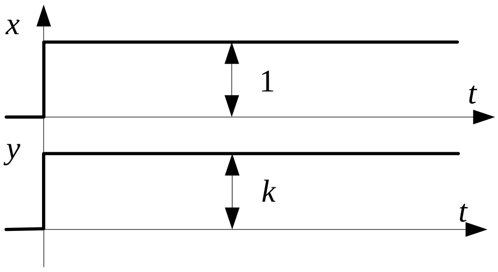
Типови динамични звена
Пропорционално звено

Диференциално уравнение: $y = k.x$

Предавателна функция: $W(p) = k$

Преходна характеристика : $y(t) = L^{-1}\left[\frac{k}{p}\right] = k$

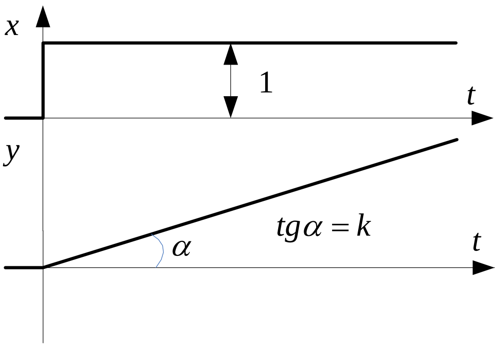


Интегриращо звено

Диференциално уравнение: $\frac{dy}{dt} = k.x$

Предавателна функция: $W(p) = \frac{k}{p}$

Преходна характеристика : $y(t) = L^{-1}\left[\frac{k}{p^2}\right] = k.t$

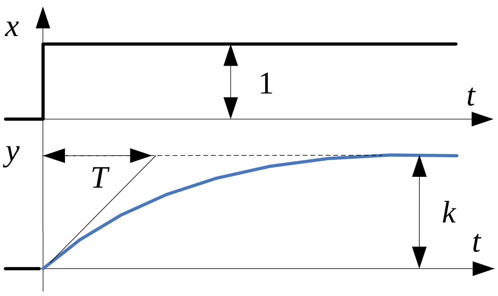


Инерционно звено от първи ред

Диференциално уравнение: $T\frac{dy}{dt} + y = k.x$

Предавателна функция: $W(p) = \frac{k}{Tp+1}$

Преходна характеристика : $y(t) = L^{-1}\left[\frac{k}{p.(T.p+1)}\right] = k.\left(1 - e^{-\frac{t}{T}}\right)$

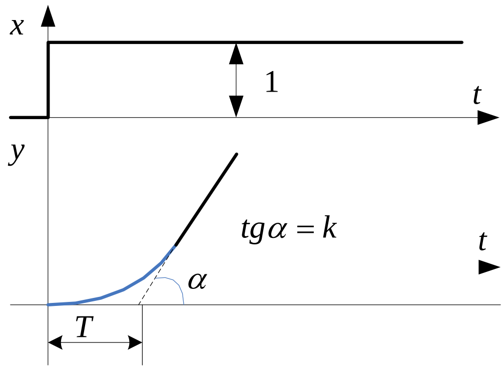


Инерционно интегриращо

Диференциално уравнение: $T\dfrac{d^2y}{dt^2}+\dfrac{dy}{dt}=k.x$

Предавателна функция: $W(p)=\dfrac{k}{p.(Tp+1)}$

Преходна характеристика : $y(t)=L^{-1}\left[\dfrac{k}{p^2.(T.p+1)}\right]=k.\left[t-T\left(1-e^{-\frac{t}{T}}\right)\right]$

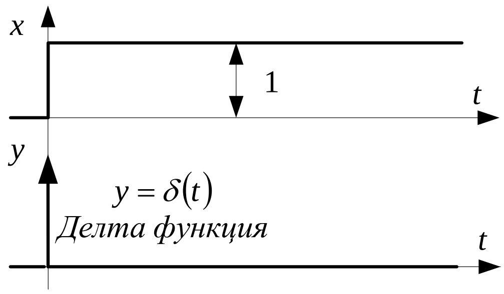


Идеално диференциращо

Диференциално уравнение: $y=k.\dfrac{dx}{dt}$

Предавателна функция: $W(p)=k.p$

Преходна характеристика : $y(t)=L^{-1}[k]$

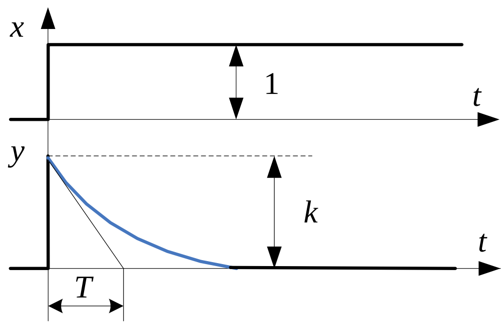


Реално диференциращо

Диференциално уравнение: $T.\dfrac{dy}{dt}+y=k.\dfrac{dx}{dt}$

Предавателна функция: $W(p)=\dfrac{k.p}{T.p+1}$

Преходна характеристика : $y(t)=L^{-1}\left[\dfrac{k}{T.p+1}\right]=\dfrac{k}{T}.e^{-\frac{t}{T}}$

