Математични модели на линейни непрекъснати САУ

Математични модели

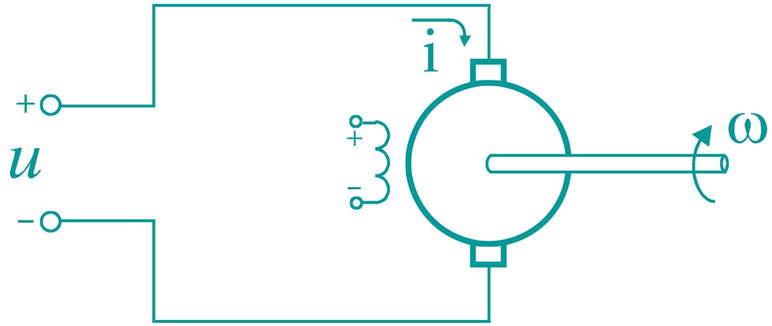
- Изисквания
  - Точно описание на свойствата на обекта
  - Прост модел (за по-лесно изследване)
- Видове модели
  - Аналитични
  - Графоаналитични
  - Графични
- Получаване на модели
  - Закони на механиката
  - Закони на електротехниката
  - Закони на аеродинамиката

Цели на лекцията

Запознаване с

- Диференциално уравнение
  - Линеаризация
    - На статични характеристики
    - Линеаризация на диференциални уравнения
  - Линейно диференциално уравнения от общ вид
- Предавателна функция
- Структурна схема
  - Преобразуване на структурни схеми
  - Системи със смущаващи въздействия
- Многомерни системи

Постояннотоков двигател с независимо възбуждане



- Втори закон на Нютон

$$J\frac{d\omega(t)}{dt} = M_{\partial}(t) - M_T(t)$$

- От електромеханиката

$$M_{\partial}(t) = k_M i(t)$$

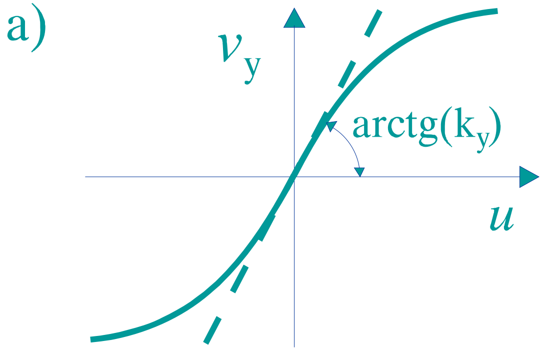
$$Ri(t) + k_e\omega(t) = u(t)$$

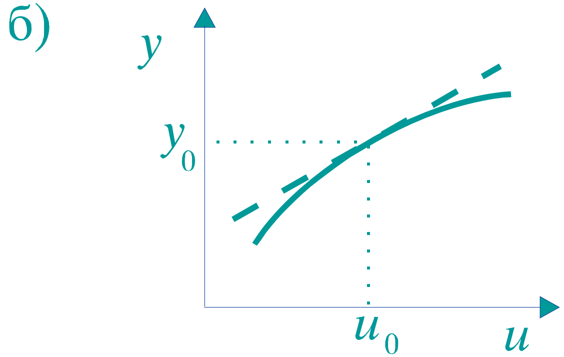
$$J\frac{d\omega(t)}{dt} + \frac{k_M k_e}{R}\omega(t) = \frac{k_M}{R}u(t) - M_T(t)$$

$$T_{\partial в} = \frac{JR}{k_M k_e} \qquad k_{\partial в} = \frac{1}{k_e}$$

$$T_{\partial в}\frac{d\omega(t)}{dt} + \omega(t) = k_{\partial в}u(t)$$

Линеаризация на статични характеристики





$$v_y(t) = k_y u(t)$$

$$u(t) = u_0 + \Delta u(t)$$

$$y(t) = y_0 + \Delta y(t)$$

$$y = f(u_0) + \left.\frac{\partial f}{\partial u}\right|_{u=u_0} \Delta u + \frac{1}{2!}\left.\frac{\partial^2 f}{\partial u^2}\right|_{u=u_0} \Delta u^2 + \frac{1}{3!}\left.\frac{\partial^3 f}{\partial u^3}\right|_{u=u_0} \Delta u^3 + \cdots$$

$$f(u_0) = y_0$$

$$y - y_0 = \Delta y$$

$$\Delta y = k\Delta u$$

Линеаризация на диференциални уравнения

$$F(\ddot{y},\dot{y},y,\dot{u},u)=0 \quad . \qquad a_0\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_2y=b_0\frac{du}{dt}+b_1u$$

$$F(0,0,y_0,0,u_0)=0$$

$$\ddot{y}=0 \quad , \quad \dot{y}=0 \quad , \quad y=y_0 \quad , \quad \dot{u}=0 \quad , \quad u=u_0 \quad .$$

$$F(\ddot{y},\dot{y},y,\dot{u},u) \;=\; F(0,0,y,0,u) \;+\; \left(\frac{\partial F}{\partial \ddot{y}}\right)^{*}\Delta\ddot{y} \;+\; \left(\frac{\partial F}{\partial \dot{y}}\right)^{*}\Delta\dot{y} \;+$$

$$+\; \left(\frac{\partial F}{\partial y}\right)^{*}\Delta y \;+\; \left(\frac{\partial F}{\partial \dot{u}}\right)^{*}\Delta\dot{u} \;+\; \left(\frac{\partial F}{\partial u}\right)^{*}\Delta u \;+\; \cdots \quad .$$

$$\left(\frac{\partial F}{\partial \ddot{y}}\right)^{*}=a_0 \quad \left(\frac{\partial F}{\partial \dot{y}}\right)^{*}=a_1 \quad \left(\frac{\partial F}{\partial y}\right)^{*}=a_2 \qquad \left(\frac{\partial F}{\partial \dot{u}}\right)^{*}=-b_0 \quad \left(\frac{\partial F}{\partial u}\right)^{*}=-b_1$$

$$a_0\Delta\ddot{y}+a_1\Delta\dot{y}+a_2\Delta y-b_0\Delta\dot{u}-b_1\Delta u=0$$

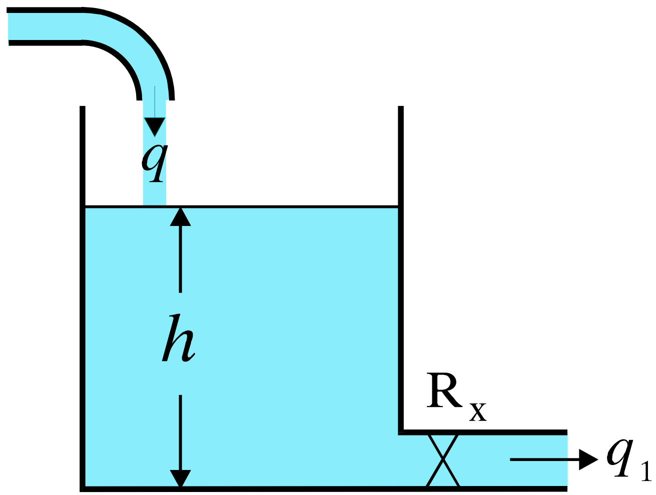
Диференциално уравнение на резервоар

$$S\frac{dh}{dt} = q - q_1$$

$$q_1 = \frac{1}{R_x}\sqrt{h}$$

$$S\frac{dh}{dt} + \frac{1}{R_x}\sqrt{h} = q$$

$$\frac{dh}{dt} = 0 \qquad \sqrt{h_0} = R_x q_0$$



$$S\frac{dh}{dt} + \frac{1}{2R_x^2 q_0}h = q$$

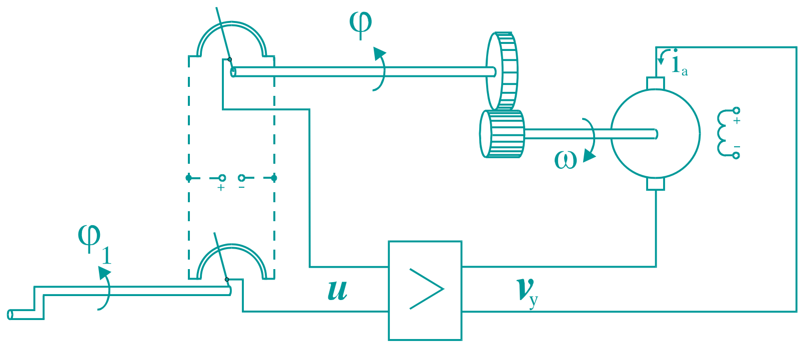
Линейно диференциално уравнение от общ вид

$$a_0 \frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_n y(t) = b_0 \frac{d^m u(t)}{dt^m} + \cdots + b_m u(t)$$

