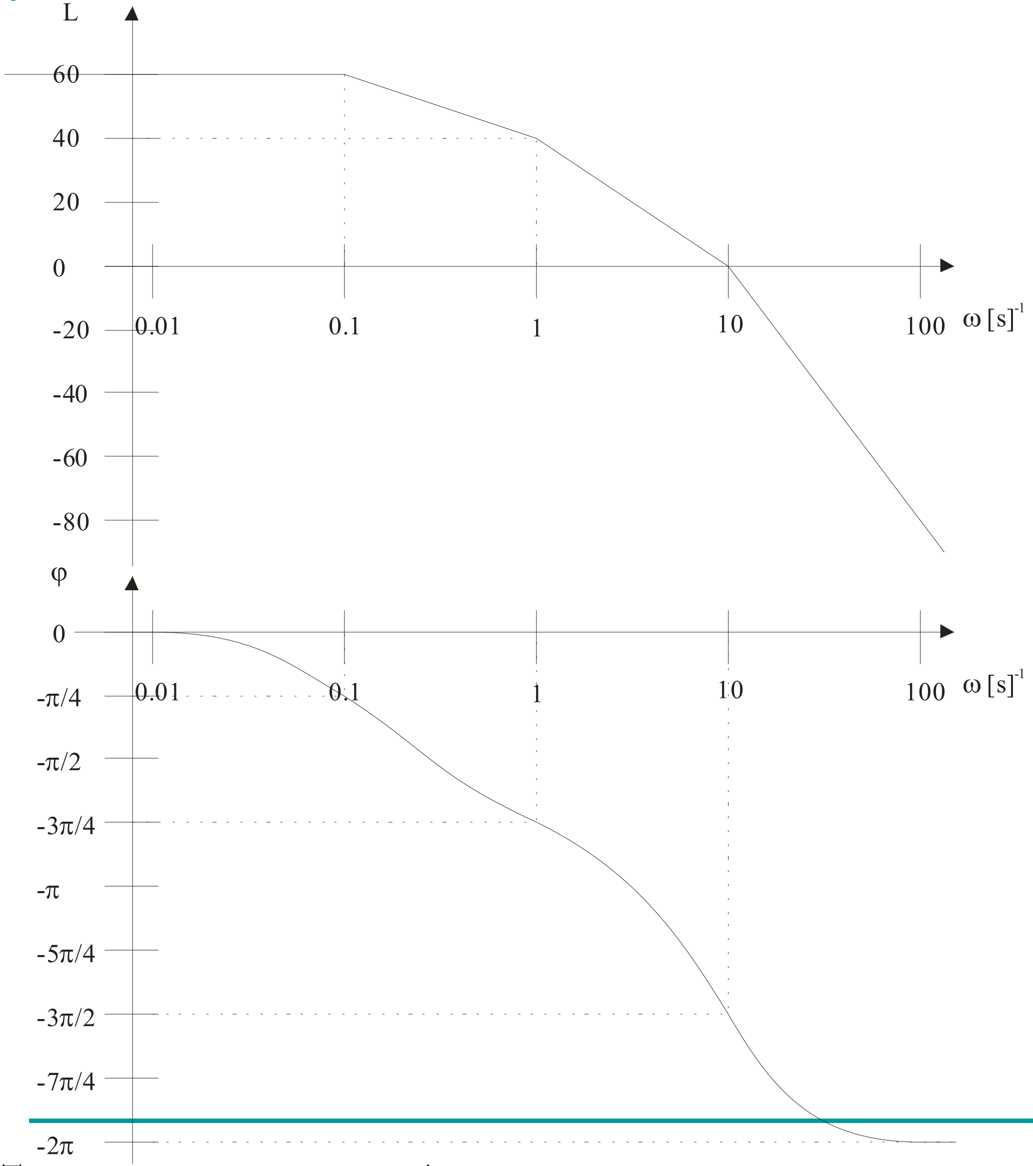
$$\omega_3 = 1 s^{-1}$$

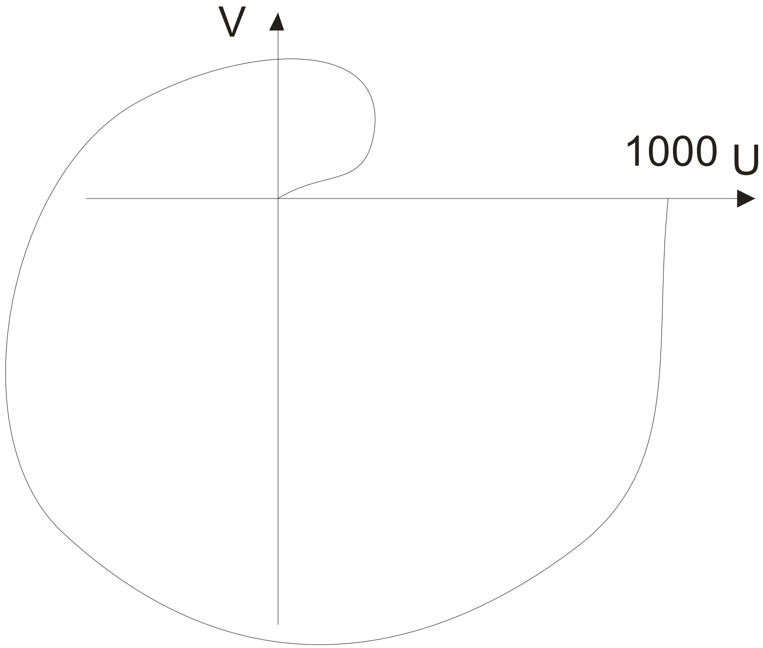
$$W_4(p) = \frac{1}{T_4^2 p^2 + 2\xi T_4 p + 1}$$

