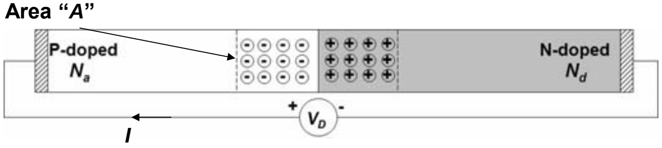
Lecture 7

Large and Small Signal Modelling of PN Junction Diodes

In this lecture you will learn:

- Circuit models of PN junction diodes
- Small signal modeling of nonlinear circuit elements
- Small signal models of PN junction diodes
- Junction resistance and capacitances

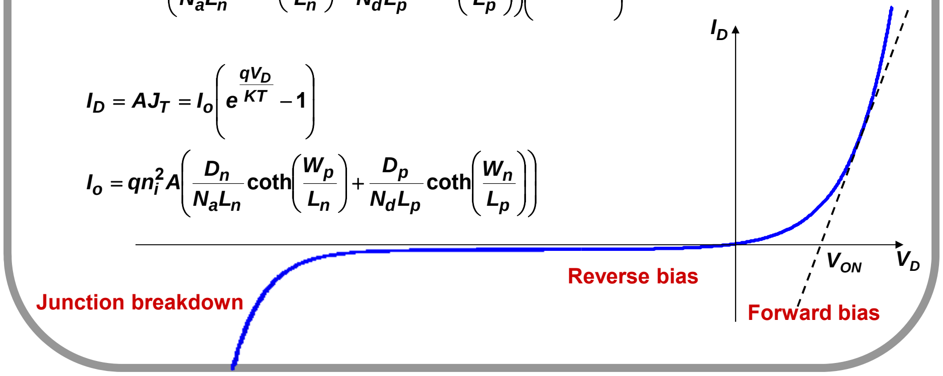
Current Flow in a PN Junction Diode



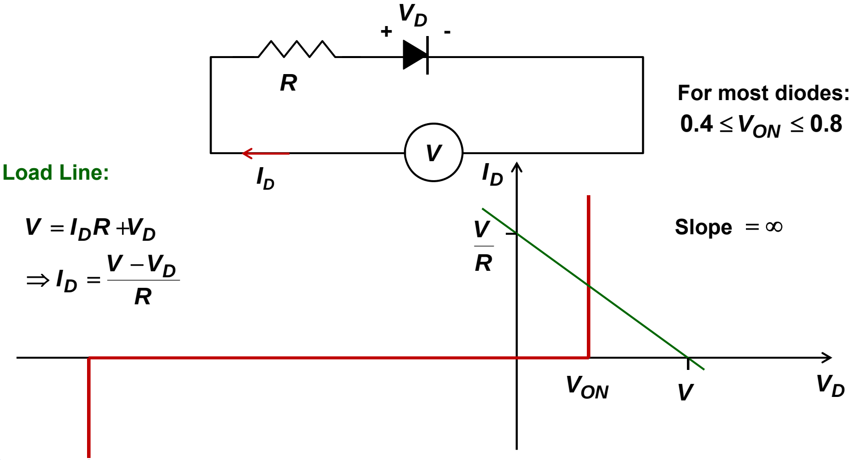
$$J_T = q n_i^2 \left(\frac{D_n}{N_a L_n} \coth\left(\frac{W_p}{L_n}\right) + \frac{D_p}{N_d L_p} \coth\left(\frac{W_n}{L_p}\right) \right) \left(e^{\frac{qV_D}{KT}} - 1 \right)$$

$$I_D = A J_T = I_o \left(e^{\frac{qV_D}{KT}} - 1 \right)$$

$$I_o = q n_i^2 A \left(\frac{D_n}{N_a L_n} \coth\left(\frac{W_p}{L_n}\right) + \frac{D_p}{N_d L_p} \coth\left(\frac{W_n}{L_p}\right) \right)$$



