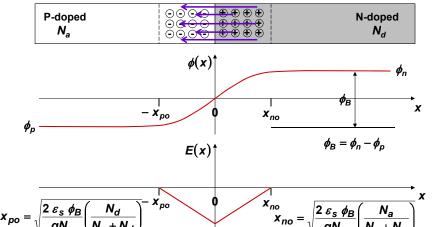
Lecture 6

Biased PN Junction Diodes and Current Flow

In this lecture you will learn:

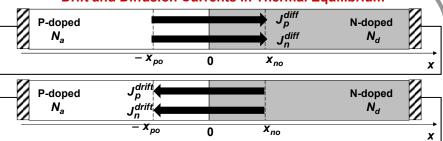
- Biased PN junction diodes (forward biased and reverse biased PN diodes)
- Depletion capacitance of PN junction diodes
- Minority and majority carrier distributions in a biased PN junction diodes
- Carrier transport and current flow in biased PN junction diodes

Review: A PN Junction Diode in Thermal Equilibrium • You have already seen a PN Junction diode in thermal equilibrium:



 \bullet In thermal equilibrium no net current flows in either left or right direction

Drift and Diffusion Currents in Thermal Equilibrium



In thermal equilibrium:

- The electron diffusion current is balanced by the equal and opposite electron drift current
- The hole diffusion current is balanced by the equal and opposite hole drift current

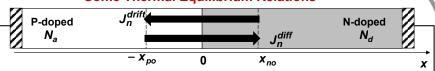
So the net currents of both the electrons as well as the holes are zero!

In thermal equilibrium:

$$J_{p}(x) = J_{p}^{drift}(x) + J_{p}^{diff}(x) = 0$$

$$J_n(x) = J_n^{drift}(x) + J_n^{diff}(x) = 0$$

Some Thermal Equilibrium Relations



• Total electron current is zero in thermal equilibrium:

$$J_n(x) = q n(x) \mu_n E(x) + q D_n \frac{d n(x)}{dx} = 0$$

In thermal equilibrium, these two components balance each other exactly at every point in space so that there is no total electron current anywhere

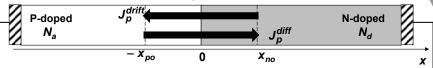
$$\Rightarrow q n(x) \mu_n E(x) = -q D_n \frac{d n(x)}{dx}$$

$$\Rightarrow -n(x) \mu_n \frac{d \phi(x)}{dx} = -D_n \frac{d n(x)}{dx}$$

$$\Rightarrow \frac{\mu_n}{D_n} \frac{d \phi(x)}{dx} = \frac{d \log[n(x)]}{dx}$$

$$\Rightarrow \frac{q}{KT} \frac{d \phi(x)}{dx} = \frac{d \log[n(x)]}{dx} \Rightarrow n(x) = n_i e^{\frac{q\phi(x)}{KT}}$$

Some Thermal Equilibrium Relations



• Total hole current is zero in thermal equilibrium:

$$J_p(x) = q p(x) \mu_p E(x) - q D_p \frac{d p(x)}{dx} = 0$$

In thermal equilibrium, these two components balance each other exactly at every point in space so that there is no total hole current anywhere

$$\Rightarrow q p(x) \mu_{p} E(x) = q D_{p} \frac{d p(x)}{dx}$$

$$\Rightarrow -p(x) \mu_{p} \frac{d \phi(x)}{dx} = D_{p} \frac{d p(x)}{dx}$$

$$\Rightarrow -\frac{\mu_{p}}{D_{p}} \frac{d \phi(x)}{dx} = \frac{d \log[p(x)]}{dx}$$

$$\Rightarrow -\frac{q}{KT} \frac{d \phi(x)}{dx} = \frac{d \log[p(x)]}{dx} \Rightarrow p(x) = n_{i} e^{\frac{-q\phi(x)}{KT}}$$