- Ред на звеното (системата) n
- Условие за физическа реализуемост $m \leq n$
- Коефициент на пропорционалност $k = \frac{b_m}{a_n}$

$$a_0^* \frac{d^n y(t)}{dt^n} + a_1^* \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + y(t) = k \left(b_0^* \frac{d^m u(t)}{dt^m} + \cdots + u(t)\right)$$

Потенциометрична следяща система



- Сравняващ елемент

$$u = k_n(\varphi_1 - \varphi)$$

- Усилвател

$$v_y = k_y u$$

- Двигател

$$T_{дв}\frac{d\omega}{dt} + \omega = k_{дв} v_y$$

- Редуктор

$$\frac{d\varphi}{dt} = k_{ред}\omega \quad (\omega_{ред} = k_{ред}\omega)$$

- Диференциално уравнение на системата

$$T_{дв}\frac{d^2\varphi}{dt^2} + \frac{d\varphi}{dt} + k_n k_y k_{дв} k_{ред}\varphi = k_n k_y k_{дв} k_{ред}\varphi_1$$

Предавателна функция

$$W(p) = \frac{Y(p)}{U(p)}$$

- ***е отношение на Лапласовото изображение на изходната променлива към Лапласовото изображение на входната променлива, при нулеви начални условия.***

Връзка между предавателна функция и диференциално уравнение

$$a_0 \frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_n y(t) = b_0 \frac{d^m u(t)}{dt^m} + \cdots + b_m u(t)$$

$$(a_0 p^n + a_1 p^{n-1} + \cdots + a_n) Y(p) = (b_0 p^m + b_1 p^{m-1} + \cdots + b_m) U(p)$$

$$W(p) = \frac{b_0 p^m + b_1 p^{m-1} + \cdots + b_m}{a_0 p^n + a_1 p^{n-1} + \cdots + a_n}$$

Потенциометрична следяща система

- Усилвател

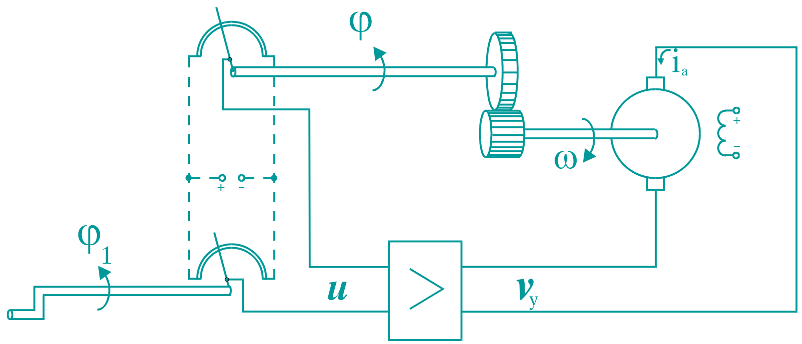
$$W_y(p) = k_y$$

- Двигател

$$W_{\partial в}(p) = \frac{k_{\partial в}}{T_{\partial в}p + 1}$$

- Редуктор

$$W_{ред}(p) = \frac{k_{ред}}{p}$$



- Затворена САУ

$$W_{зс}(p) = \frac{k}{T_{\partial в}p^2 + p + k}$$

$$k = k_n k_y k_{\partial в} k_{ред}$$

Преминаване от диференциално уравнение към предавателна функция за звена от/до втори ред

$$a_0 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_2 y(t) = b_0 \frac{du(t)}{dt} + b_1 u(t)$$

$$a_0 p^2 Y(p) + a_1 p Y(p) + a_2 Y(p) = b_0 p U(p) + b_1 U(p)$$

$$(a_0 p^2 + a_1 p + a_2) Y(p) = (b_0 p + b_1) U(p)$$

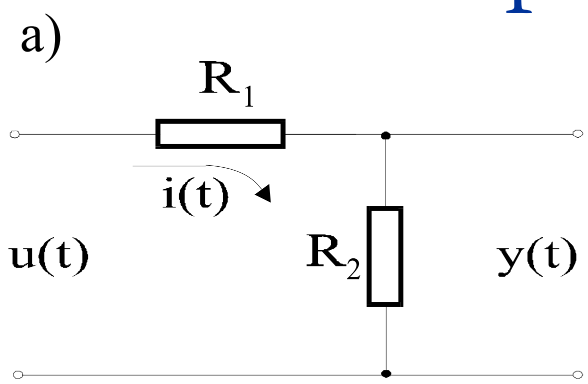
$$Y(p) = \frac{b_0 p + b_1}{a_0 p^2 + a_1 p + a_2} U(p)$$

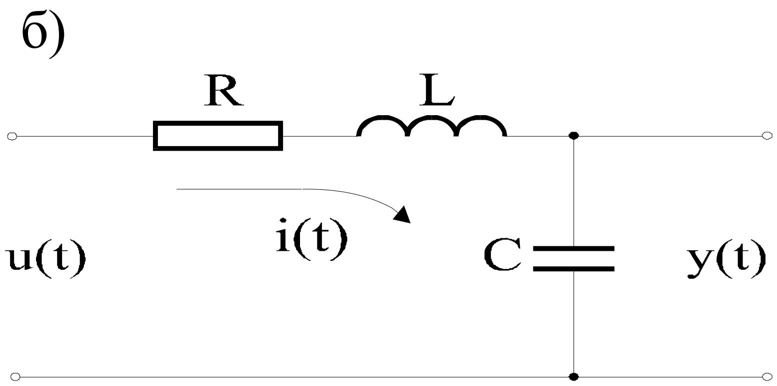
$$W(p) = \frac{b_0 p + b_1}{a_0 p^2 + a_1 p + a_2}$$

Потенциометрична следяща система

наименование	уравнение	предавателна функция
пропорционално	$y(t) = ku(t)$	$W(p) = k$
апериодично	$T\frac{dy(t)}{dt} + y(t) = ku(t)$	$W(p) = \frac{k}{Tp+1}$
интегриращо	$T\frac{dy(t)}{dt} = ku(t)$	$W(p) = \frac{k}{Tp}$
колебателно	$T^2\frac{d^2y(t)}{dt^2} + 2\xi T\frac{dy(t)}{dt} + y(t) = ku(t)$	$W(p) = \frac{k}{T^2p^2 + 2\xi Tp + 1}$
реално диференциращо	$T_1\frac{dy(t)}{dt} + y(t) = kT_2\frac{du(t)}{dt}$	$W(p) = \frac{kT_2p}{T_1p+1}$
реално форсиращо	$T_1\frac{dy(t)}{dt} + y(t) = k\left[T_2\frac{du(t)}{dt} + u(t)\right]$	$W(p) = \frac{k(T_2p+1)}{T_1p+1}$

Пасивни вериги





$$y(t) = R_2 i(t)$$

$$i(t) = \frac{u(t)}{R_1 + R_2}$$

$$y(t) = k u(t) \qquad k = \frac{R_2}{R_1 + R_2}$$

$$W(p) = k$$

$$u(t) = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(\tau)d\tau \qquad y(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$$

$$U(p) = \left(R + pL + \frac{1}{pC}\right) I(p) \qquad Y(p) = \frac{1}{pC} I(p)$$