Simplest Circuit Model for a PN Junction Diode



For most diodes:
$0.4 \le V_{ON} \le 0.8$

Load Line:

$$V = I_D R + V_D$$
$$\Rightarrow I_D = \frac{V - V_D}{R}$$

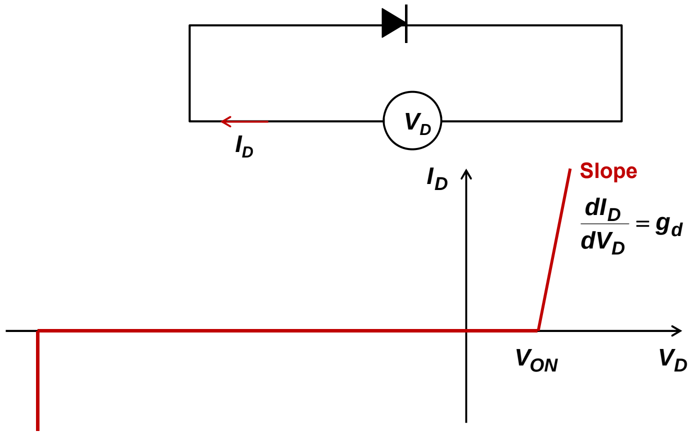
Slope $= \infty$

Solution for current:

If $V \le V_{ON}$:
$$\begin{cases} V_D = V \\ I_D \approx 0 \end{cases}$$

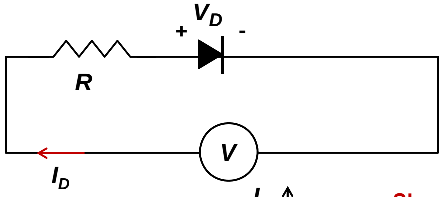
If $V > V_{ON}$:
$$\begin{cases} V_D = V_{ON} \\ I_D = \frac{V - V_{ON}}{R} \end{cases}$$

Better Circuit Model a PN Junction Diode



Slope
$$\frac{dI_D}{dV_D} = g_d$$

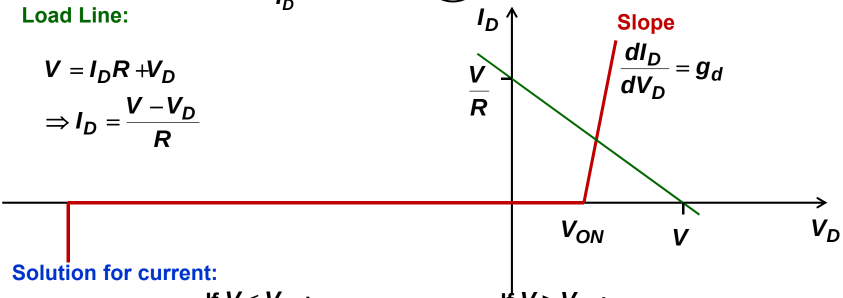
Circuit Example for a PN Junction Diode



Load Line:

$$V = I_D R + V_D$$

$$\Rightarrow I_D = \frac{V - V_D}{R}$$



Solution for current:

If $V \le V_{ON}$:

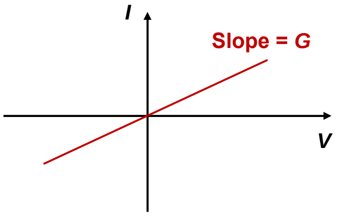
$$\begin{cases} V_D = V \\ I_D \approx 0 \end{cases}$$

If $V > V_{ON}$:

$$\begin{cases} V_D = \dfrac{V}{1+g_d R} + V_{ON}\dfrac{g_d R}{1+g_d R} \\ I_D = (V - V_{ON})\dfrac{g_d}{1+g_d R} \end{cases}$$

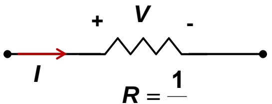
Linear Circuit Elements

Ohm's Law:



$$I = \frac{V}{R} = GV$$

Ohm's law implies a LINEAR relationship between current and voltage

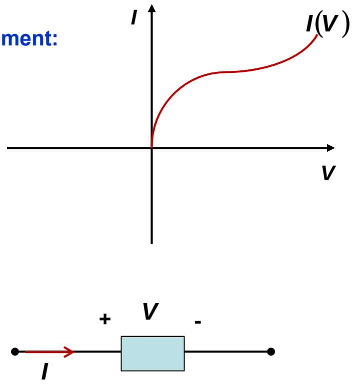


$$R = \frac{1}{G}$$

The current-voltage relationship of resistors is linear

Nonlinear Circuit Elements

Nonlinear Element:



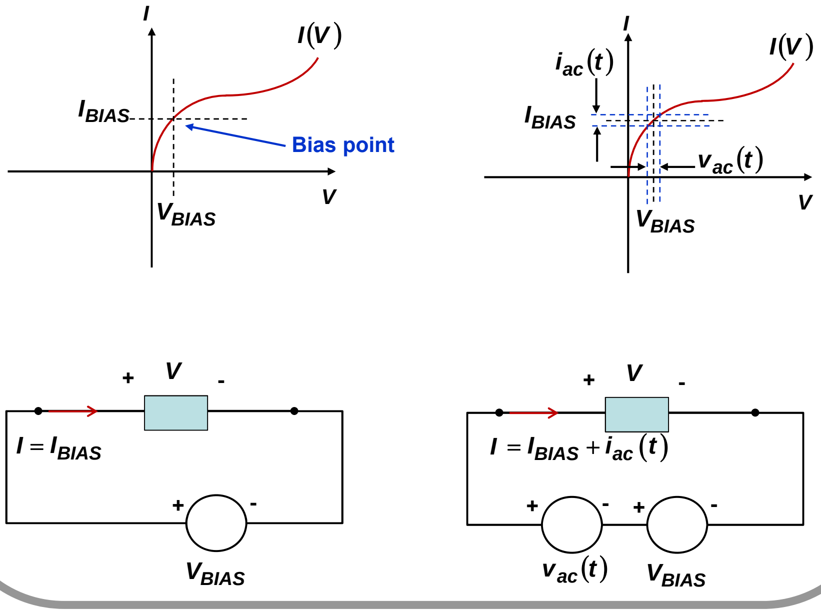
Current is a function of the voltage (but the current-voltage relationship is not linear)

For example:

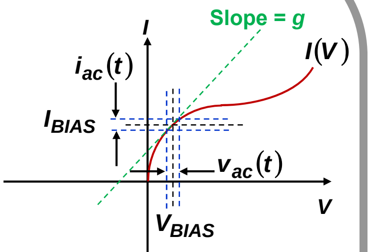
$$I(V) = A\sqrt{|V|} + Be^{CV}$$

The current-voltage relationship of most devices is not linear!

Small Signal Modeling of Nonlinear Circuit Elements



Small Signal Modeling of Nonlinear Circuit Elements



Taylor expand the current-voltage relation around the bias voltage:

$$I_{BIAS} + i_{ac}(t) = I(V_{BIAS} + v_{ac}(t))$$

$$= I(V_{BIAS}) + \left.\frac{dI}{dV}\right|_{V=V_{BIAS}} v_{ac}(t) + \overbrace{\frac{1}{2}\left.\frac{d^2I}{dV^2}\right|_{V=V_{BIAS}} v_{ac}^2(t)}^{\text{Assume small}} + \ldots\ldots$$

$$I_{BIAS} + i_{ac}(t) \approx I(V_{BIAS}) + \left.\frac{dI}{dV}\right|_{V=V_{BIAS}} v_{ac}(t)$$

$$= I(V_{BIAS}) + g v_{ac}(t)$$

$$\Rightarrow i_{ac}(t) \approx g v_{ac}(t)$$

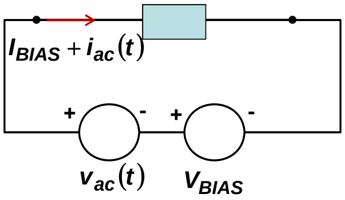
Differential resistance or differential conductance

$$\frac{1}{r} = g = \left.\frac{dI}{dV}\right|_{V=V_{BIAS}}$$

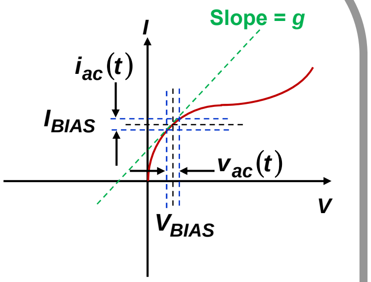
Incremental resistance or incremental conductance

Small Signal Modeling of Nonlinear Circuit Elements

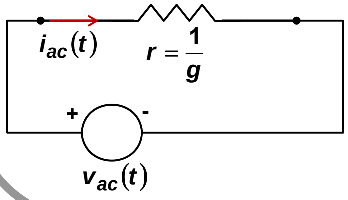
Complete circuit is:



$$i_{ac}(t) \approx g v_{ac}(t)$$



Small signal equivalent circuit is:

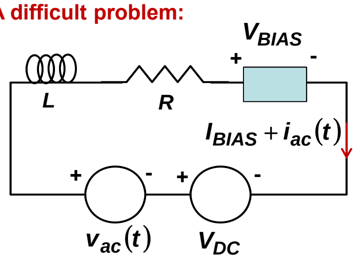


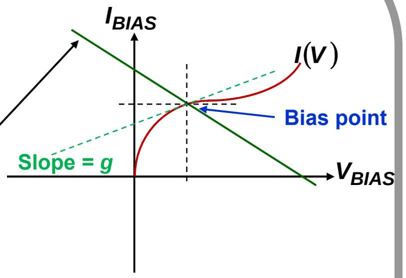
$$i_{ac}(t) \approx g v_{ac}(t)$$

In small signal models, nonlinear circuit elements are replaced by their linearized models that are valid over a limited range of excursion around the bias point

Small Signal Modeling of Nonlinear Circuit Elements

A difficult problem:



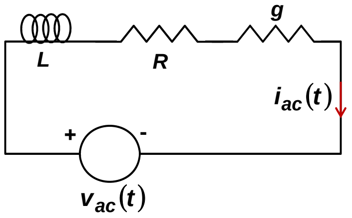


DC Load Line:

$$V_{DC} = I_{BIAS}R + V_{BIAS}$$

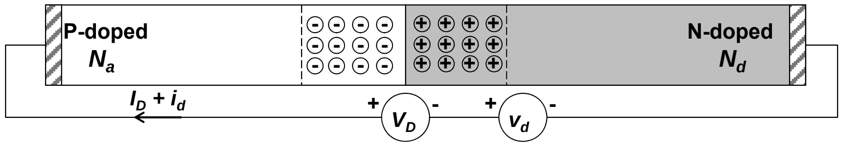
$$\Rightarrow I_{BIAS} = \frac{V_{DC} - V_{BIAS}}{R}$$

A simpler problem:



$$\left\{ \frac{1}{r} = g = \left.\frac{dI}{dV}\right|_{V=V_{BIAS}} \right.$$

Small Signal Model of a PN Junction Diode: Junction Conductance



$$I_D = I_o\left(e^{\frac{qV_D}{KT}} - 1\right)$$

$$\Rightarrow I_D + i_d = I_o\left(e^{\frac{q(V_D + v_d)}{KT}} - 1\right) \approx I_D + \frac{\partial I_D}{\partial V_D} v_d + \dots\dots\dots = I_D + g_d v_d$$

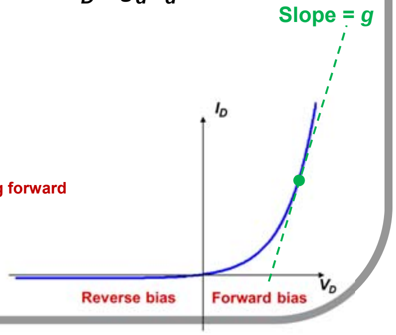
$$\Rightarrow i_d = g_d v_d$$

$$g_d = \frac{1}{r_d} = \frac{\partial I_D}{\partial V_D} = \frac{qI_o}{KT} e^{\frac{qV_D}{KT}} = \frac{q(I_D + I_o)}{KT} \approx \underbrace{\frac{qI_D}{KT}}$$

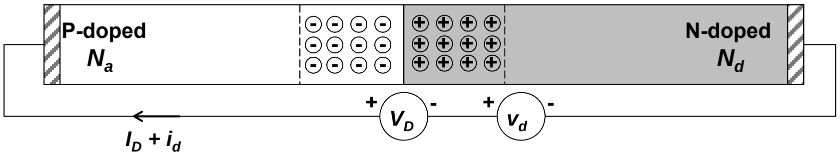
In strong forward bias

Differential resistance

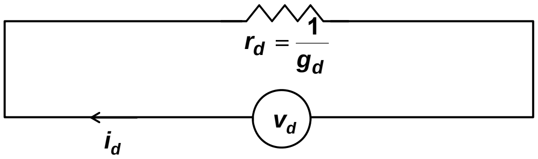
Differential conductance



Small Signal Model of a PN Junction Diode: Junction Conductance



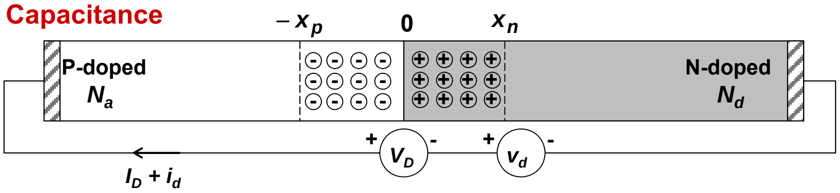
$$I_D + i_d \approx I_D + g_d v_d$$
$$\Rightarrow i_d \approx g_d v_d$$