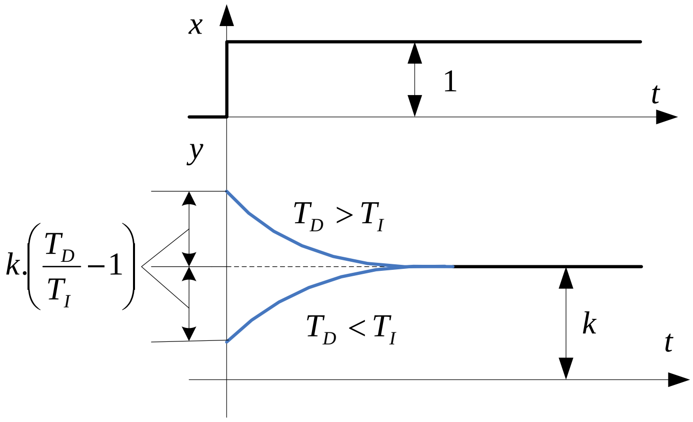


Интегро-диференциращо

Диференциално уравнение: $T_I.\dfrac{dy}{dt}+y=k.\left(T_D.\dfrac{dx}{dt}+x\right)$

Предавателна функция: $W(p)=\dfrac{k.(T_D.p+1)}{T_I.p+1}$

Преходна характеристика : $y(t)=L^{-1}\left[\dfrac{k.(T_D.p+1)}{p.(T_I.p+1)}\right]=k.\left[1+e^{-\frac{t}{T_I}}.\left(\dfrac{T_D}{T_I}-1\right)\right]$

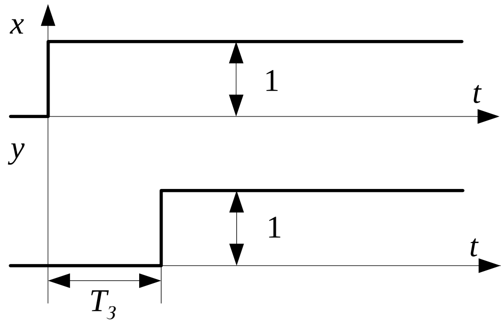


Закъснително

Предавателна функция: $W(p)=e^{-p.T_3}$

Преходна характеристика : $y(t)=L^{-1}\left[\dfrac{1}{p}.e^{-p.T_3}\right]$

Теорема за закъснението : $L[f(t-T_3)]=e^{-p.T_3}.L[f(t)]$



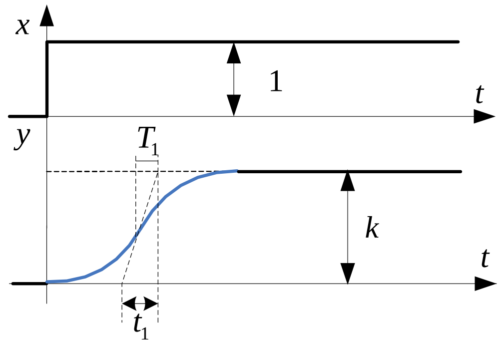
Инерционно звено от втори ред

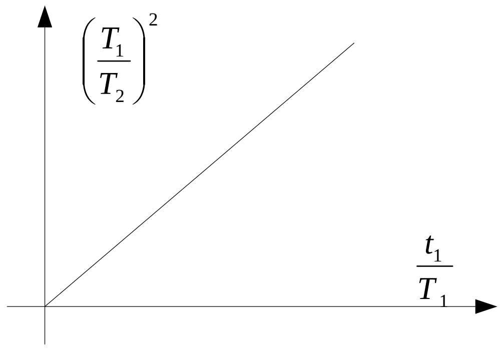
Диференциално уравнение: $T_2^2.\dfrac{d^2y}{dt^2}+T_1.\dfrac{dy}{dt}+y=k.x$ при $T_1>2.T_2$ - инерционно звено; два реални корена - $-b\pm a$, където $b=\dfrac{T_1}{2.T_2^2}$; $a=\dfrac{1}{2.T_2^2}\sqrt{T_1^2-4.T_2^2}$

Предавателна функция: $W(p)=\dfrac{k}{T_2^2.p^2+T_1p+1}$

Преходна характеристика : $y(t)=k.[1+C_1.e^{(a-b).t}+C_2.e^{-(a+b).t})]$,
където

$$C_1=\frac{k}{3.T_2^2(a-b)^2+2.T_1(a-b)+1};\quad C_2=\frac{k}{3.T_2^2(-a-b)^2+2.T_1(-a-b)+1}$$