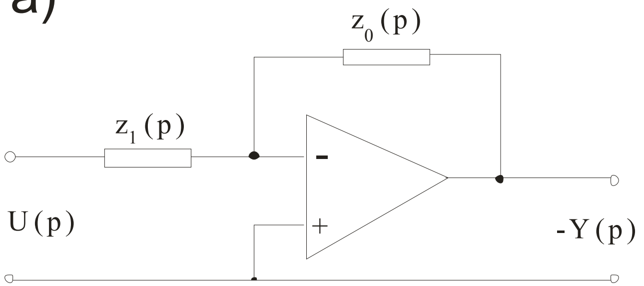
$$W(p) = \frac{Y(p)}{U(p)} = \frac{1}{LC\,p^2 + RC\,p + 1}$$

$$W(p) = \frac{k}{T^2 p^2 + 2\xi T p + 1} \qquad T = \sqrt{LC} \qquad 2\xi T = RC$$

Операционен усилвател с обратна връзка

$$W(p) = \frac{-Y(p)}{U(p)} = \frac{z_0(p)}{z_1(p)}$$

а)



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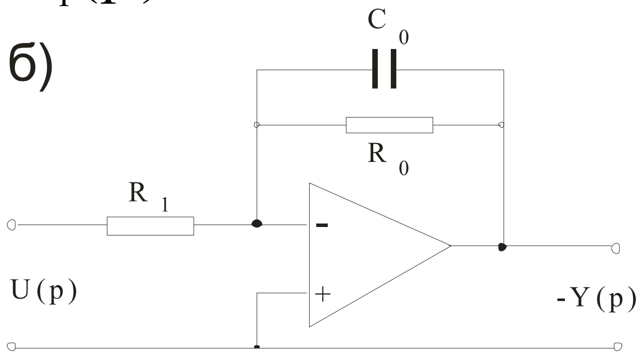


$$z_1(p) = R_1$$

$$z_0(p) = R_0$$

$$z_1(p) = R_1$$

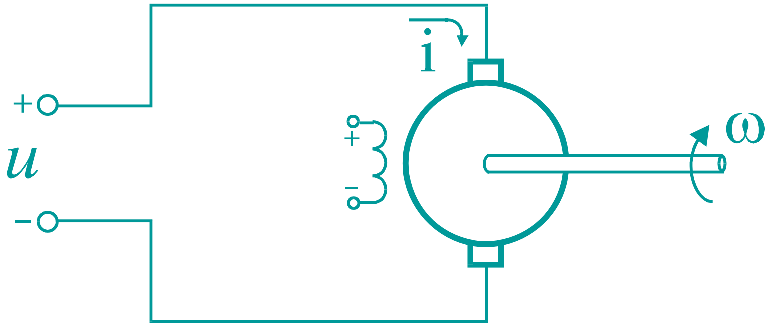
$$z_0(p) = \frac{R_0 \frac{1}{pC_0}}{R_0 + \frac{1}{pC_0}} = \frac{R_0}{R_0 C_0 p + 1}$$

$$k = \frac{R_0}{R_1}$$

$$W(p) = \frac{R_0}{R_1} = k \qquad W(p) = \frac{R_0}{R_1} \frac{1}{R_0 C_0 p + 1} = \frac{k}{Tp + 1}$$

$$T = R_0 C_0$$

Уравнения и предавателни функции на двигател



- Втори закон на Нютон

$$J\frac{d\omega(t)}{dt} = M_{\partial}(t) - M_{T}(t)$$

- От електромеханиката

$$M(t) = k_{\mathrm{M}} i(t);$$

$$L\frac{di(t)}{dt} + R i(t) + k_{\mathrm{E}}\omega(t) = u(t),$$

$$T_{\mathrm{A}} = \frac{L}{R} \qquad T_{\mathrm{M}} = \frac{JR}{k_{\mathrm{M}} k_{\mathrm{E}}} \qquad k_D = \frac{1}{k_E}$$

$$T_A \ll T_M$$

серводвигател

$$T_A T_M \frac{d^2\omega(t)}{dt^2} + T_M\frac{d\omega(t)}{dt} + \omega(t) = \kappa_D u(t).$$

$$W(p) = \frac{k_D}{T_A T_M p^2 + T_M p + 1}$$

$$T_{\mathrm{M}}\frac{d\omega(t)}{dt} + \omega(t) = k_{\mathrm{D}} u(t)$$

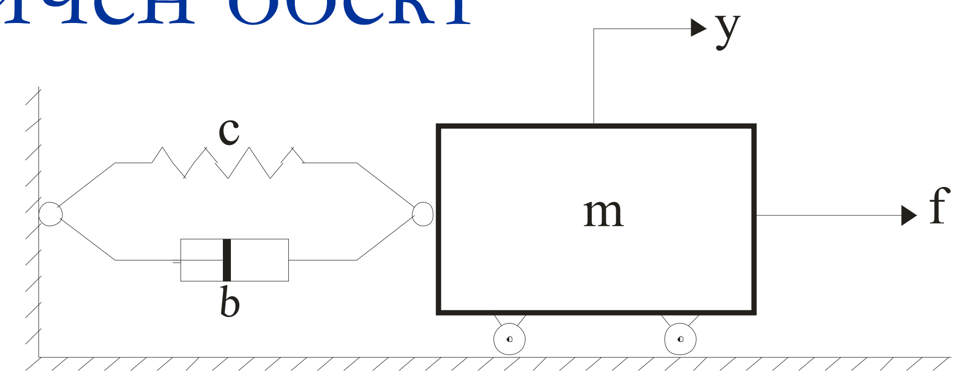
$$W(p) = \frac{k_D}{T_M p + 1}$$

$$T_M = 0 \quad \omega(t) = \frac{d\varphi(t)}{dt}$$

$$\frac{d\varphi(t)}{dt} = k_D u(t)$$

$$W(p) = \frac{k_D}{p}$$

Механичен обект



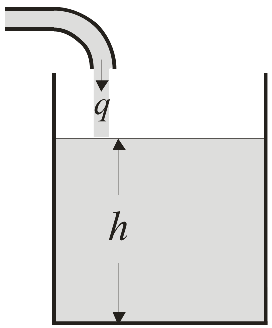
$$f(t) = m\frac{d^2y(t)}{dt^2} + b\frac{dy(t)}{dt} + cy(t)$$

$$W(p) = \frac{Y(p)}{F(p)} = \frac{1}{m\,p^2 + b\,p + c} = \frac{k}{T^2p^2 + 2\xi T\,p + 1}$$

$$T = \sqrt{\frac{m}{c}} \qquad 2\xi T = \frac{b}{c} \qquad k = \frac{1}{c}$$

Резервоари

а)



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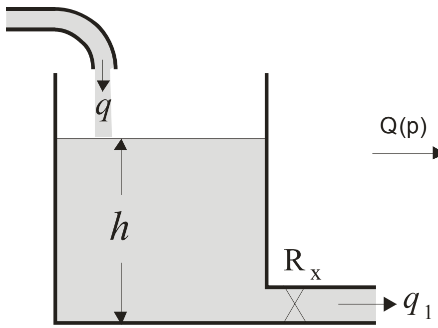
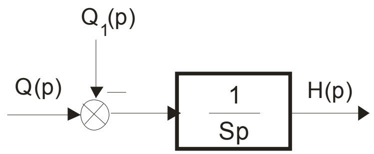


в)



$$H(p) = \frac{1}{S\,p}\left[Q(p) - Q_1(p)\right]$$

$$\frac{d\,h(t)}{dt} = \frac{1}{S}q(t)$$

$$\frac{d\,h(t)}{dt} = \frac{1}{S}\left[q(t) - q_1(t)\right]$$

$$W(p) = \frac{1}{S\,p}$$

$$q_1(t) = \frac{1}{R_x}\sqrt{h(t)}$$

$$S\frac{dh(t)}{dt} + \frac{1}{2R_x^2 q_0}h(t) = q(t)$$

$$S\frac{dh(t)}{dt} + \frac{1}{R_x}\sqrt{h(t)} = q(t)$$

$$W(p) = \frac{k}{Tp+1}$$

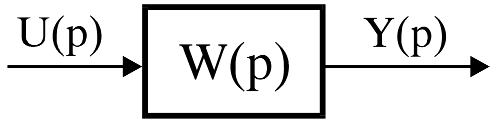
$$\frac{dh}{dt} = 0 \qquad q_1 = q = q_0$$

$$k = 2R_x^2 q_0 \qquad T = 2R_x^2 q_0 S$$

Структурна схема

$$Y(p) = W(p)U(p)$$

а)