$$g_d \approx \frac{qI_D}{KT}$$

Small signal circuit model of a PN diode

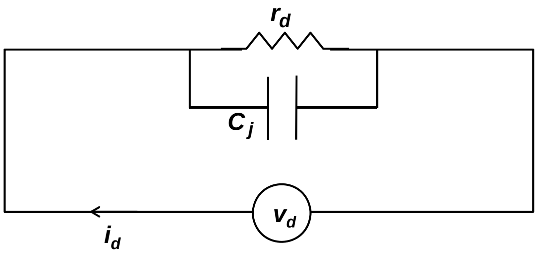
Small Signal Model of a PN Junction Diode: Junction Depletion Capacitance



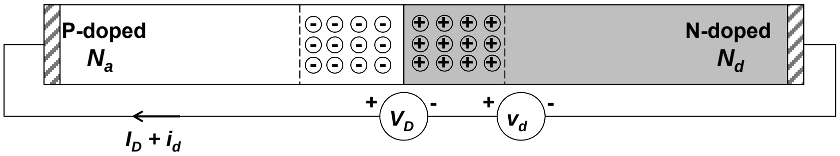
At high frequencies, part of the current i_d flows through the junction but part of it also charges up the junction capacitance

$$i_d \approx g_d v_d + C_j \frac{dv_d}{dt}$$

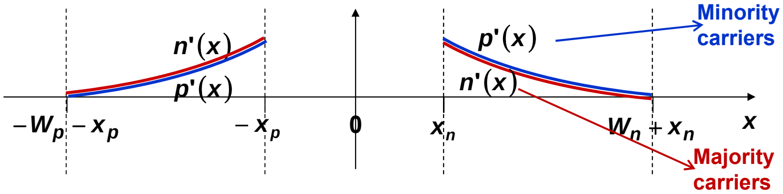
$$C_j = \frac{\varepsilon_s A}{(x_p + x_n)}$$



Small Signal Model of a PN Junction Diode: Diffusion Capacitance



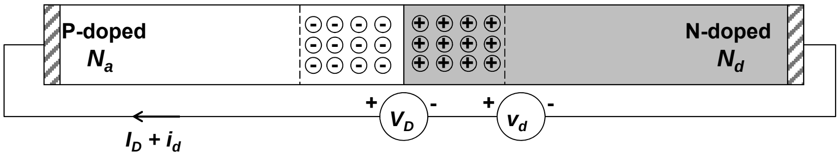
There is also charge stored in the quasi-neutral regions that changes as the junction voltage is varied (negative and positive charge stored at the same location!!)



Charge stored:

$$Q_d = qA\int_{x_n}^{W_n+x_n} p'(x)dx + qA\int_{-W_p-x_p}^{-x_p} p'(x)dx$$

Small Signal Model of a PN Junction Diode: Diffusion Capacitance



Charge stored: $Q_d = qA\int_{x_n}^{W_n+x_n} p'(x)dx + qA\int_{-W_p-x_p}^{-x_p} p'(x)dx$

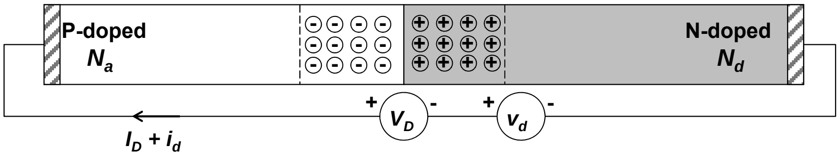
Diffusion Capacitance: $C_d = \dfrac{\partial Q_d}{\partial V_D}$

$$C_d = \frac{q^2 A}{KT} e^{\frac{qV_D}{KT}} \left[\frac{n_i^2}{N_a} L_n \frac{\cosh\left(\frac{W_p}{L_n}\right) - 1}{\sinh\left(\frac{W_p}{L_n}\right)} + \frac{n_i^2}{N_d} L_p \frac{\cosh\left(\frac{W_n}{L_p}\right) - 1}{\sinh\left(\frac{W_n}{L_p}\right)} \right]$$