$$\omega_4 = 10 s^{-1}$$

АФХ

$$W(p)=\frac{1000}{(10p+1)(p+1)(0.01p^2+0.1p+1)}$$





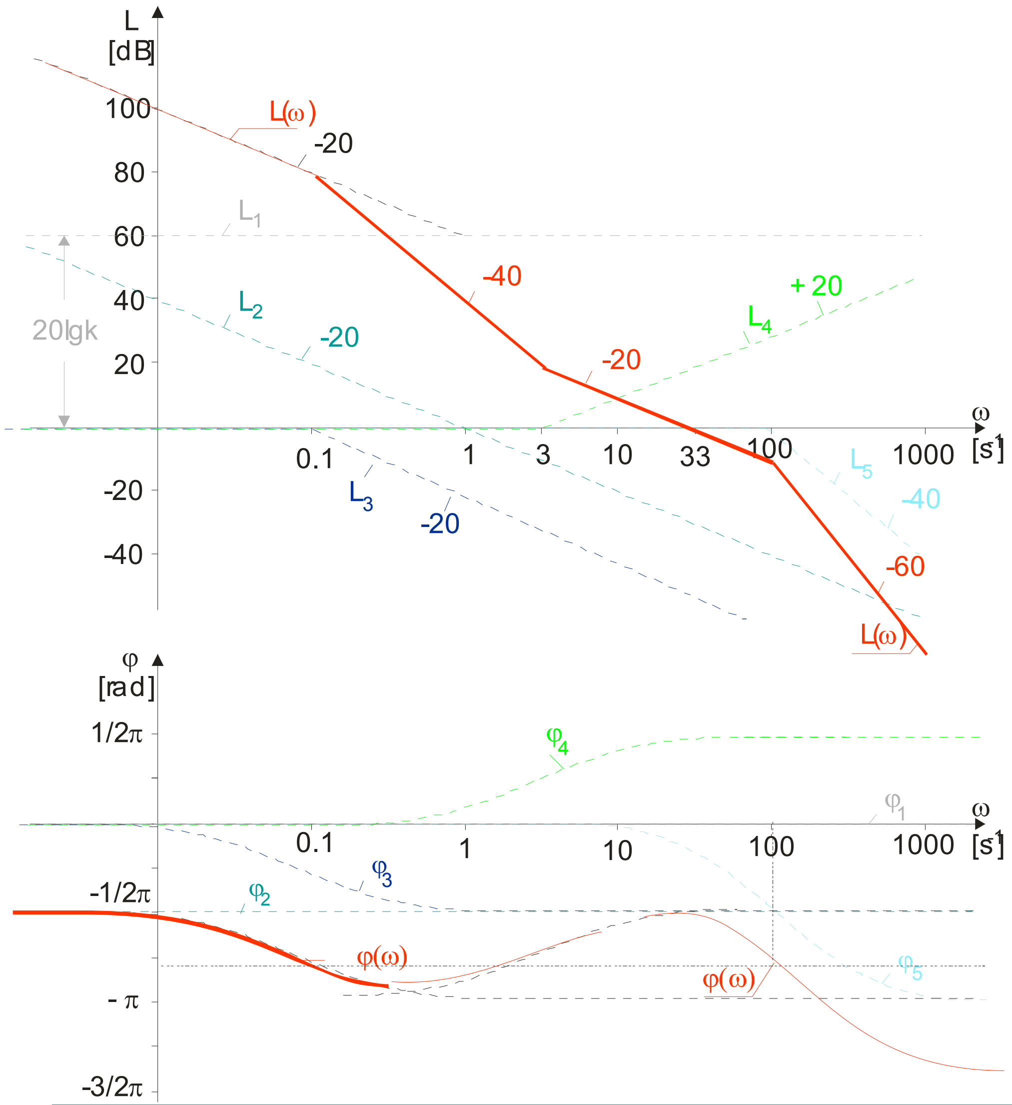
Пример 2

$$W(p) = \frac{1000(0.333p+1)}{p(10p+1)(0.0001p^2+0.01p+1)}$$

1. Пропорционално $W_1(p) = k = 1000$ $\quad 20\lg 1000 = 60[dB]$
2. Интегриращо $W_2(p) = \frac{1}{p}$ $\quad \omega_2 = 1[s^{-1}]$
3. Апериодично $W_3(p) = \frac{1}{T_3 p + 1}$ $\quad T_3 = 10s$ $\quad \omega_3 = \frac{1}{T_3} = 0.1\ s^{-1}$
4. Идеално форсиращо $W_4(p) = T_4 p + 1$ $\quad T_4 = 0.333s$ $\quad \omega_4 = \frac{1}{T_4} = 3s^{-1}$
5. Колебателно $W_5(p) = \frac{1}{T_5^2 p^2 + 2\xi T_5 p + 1}$ $\quad T_5 = 0.01s$ $\quad \xi = 0.5$ $\quad \omega_5 = \frac{1}{T_5} = 100s^{-1}$