блок (звено)

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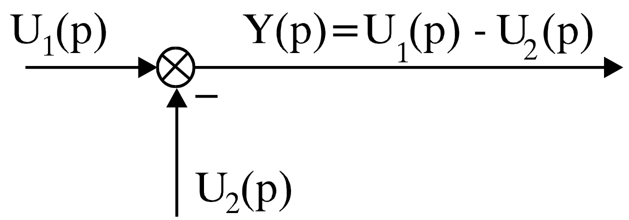

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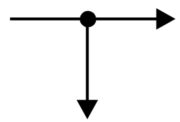

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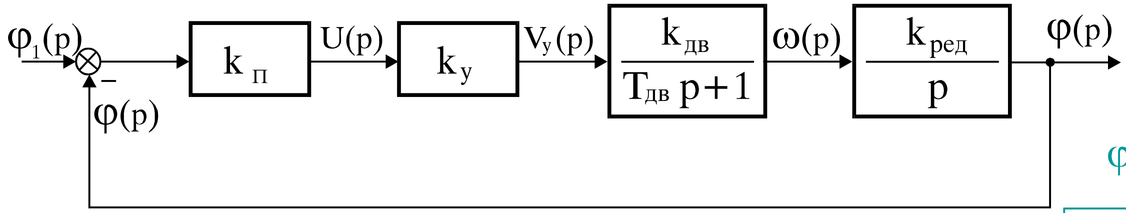
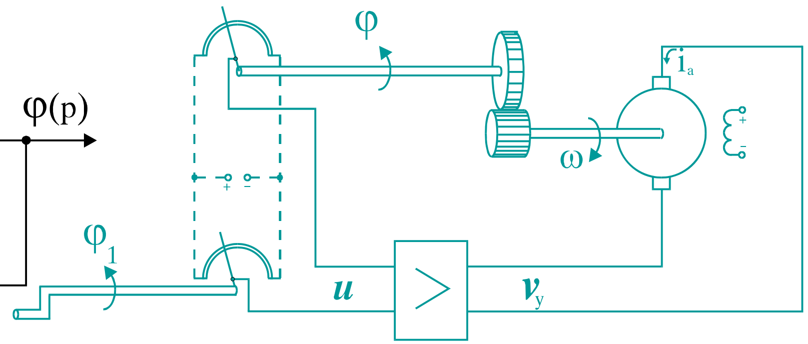
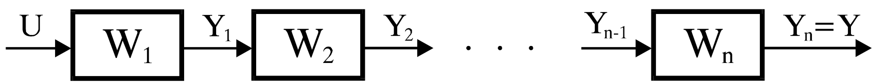




Преобразуване на структурни схеми

- Последователно свързване

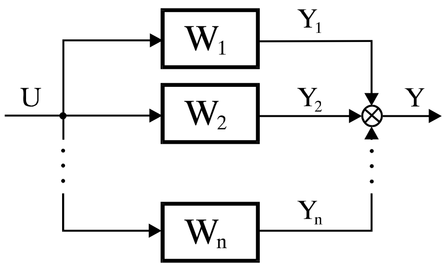


$$Y_1(p) = W_1(p)\,U(p)$$
$$Y_2(p) = W_2(p)\,Y_1(p)$$
$$\vdots$$
$$Y_n(p) = W_n(p)\,Y_{n-1}(p) \quad ,$$

$$Y(p) = Y_n(p) =$$
$$W_n(p)W_{n-1}(p)\cdots W_2(p)W_1(p)U(p)$$

$$W(p) = \prod_{i=1}^{n} W_i(p)$$

- Паралелно свързване

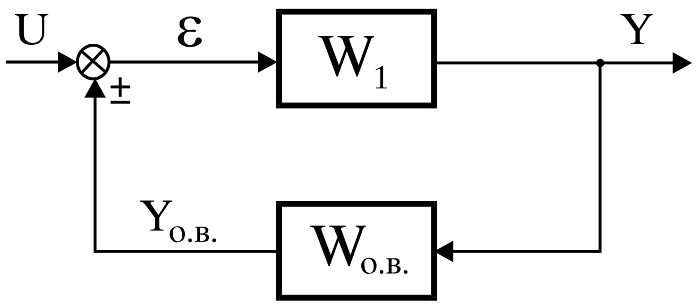


$$Y_1(p) = W_1(p)\,U(p)$$
$$Y_2(p) = W_2(p)\,U(p)$$
$$\vdots$$
$$Y_n(p) = W_n(p)\,U(p) \quad .$$
$$Y(p) = \sum_{i=1}^{n} Y_i(p)$$

$$W(p) = \sum_{i=1}^{n} W_i(p)$$

Преобразуване на структурни схеми

- Свързване с обратна връзка



$$Y(p) = W_1(p)\varepsilon(p);$$
$$Y_{OB}(p) = W_{OB}(p)Y(p);$$
$$\varepsilon(p) = U(p) \pm Y_{OB}(p).$$

$$Y(p) = W_1(p)[U(p) \pm W_{OB}(p)Y(p)]$$
$$[1 \mp W_1(p)W_{OB}(p)]Y(p) = W_1(p)U(p)$$

$$W(p) = \frac{W_1(p)}{1 \mp W_1(p)W_{OB}(p)}$$

САУ с типов регулатор

$$\varepsilon(t) = v(t) - y(t)$$

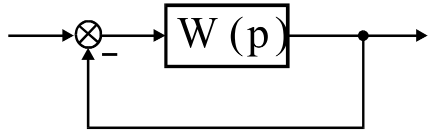
$$\varepsilon(p) = V(p) - Y(p)$$

$$u(t) = 25\left[\varepsilon(t) + \frac{1}{2}\int_0^t \varepsilon(\tau)d\tau\right]$$

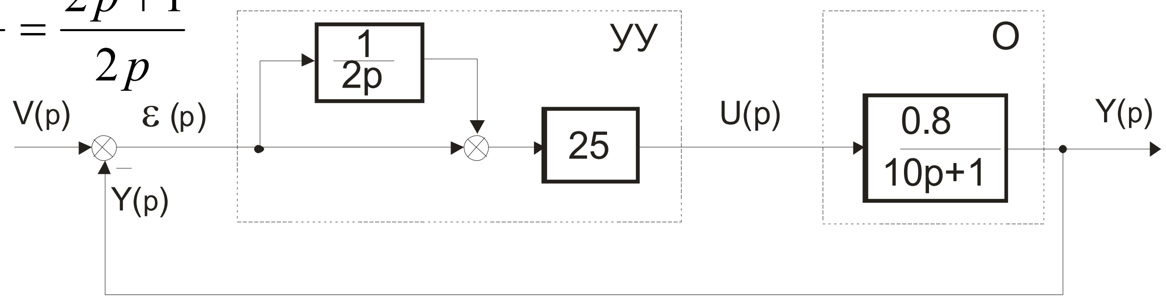
$$U(p) = 25\left[\varepsilon(p) + \frac{1}{2p}\varepsilon(p)\right]$$