↑

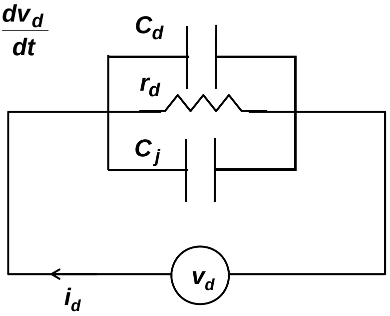
Increases exponentially with bias!

Small Signal Model of a PN Junction Diode: Total Capacitance



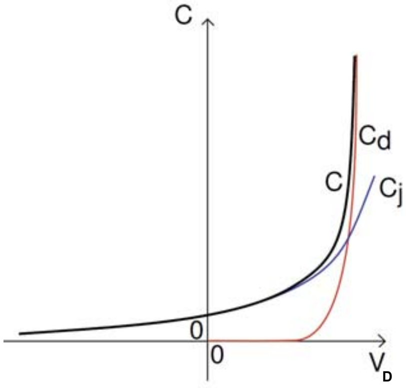
At high frequencies, part of the current i_d flows through the junction but part of it also charges up the junction capacitance and the diffusion capacitance

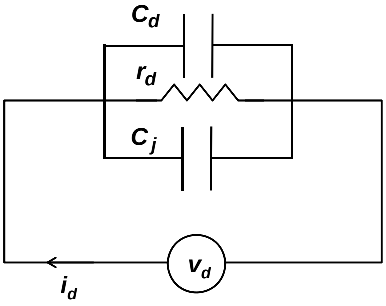
$$i_d \approx g_d v_d + \left(C_j + C_d\right)\frac{dv_d}{dt}$$



Capacitances of a PN Junction Diode

Total Capacitance: $C = C_j + C_d$

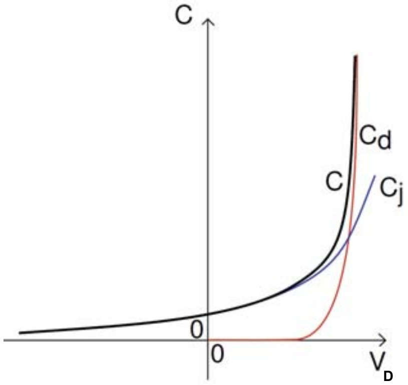


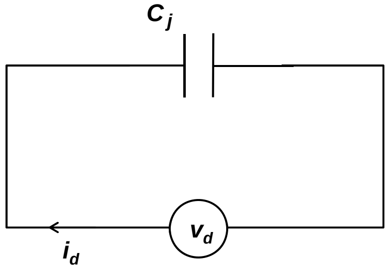


Small Signal Model of a PN Junction Diode in Revere Bias

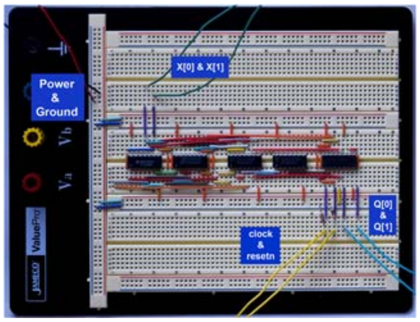
$$g_d = \frac{1}{r_d} = \frac{q(I_D + I_o)}{KT} \approx 0$$

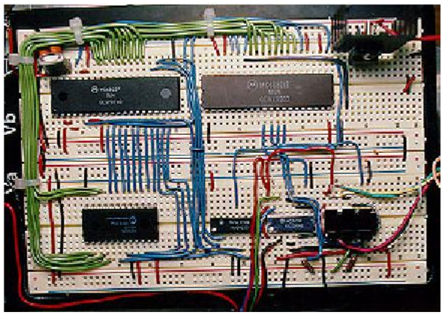
$$C_d \approx 0$$





Breadboard Wiring: Good Wiring





Breadboard Wiring: Bad Wiring