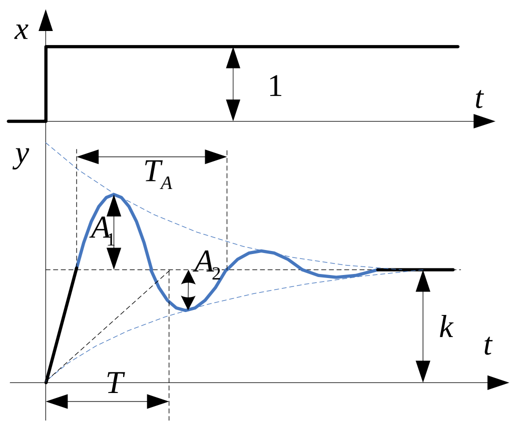
Колебателно звено

Диференциално уравнение: $T_2^2.\frac{d^2y}{dt^2}+T_1.\frac{dy}{dt}+y=k.x$ при $T_1<2.T_2$ - колебателно звено; два комплексни корена $-b\pm i.a$, където $b=\frac{T_1}{2.T_2^2}$; $a=\frac{1}{2.T_2^2}\sqrt{T_1^2-4.T_2^2}$

Предавателна функция: $W(p)=\frac{k}{T_2^2.p^2+T_1p+1}$

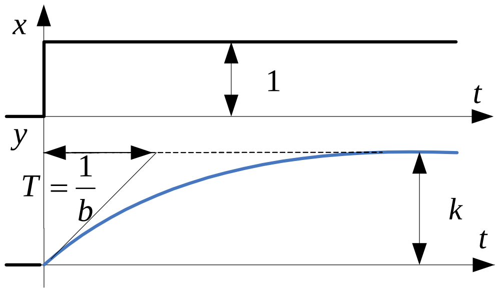
Преходна характеристика : $y(t)=L^{-1}\left[\frac{k}{p.(T_2^2.p^2+T_1p+1)}\right]=k.\left[1-e^{-b.t}\left(\cos a.t+\frac{b}{a}.\sin a.t\right)\right]$

Ако $b=\frac{1}{T}$; $a=\omega$ $\quad y(t)=k.\left[1-e^{-\frac{t}{T}}\left(\cos\omega.t+\frac{1}{\omega.T}.\sin\omega.t\right)\right]$



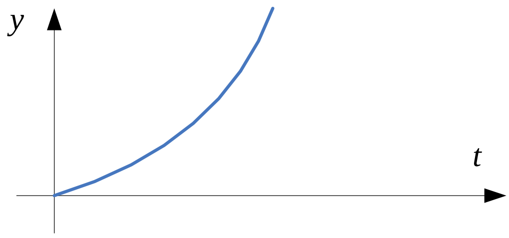
$$a=\frac{2.\pi}{T_A};\quad \frac{A_2}{A_1}=e^{-\frac{b.T_A}{2}}\rightarrow b=\frac{a}{\pi}\ln\frac{A_1}{A_2}$$

При $T_1 = 2.T_2 \rightarrow a = 0,\, b = \dfrac{1}{T_2} = \dfrac{1}{T}$ един реален двукратен корен – инерционно звено от първи ред $\quad y(t) = k.\left(1 - e^{-\frac{t}{T}}\right)$

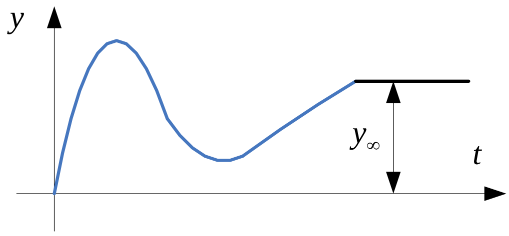


Статични и астатични обекти

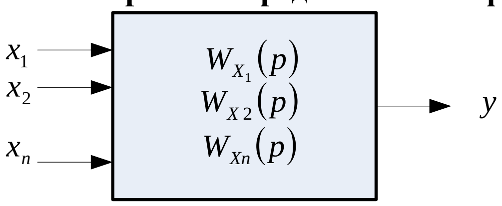
Астатичен: $y(t \rightarrow \infty) = y_\infty \rightarrow \infty$



Статичен : $y(t \rightarrow \infty) = y_\infty = y_{установено}$



Методи за намиране на предавателните функции



Пълен диференциал и частни производни

$$y = F(x_1, x_2 \ldots\ldots x_n) \rightarrow \quad \frac{dy}{dt} = \frac{\partial y}{\partial x_1}.\frac{dx_1}{dt} + \frac{\partial y}{\partial x_2}.\frac{dx_2}{dt} + \ldots + \frac{\partial y}{\partial x_n}.\frac{dx_n}{dt}$$

$$W_{X_1}(p) = \frac{\partial y}{\partial x_1}; \quad W_{X_2}(p) = \frac{\partial y}{\partial x_2} \ldots\ldots W_{X_n}(p) = \frac{\partial y}{\partial x_n}$$