$$10\frac{d\,y(t)}{dt} + y(t) = 0.8u(t)$$

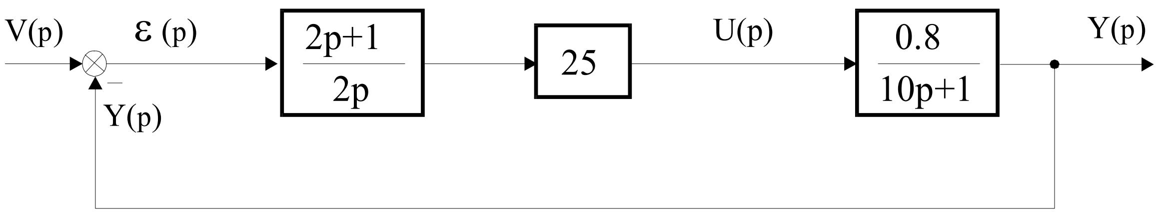
$$W(p) = \frac{Y(p)}{U(p)} = \frac{0.8}{10p+1}$$



$$1 + \frac{1}{2p} = \frac{2p+1}{2p}$$

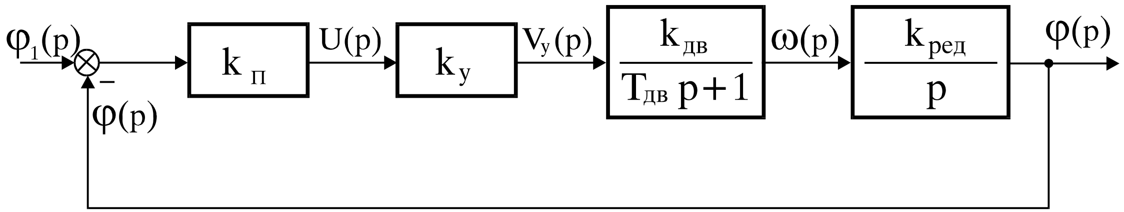


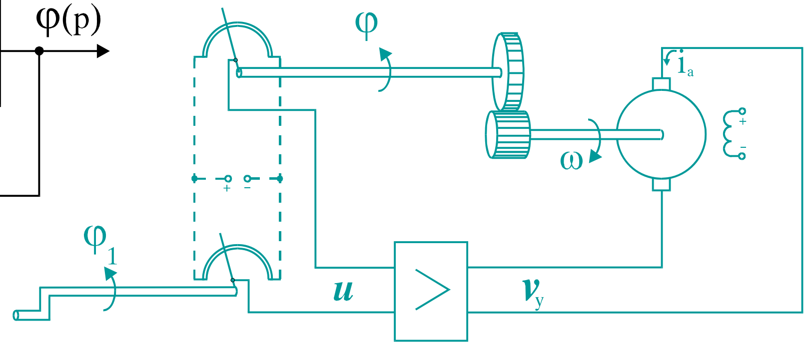
$$W_{y,v}(p) = \frac{W(p)}{1+W(p)} = \frac{\dfrac{10(2p+1)}{p(10p+1)}}{1+\dfrac{10(2p+1)}{p(10p+1)}} = \frac{20p+1}{10p^2+21p+10}$$



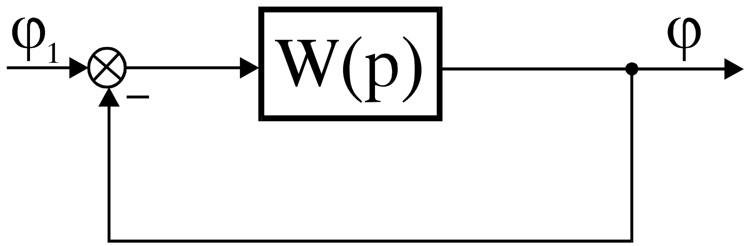
$$W(p) = \frac{2p+1}{2p}25\frac{0.8}{10p+1} = \frac{10(2p+1)}{p(10p+1)}$$

Потенциометрична следяща система





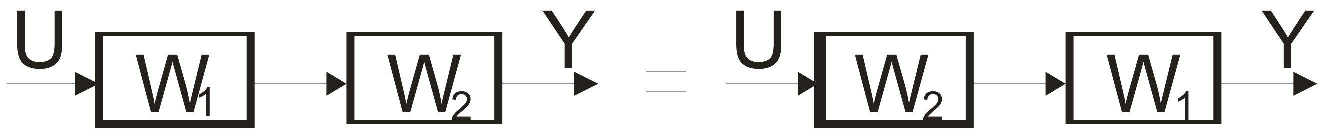
$$W(p) = \frac{k}{p(T_{\partial e} p + 1)}$$

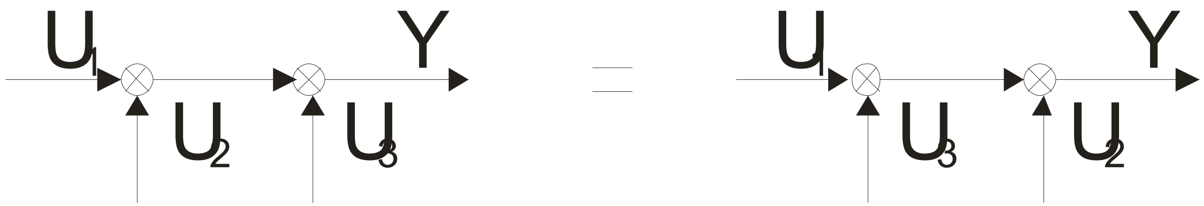


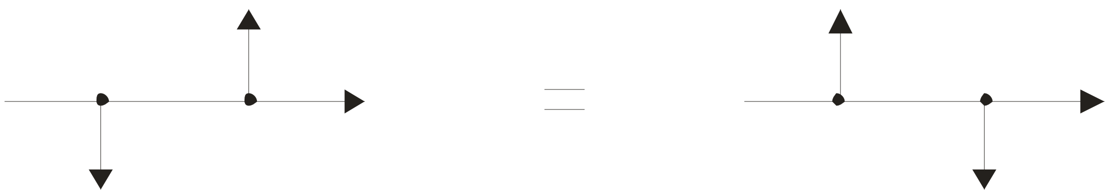
$$k = k_n k_y k_{\partial e} k_{pe\partial}$$