Carrier Concentrations in Thermal Equilibrium

$$n(x) = n_i e^{\frac{q\phi(x)}{KT}}$$

$$n(x_1) = \frac{q\phi(x)}{q\phi(x)}$$

Another way to write the same equations is:

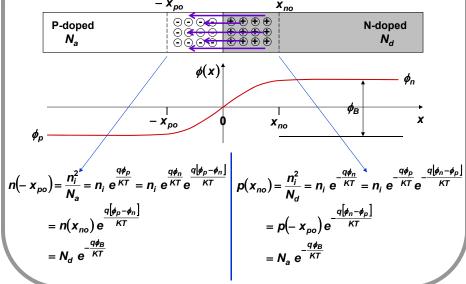
$$n(x_2) = n(x_1) e^{\frac{q[\phi(x_2) - \phi(x_1)]}{KT}}$$

$$p(x_2) = p(x_1) e^{-\frac{q[\phi(x_2) - \phi(x_1)]}{KT}}$$

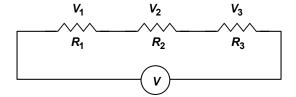
In thermal equilibrium, electron and hole concentrations at different points are related exponentially to the potential difference at these points

A PN Junction Diode in Thermal Equilibrium

• Minority carrier concentrations at the edges of the depletion region:



Voltage Drops in Resistive Networks



Most of the voltage drops across the largest resistor in series

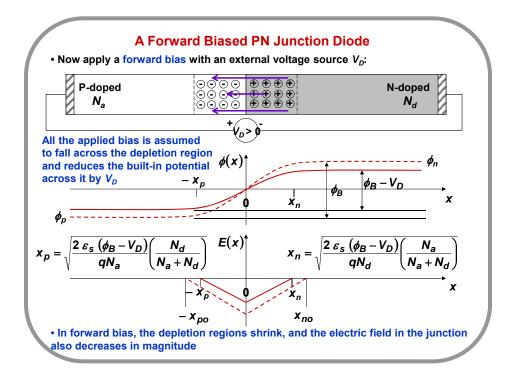
Suppose:

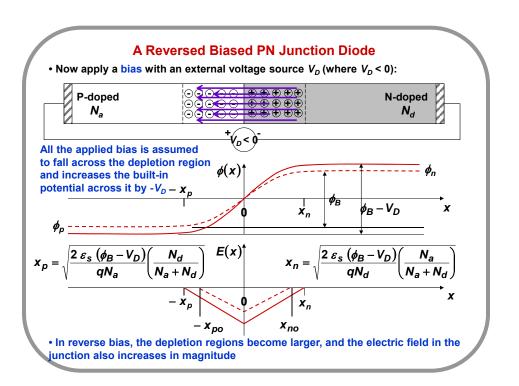
$$R_2 >> R_1$$

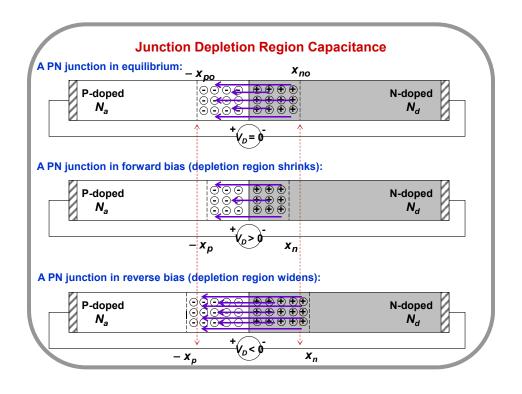
$$R_2 >> R_3$$

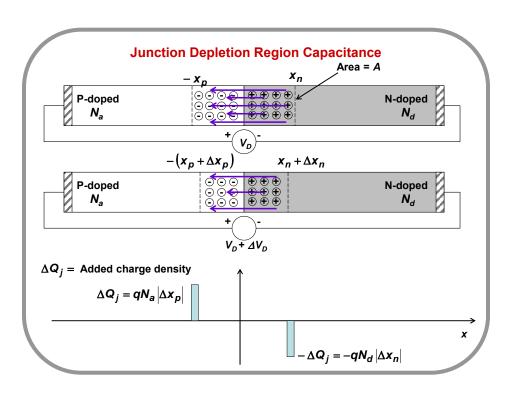
Then:

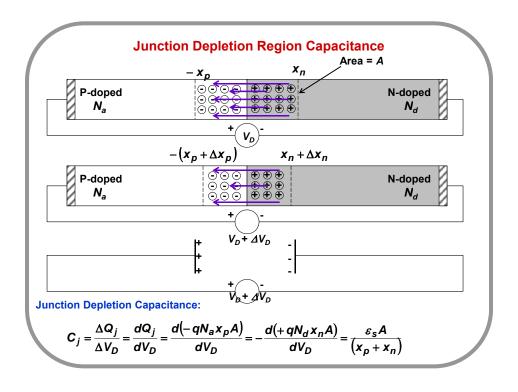
$$V_2 = V \frac{R_2}{R_1 + R_2 + R_3} \approx V$$

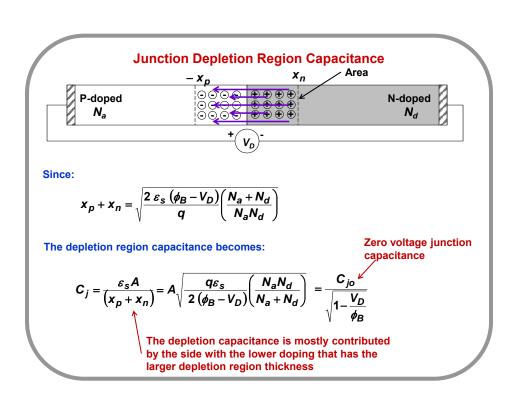


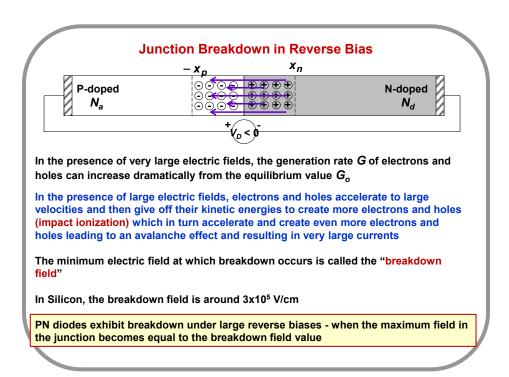


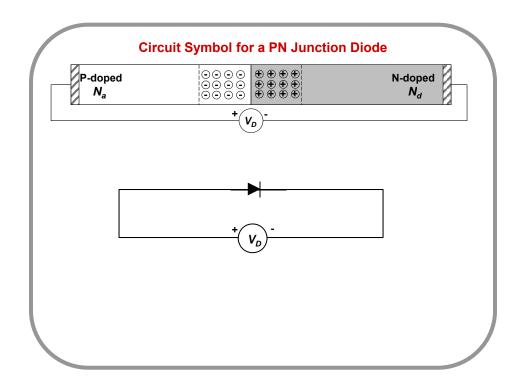


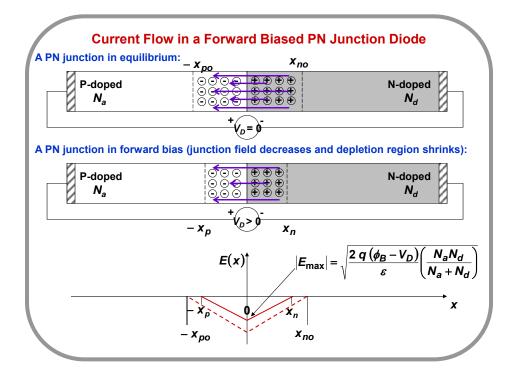


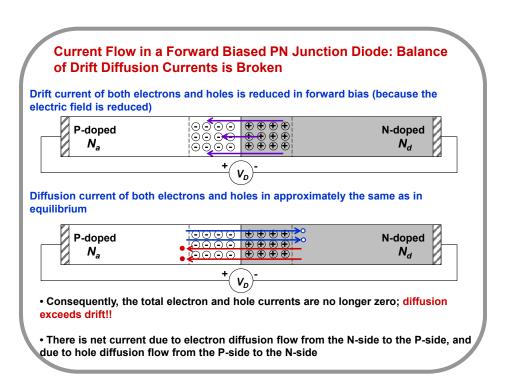






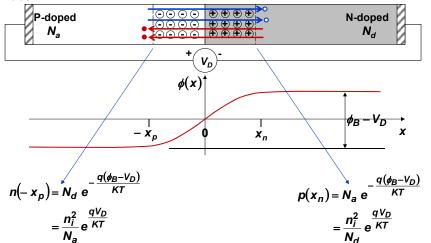






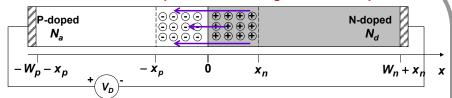
Carrier Injection in a Forward Biased PN Junction Diode

• Minority carrier concentrations at the edges of the depletion region in forward bias:



In forward bias, the minority carrier concentrations increase exponentially at the edges of the depletion region

Fundamental Assumptions In Modeling Carrier Transport



• In forward bias, the main obstacle to current flow is not the depletion region but carrier diffusion in the N- and the P-sides

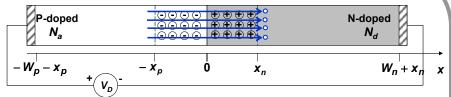
This assumption implies that it is enough to study current flow in the N- and P-sides and not worry about what happens inside the depletion region

• The N-side and the P-side are "quasi-neutral"