Линеаризация на обектите

$$y_{стац.} + \hat{y}(t) = F\left(x_{1_{стац.л}} + \hat{x}_1(t), x_{2_{стац.}} + \hat{x}_2(t), \ldots x_{n_{стац.}} + \hat{x}_n(t)\right)$$

$$W_{X_1}(p) \approx \frac{\hat{y}(t)}{\hat{x}_1(t)}; \quad W_{X_2}(p) \approx \frac{\hat{y}(t)}{\hat{x}_2(t)} \ldots\ldots W_{X_n}(p) \approx \frac{\hat{y}(t)}{\hat{x}_n(t)}$$

Примери

Понижаващ DC/DC преобразувател

$$U_O = \delta.U_I \rightarrow \frac{dU_O}{dt} = U_I.\frac{d\delta}{dt} + \delta.\frac{dU_I}{dt}$$

$$W_\delta(p) = U_I; \quad W_{U_I}(p) = \delta$$

$$U_O + \hat{U}_O(t) = \left(\delta + \hat{\delta}(t)\right).\left(U_I + \hat{U}_I(t)\right) \rightarrow \hat{U}_O(t) = U_I.\hat{\delta}(t) + \delta.\hat{U}_I(t)$$

Повишаващ DC/DC преобразувател

$$U_O = \frac{1}{1-\delta}.U_I \rightarrow \frac{dU_O}{dt} = \frac{1}{1-\delta}.\frac{dU_I}{dt} + U_I.\frac{1}{(1-\delta)^2}.(-1).\frac{d\delta}{dt}$$

$$W_\delta(p) = -U_I.\frac{1}{(1-\delta)^2}; \quad W_{U_I}(p) = \frac{1}{1-\delta}$$

$$U_O + \hat{U}_O(t) = \frac{1}{1-(\delta+\hat{\delta}(t))}.(U_I + \hat{U}_I(t)) \rightarrow \hat{U}_O(t) = \frac{1}{1-\delta}.\hat{U}_I(t) - U_I.\frac{1}{(1-\delta)^2}.\hat{\delta}(t)$$

Управляем тиристорен токоизправител с активно-индуктивен товар

$$U_{D_\alpha} = U_{D_0}.\cos\alpha - \left(R_I + m.f.L_S\right).I_D = K_1.U_{2F}.\cos\alpha - K_2.I_D$$

$$\frac{dU_{D\alpha}}{dt} = -K_1.U_{2F}.\sin\alpha.\frac{d\alpha}{dt} + K_1.\cos\alpha.\frac{dU_{2F}}{dt} - K_2.\frac{dI_D}{dt}$$

$$W_\alpha(p) = -K_1.U_{2F}.\sin\alpha; \quad W_{U_{2F}}(p) = K_1.\cos\alpha; \quad W_{I_D}(p) = -K_2$$

Еднофазен тиристорен променливотоков регулатор с активен товар

$$U_L = U_2.\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2.\pi}}$$

$$\frac{dU_L}{dt} = \sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2.\pi}}.\frac{dU_2}{dt} + U_2.\frac{1}{2}.\frac{1}{\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2.\pi}}}\left(-\frac{1}{\pi}+\frac{1}{2.\pi}\cos 2\alpha.2\right).\frac{d\alpha}{dt}$$

$$\frac{dU_L}{dt} = \sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2.\pi}}.\frac{dU_2}{dt} + U_2.\frac{1}{2.\pi}.\frac{(\cos 2\alpha - 1)}{\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2.\pi}}}.\frac{d\alpha}{dt}$$

$$W_{U_2}(p) = \sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2.\pi}}; \quad W_\alpha(p) = U_2.\frac{1}{2.\pi}.\frac{(\cos 2\alpha - 1)}{\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2.\pi}}}$$

Еднофазен регулируем транзисторен инвертор на напрежение

$$U_L = U_D.\sqrt{1-\frac{\alpha}{\pi}} \rightarrow \frac{dU_L}{dt} = \sqrt{1-\frac{\alpha}{\pi}}.\frac{dU_D}{dt} + U_D.\frac{1}{2}.\frac{1}{\sqrt{1-\frac{\alpha}{\pi}}}\left(-\frac{1}{\pi}\right).\frac{d\alpha}{dt}$$

$$W_{U_D}(p) = \sqrt{1-\frac{\alpha}{\pi}}.; \quad W_\alpha(p) = -U_D.\frac{1}{2.\pi}.\frac{1}{\sqrt{1-\frac{\alpha}{\pi}}}$$