$$W_{зс}(p) = \frac{W(p)}{1 + W(p)} = \frac{k}{T_{\partial e} p^2 + p + k}$$

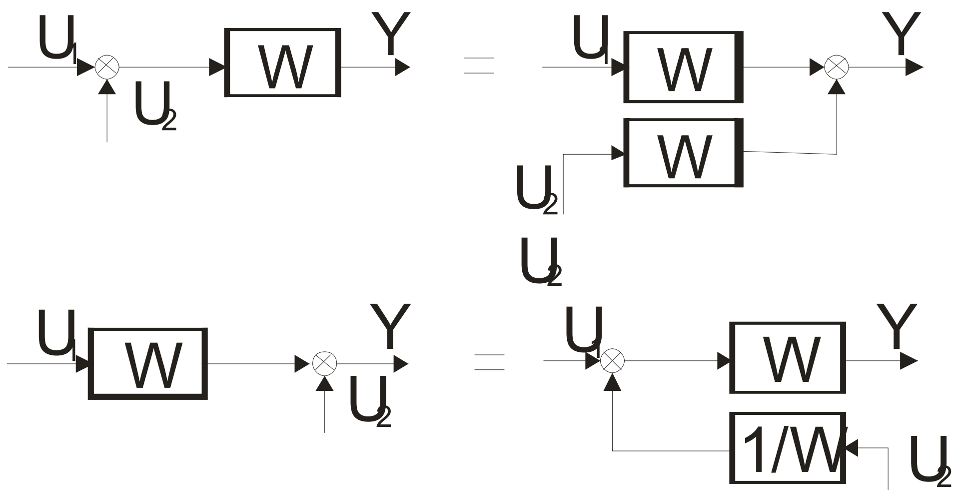
Правила за еквивалентни преобразувания





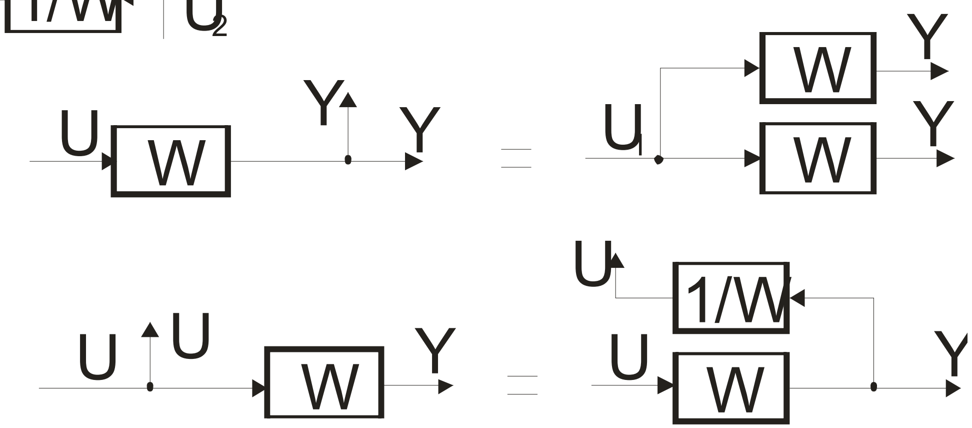


Правила за еквивалентни преобразувания

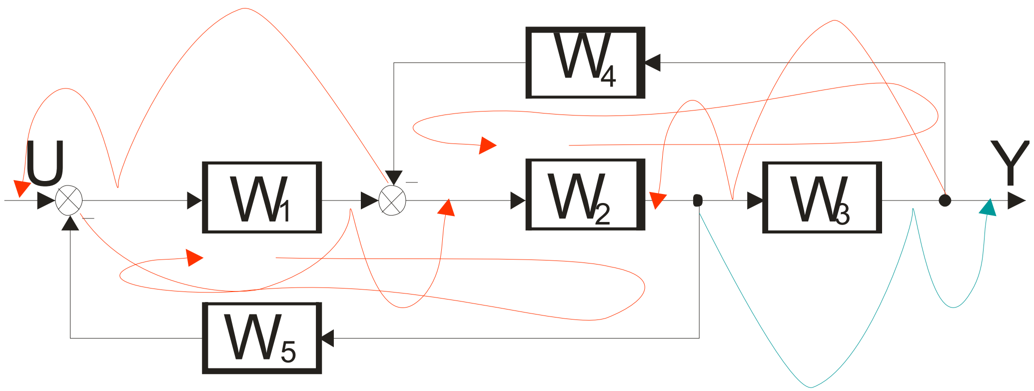


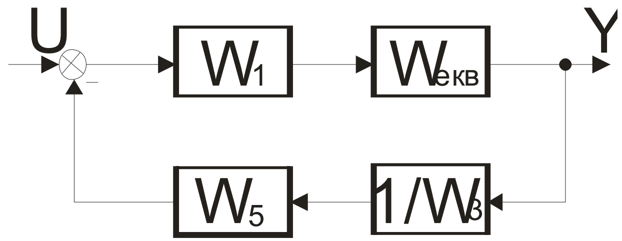
- Пренасяне на възел на разклонение през звено

- Пренасяне на суматор през звено

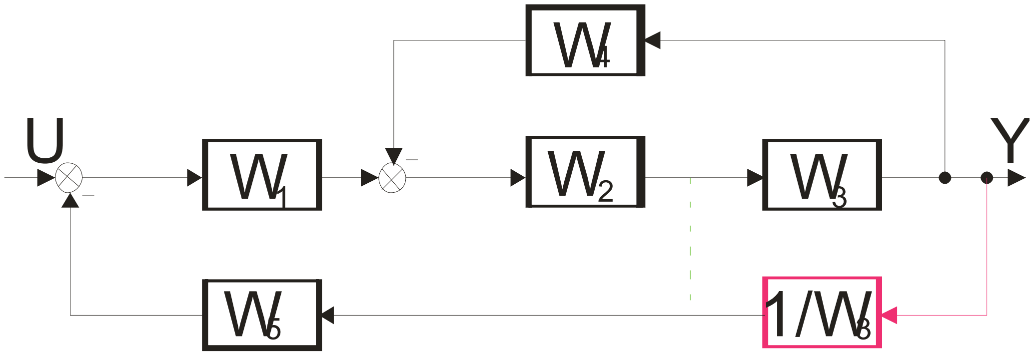


Правила за еквивалентни преобразувания



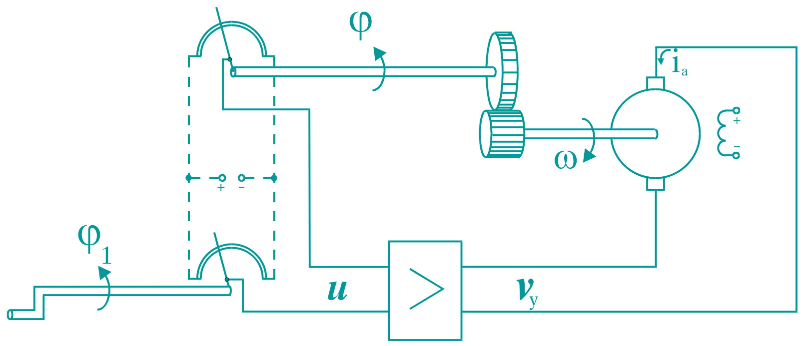


$$W_{екв}(p) = \frac{W_2(p)W_3(p)}{1 + W_2(p)W_3(p)W_4(p)}$$



$$W_{yu}(p) = \frac{W_1(p)W_2(p)W_3(p)}{1 + W_1(p)W_2(p)W_5(p) + W_2(p)W_3(p)W_4(p)}$$

Потенциометрична следяща система

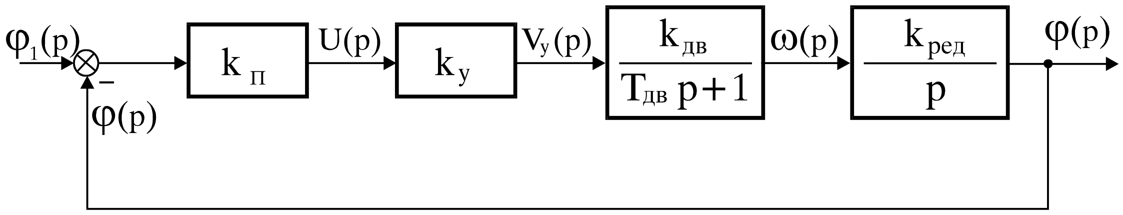


- Сравняващ елемент

$$u = k_n(\varphi_1 - \varphi) \qquad W_n(p) = k_n$$

- Усилвател

$$v_y = k_y u \qquad W_y(p) = k_y$$



- Двигател

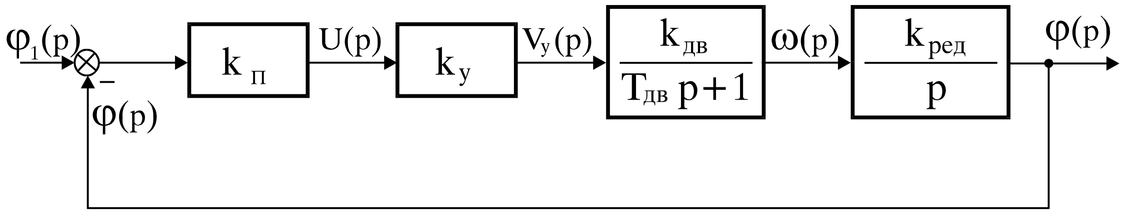
$$T_{дв}\frac{d\omega}{dt} + \omega = k_{дв} v_y$$

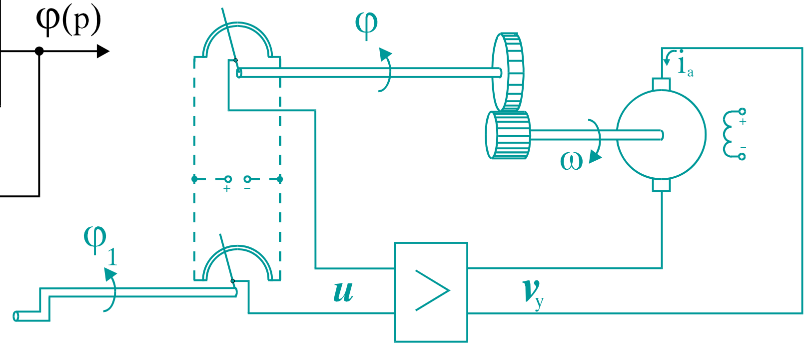
$$W_{дв}(p) = \frac{k_{дв}}{T_{дв}p + 1}$$