The word "quasi-neutral" implies that there is almost no net charge densities inside these regions and, by Gauss's law, almost zero electric fields inside these regions

This assumption is not 100% accurate. The physical reason behind this assumption is that when a material is highly conducting (like the N- and P-sides) the electric field inside it is usually small. How small is small......see the next slide....

Modeling Minority Carrier Diffusion (N-side)



Consider the N-side first:

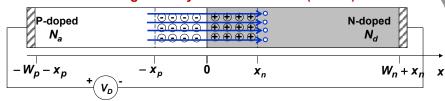
- The holes diffuse from the P-side, cross the depletion region, and enter the N-side
- On the N-side, the holes are the minority carriers
- The dynamics of holes on the N-side in steady state are described by the Shockley equations:

$$\frac{\partial \stackrel{\circ}{p}(x)}{\partial t} = G - R - \frac{1}{q} \frac{\partial J_{p}(x)}{\partial x}$$

$$J_{p}(x) = q p(x) \mu_{p} \stackrel{\circ}{E}(x) - q D_{p} \frac{\partial p(x)}{\partial x} \approx -q D_{p} \frac{\partial p(x)}{\partial x}$$

• The electric fields in the quasi-neutral regions are assumed to be small enough that they may be neglected in modeling minority carrier transport. Therefore, minority carriers flow by diffusion (not drift).