ЛАЧХ и ЛФЧХ

$$W(p) = \frac{1000\,(0.333p+1)}{p(10p+1)(0.0001p^2+0.01p+1)}$$



$$W_1(p) = k = 1000$$

$$20\lg 1000 = 60\,[dB]$$

$$W_2(p) = \frac{1}{p} \quad \omega_2 = 1\,[s^{-1}]$$

$$W_3(p) = \frac{1}{T_3 p + 1}$$

$$\omega_3 = 0.1 s^{-1}$$

$$W_4(p) = T_4 p + 1$$

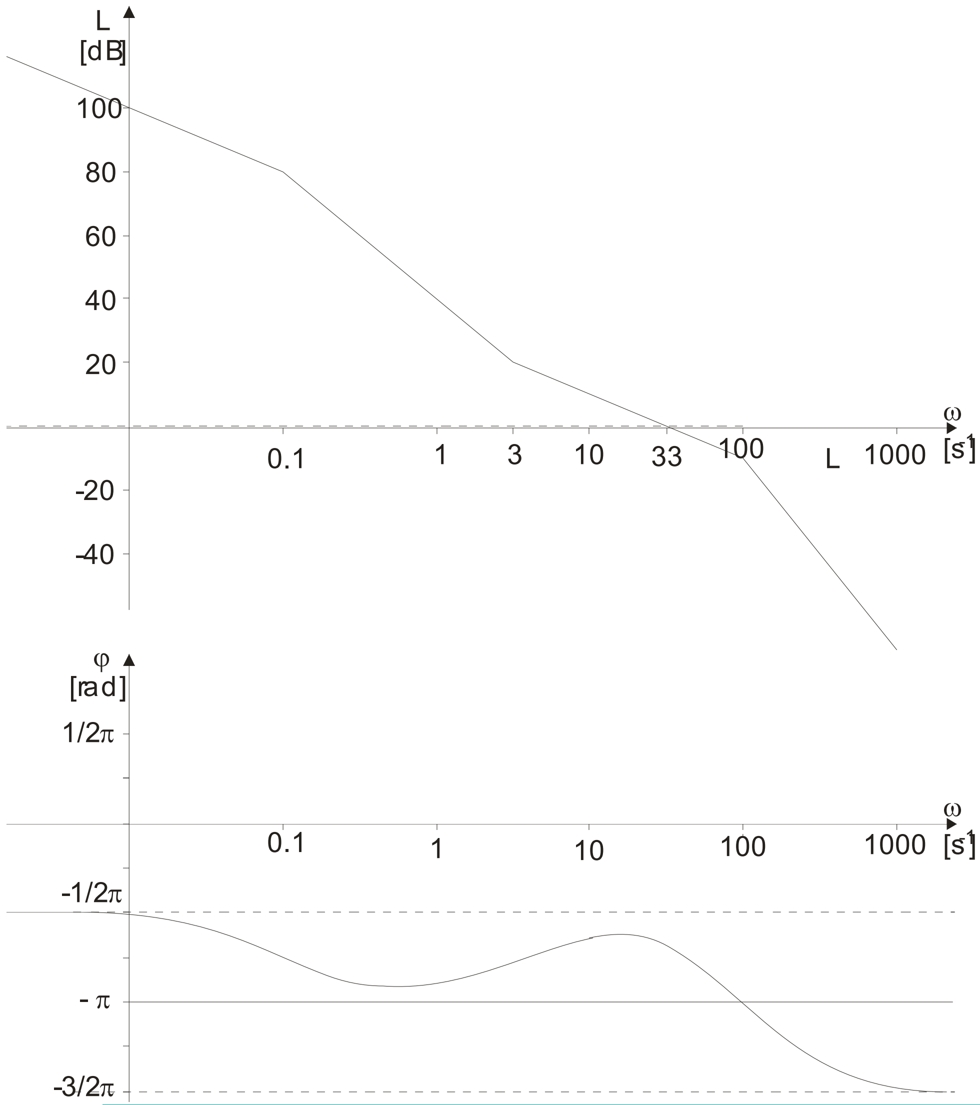
$$\omega_4 = 3 s^{-1}$$

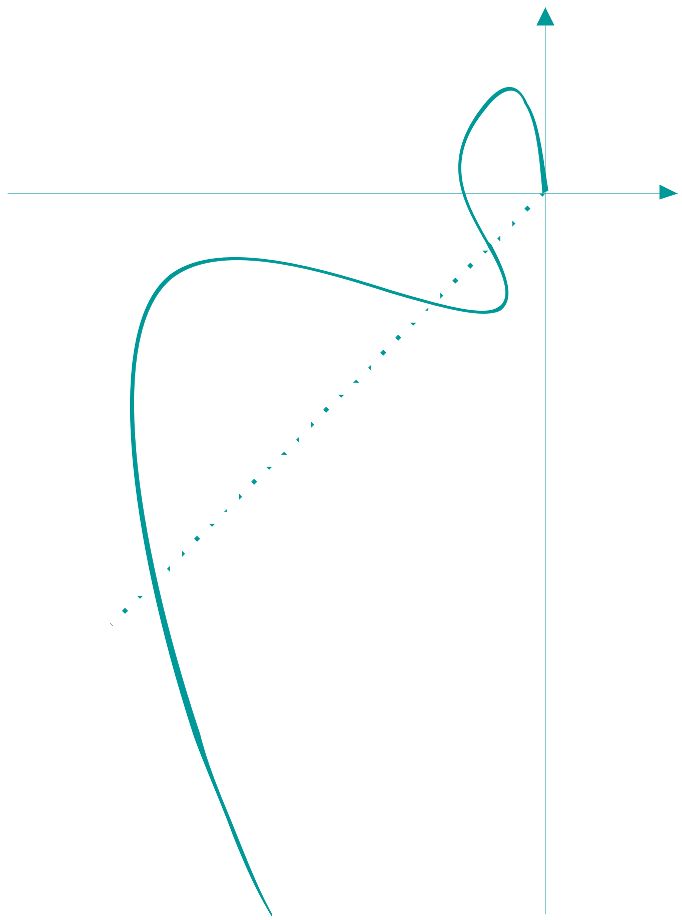
$$W_5(p) = \frac{1}{T_5^2 p^2 + 2\xi T_5 p + 1}$$