- Редуктор

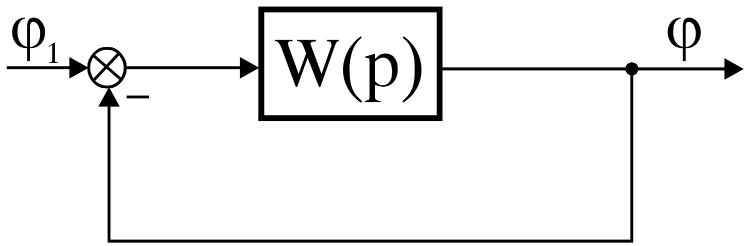
$$\frac{d\varphi}{dt} = k_{ред}\omega \quad (\omega_{ред} = k_{ред}\omega) \qquad W_{ред}(p) = \frac{k_{ред}}{p}$$

Потенциометрична следяща система





$$W(p) = \frac{k}{p(T_{\partial в} p + 1)}$$

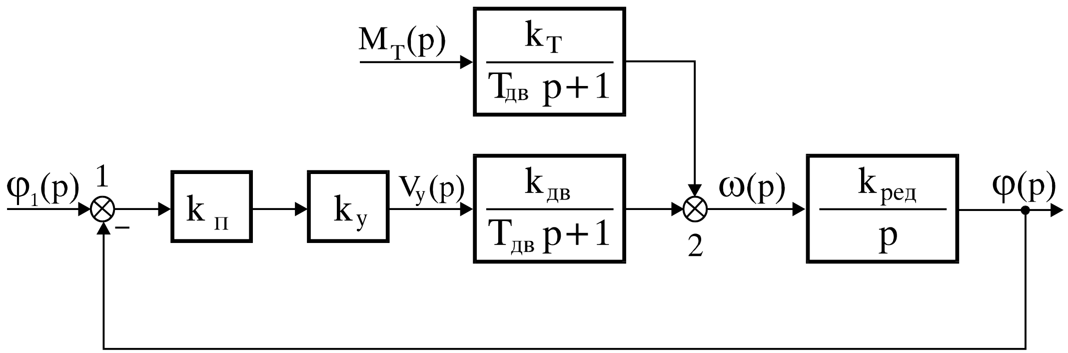


$$k = k_п k_у k_{\partial в} k_{ред}$$

$$W_{зс}(p) = \frac{W(p)}{1 + W(p)} = \frac{k}{T_{\partial в} p^2 + p + k}$$

Система със смущаващи въздействия

$$T_{\partial e}\frac{d\omega}{dt}+\omega=k_{\partial e}v_y-\frac{R}{k_M k_e}M_T$$



$$\omega(p)=\frac{k_{\partial e}}{T_{\partial e}p+1}V_y(p)+\frac{k_T}{T_{\partial e}p+1}M_T(p)$$

$$k_T=-\frac{R}{k_M k_e}$$

$$W_{\omega,V_y}(p)=\frac{k_{\partial e}}{T_{\partial e}p+1};\ W_{\omega,M_T}(p)=\frac{k_T}{T_{\partial e}p+1}$$

$$W_{\varphi,M_T}(p)=\frac{k_T k_{pe\partial}}{T_{\partial e}p^2+p+k}$$

$$\varphi(p)=W_{\varphi,\varphi_1}(p)\varphi_1(p)+W_{\varphi,M_T}(p)M_T(p)$$

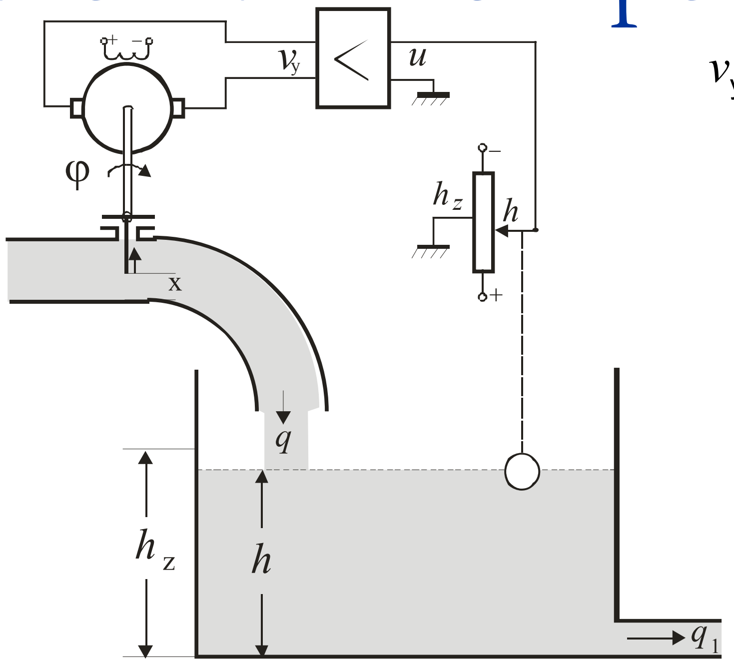
Регулиране на ниво в резервоар

$$W(p) = \frac{k_3}{p}$$

$$x(t) = k_4 \varphi(t)$$

$$q(t) = k_5 x(t)$$

$$W(p) = \frac{k_6}{Tp + 1}$$



$$v_y(t) = k_2 u(t)$$

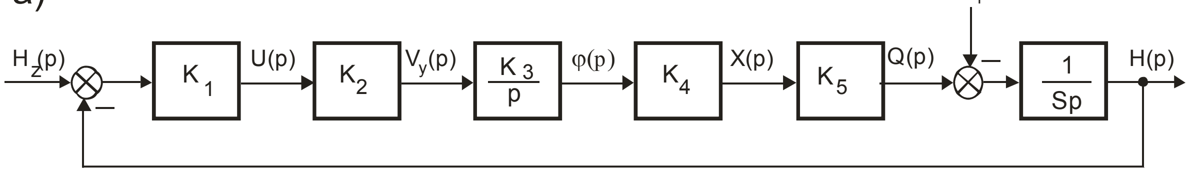
$$W(p) = k_2$$

$$u(t) = k_1 \left[h_z - h(t) \right]$$

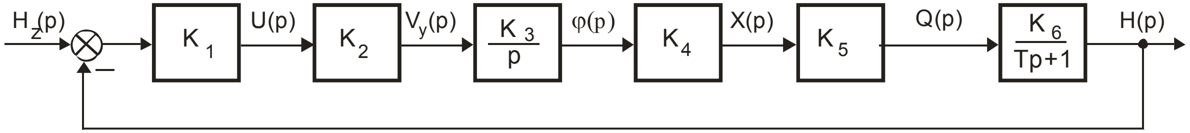
$$h_z - h(t) \quad H_z(p) - H(p)$$

$$W(p) = k_1$$

а)

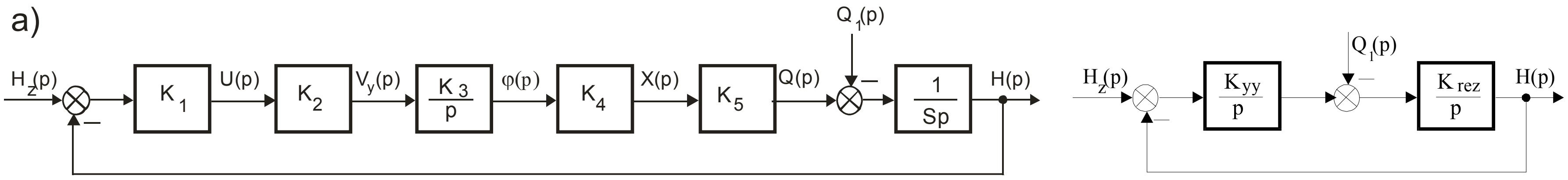


б)

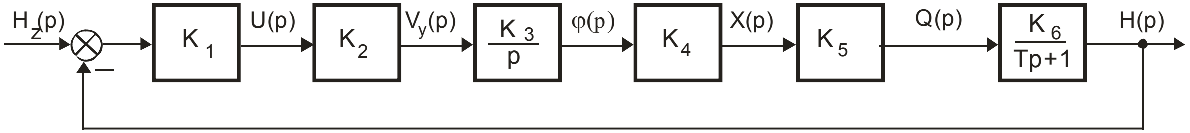


Регулиране на ниво в резервоар

а)



б)



$$K_{rez} = \frac{1}{S}$$

$$K_{yy} = K_1 K_2 K_3 K_4 K_5$$

$$W(p) = K_1 K_2 \frac{K_3}{p} K_4 K_5 \frac{K_6}{Tp+1} =$$

$$= \frac{K}{p(Tp+1)}$$

$$W_{y,v}(p) = \frac{H(p)}{H_z(p)} \qquad W_{y,v}(p) = \frac{\frac{k_{yy}k_{rez}}{p^2}}{1+\frac{k_{yy}k_{rez}}{p^2}} = \frac{k_{yy}k_{rez}}{p^2 + k_{yy}k_{rez}}$$

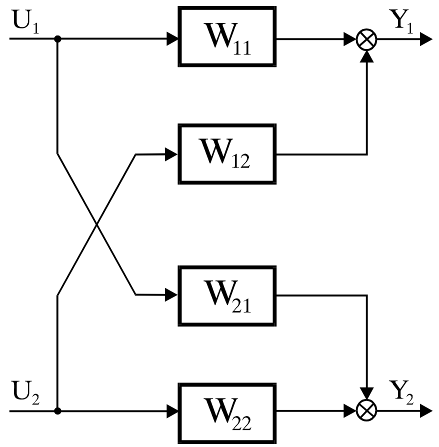
$$K = K_1 K_2 K_3 K_4 K_5 K_6$$

$$W_{y,z}(p) = \frac{H(p)}{Q_1(p)} \qquad W_{y,z}(p) = \frac{\frac{k_{rez}}{p}}{1+\frac{k_{yy}k_{rez}}{p^2}} = \frac{k_{rez}p}{p^2 + k_{yy}k_{rez}}$$

Многомерни системи

$$Y_1(p) = W_{11}(p)U_1(p) + W_{12}(p)U_2(p)$$
$$Y_2(p) = W_{21}(p)U_1(p) + W_{22}(p)U_2(p)$$

$$\mathbf{U}(p) = \begin{bmatrix} U_1(p) \\ \vdots \\ U_r(p) \end{bmatrix}$$



$$\mathbf{Y}(p) = \begin{bmatrix} Y_1(p) \\ \vdots \\ Y_l(p) \end{bmatrix}$$

$$\mathbf{Y}(p) = \mathbf{W}(p)\mathbf{U}(p)$$

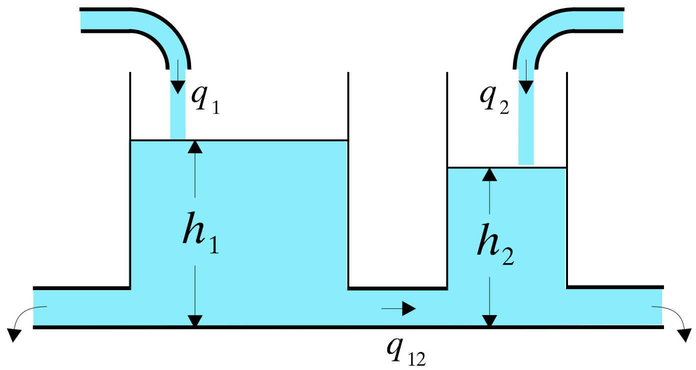
$$\mathbf{W}(p) = \begin{bmatrix} W_{11}(p) & \cdots & W_{1r}(p) \\ \vdots & \vdots & \vdots \\ W_{l1}(p) & \cdots & W_{lr}(p) \end{bmatrix}$$

Скачени резервоари

$$S_1\frac{dh_1}{dt}+c_1h_1(t)=q_1(t)-q_{12}(t)$$

$$S_2\frac{dh_2}{dt}+c_2h_2(t)=q_2(t)+q_{12}(t)$$

$$q_{12}=c_{12}(h_1-h_2)$$



$$(S_1p+c_1+c_{12})h_1(p)-c_{12}h_2(p)=q_1(p)$$

$$-c_{12}h_1(p)+(S_2p+c_2+c_{12})h_2(p)=q_2(p)$$

$$h_1(p)=\frac{1}{S_1p+c_1+c_{12}}[q_1(p)+c_{12}h_2(p)]$$

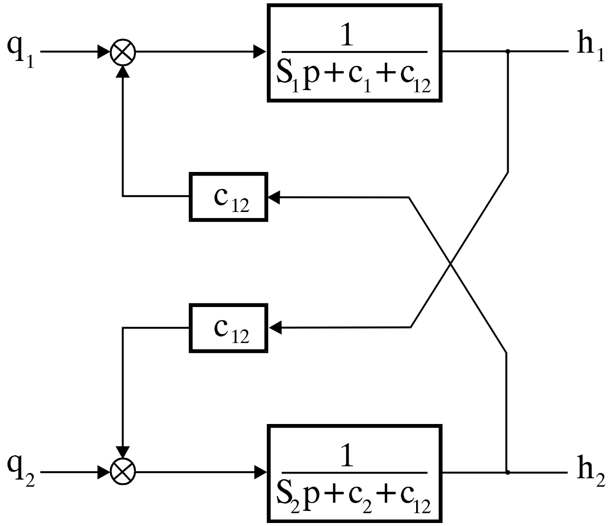
$$h_2(p)=\frac{1}{S_2p+c_2+c_{12}}[q_2(p)+c_{12}h_1(p)]\quad ,$$

Скачени резервоари

$$W_{11}(p) = \frac{h_1(p)}{q_1(p)} = \frac{S_2 p + c_2 + c_{12}}{D(p)}$$

$$W_{22}(p) = \frac{h_2(p)}{q_2(p)} = \frac{S_1 p + c_1 + c_{12}}{D(p)}$$

$$W_{12}(p) = W_{21}(p) = \frac{c_{12}}{D(p)}$$



$$S_1 = 8m^2$$

$$S_2 = 3m^2$$

$$c_1 = c_2 = c_{12} = 0.5\, m^2/s$$

$$D(p) = S_1 S_2 p^2 + [S_1(c_2 + c_{12}) + S_2(c_1 + c_{12})]p + c_1 c_2 + c_1 c_{12} + c_2 c_{12}$$

$$\mathbf{W}(p) = \begin{bmatrix} \dfrac{3p+1}{24p^2 + 11p + 0.75} & \dfrac{0.5}{24p^2 + 11p + 0.75} \\ \dfrac{0.5}{24p^2 + 11p + 0.75} & \dfrac{8p+1}{24p^2 + 11p + 0.75} \end{bmatrix}$$

Скачени резервоари

$$\mathbf{A}(p)\mathbf{Y}(p) = \mathbf{B}(p)\mathbf{U}(p)$$

$$\mathbf{W}(p) = \mathbf{A}^{-1}(p)\mathbf{B}(p)$$

$$\mathbf{A}(p) = \begin{bmatrix} S_1 p + c_1 + c_{12} & -c_{12} \\ -c_{12} & S_2 p + c_2 + c_{12} \end{bmatrix} \qquad \mathbf{Y}(p) = \begin{bmatrix} h_1(p) \\ h_2(p) \end{bmatrix}$$

$$\mathbf{B}(p) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{U}(p) = \begin{bmatrix} q_1(p) \\ q_2(p) \end{bmatrix}$$