$$\omega_5 = 100 s^{-1}$$

АФХ

$$W(p) = \frac{1000(0.333p+1)}{p(10p+1)(0.0001p^2+0.01p+1)}$$





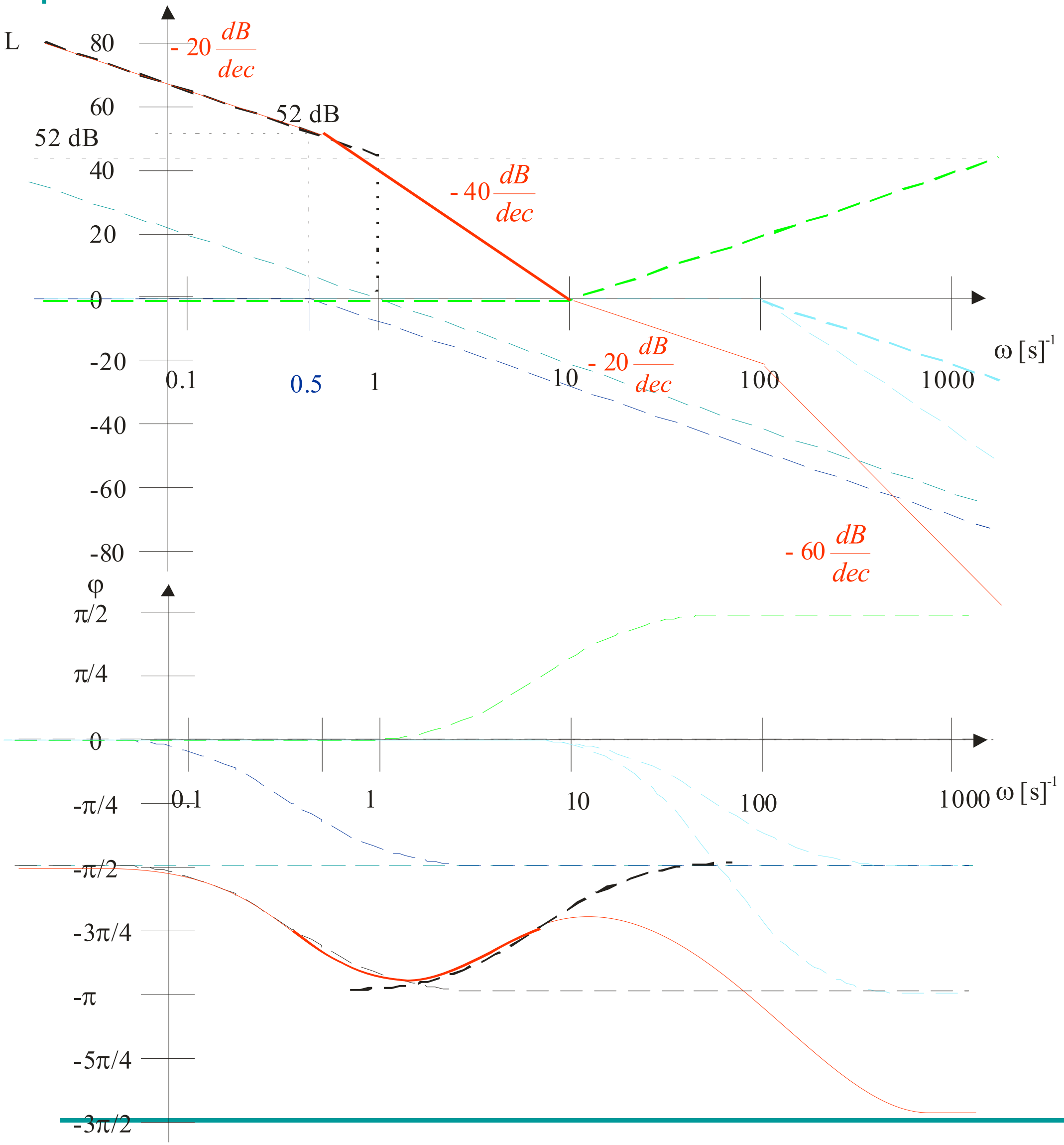
Пример 3

$$W(p)=\frac{200(0.1p+1)}{p(2p+1)(0.01p+1)^2}$$

1. Пропорционално $W_1(p)=k=200$ $20\lg 200=46[dB]$
2. Интегриращо $W_2(p)=\frac{1}{p}$ $\omega_2=1\,[s^{-1}]$
3. Апериодично $W_3(p)=\frac{1}{T_3p+1}$ $T_3=2s$ $\omega_3=\frac{1}{T_3}=0.5s^{-1}$
4. Идеално форсиращо $W_4(p)=T_4p+1$ $T_4=0.1s$ $\omega_4=\frac{1}{T_4}=10s^{-1}$
5. Апериодично $W_5(p)=\frac{1}{T_5p+1}$ $T_5=0.01s$ $\omega_5=\frac{1}{T_5}=100s^{-1}$

ЛАЧХ и ЛФЧХ

$$W(p) = \frac{200(0.1p+1)}{p(2p+1)(0.01p+1)^2}$$



$$W_1(p) = k = 200$$

$$W_2(p) = \frac{1}{p}$$

$$\omega_2 = 1\,[s^{-1}]$$

$$W_3(p) = \frac{1}{T_3 p + 1}$$

$$\omega_3 = 0.5 s^{-1}$$

$$W_4(p) = T_4 p + 1$$

$$\omega_4 = 10 s^{-1}$$

$$W_5(p) = \frac{1}{T_5 p + 1}$$

$$\omega_5 = 100 s^{-1}$$

АФХ

$$W(p) = \frac{200(0.1p+1)}{p(2p+1)(0.01p+1)^2}$$