Modeling Minority Carrier Diffusion (N-side)



$$0 = G - R - \frac{1}{q} \frac{\partial J_p(x)}{\partial x} = G - R + D_p \frac{\partial^2 p(x)}{\partial x^2}$$

• Let the total hole density on the N-side be written as:

$$p(x) = p_{no} + p'(x)$$
e density

Excess hole density

$$p_{no} = \frac{n_i^2}{N_d}$$

Equilibrium hole density

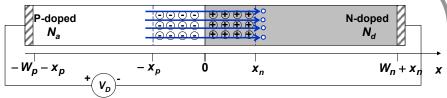
•Then the generation-recombination term becomes:

$$G = G_o$$

$$R = R_o + \frac{p'(x)}{\tau_p}$$

$$G - R = -\frac{p'(x)}{\tau_p}$$





$$0 = G - R - \frac{1}{q} \frac{\partial J_p(x)}{\partial x} = G - R + D_p \frac{\partial^2 p(x)}{\partial x^2}$$

• Let the total hole density on the N-side be written as:

$$p(x) = p_{no} + p'(x)$$
Equilibrium hole density

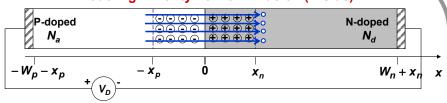
Excess hole density

$$p_{no} = \frac{n_i^2}{N_d}$$

•And we get:

$$\frac{\partial^2 p'(x)}{\partial x^2} - \frac{p'(x)}{D_p \tau_p} = 0$$
 Diffusion equation for the excess hole density

Modeling Minority Carrier Diffusion (N-side)



• We need to solve the second order diffusion equation: $\frac{\partial^2 p'(x)}{\partial x^2} - \frac{p'(x)}{D_p \tau_p} = 0$

with the boundary condition:

$$p'(x_n) = p(x_n) - p_{no} = \frac{n_i^2}{N_d} e^{\frac{qV_D}{KT}} - \frac{n_i^2}{N_d} = \frac{n_i^2}{N_d} \left(e^{\frac{qV_D}{KT}} - 1 \right)$$

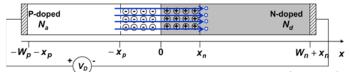
· But it is a second order differential equation, so we need a second boundary condition!

The minority carrier lifetime τ_p is assumed to be zero at the metal contacts.

· Consequently, there cannot be any excess hole density at the right metal contact. This gives us the second boundary condition:

$$p'(W_n+x_n)=0$$

Modeling Minority Carrier Diffusion (N-side)



We need to solve:

eed to solve:

$$\frac{\partial^{2} p'(x)}{\partial x^{2}} - \frac{p'(x)}{D_{p}\tau_{p}} = 0$$

$$p'(x_{n}) = \frac{n_{i}^{2}}{N_{d}} \left(e^{\frac{qV_{D}}{KT}} - 1 \right)$$

$$p'(W_{n} + x_{n}) = 0$$

Define a "minority carrier diffusion length" L_p for holes as: $L_p = \sqrt{D_p \tau_p}$

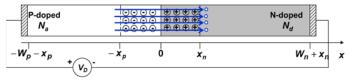
We need to solve:

$$\frac{\partial^2 p'(x)}{\partial x^2} - \frac{p'(x)}{L_p^2} = 0$$

$$p'(x_n) = \frac{n_i^2}{N_d} \left(e^{\frac{qV_D}{KT}} - 1 \right)$$

$$p'(W_n + x_n) = 0$$

Modeling Minority Carrier Diffusion (N-side)



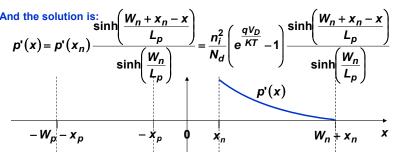
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$$\frac{\partial^2 p'(x)}{\partial x^2} - \frac{p'(x)}{L_p^2} = 0$$

$$p'(x_n) = \frac{n_i^2}{N_d} \left(e^{\frac{qV_D}{KT}} - 1 \right)$$

$$p'(W_n + x_n) = 0$$



The Minority Carrier Diffusion Length (N-Side)

The minority carrier diffusion length L_p is the average length a hole injected into the N-side will diffuse before it finds an electron and recombines with it

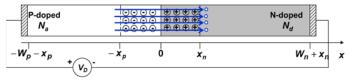
• Long Base Limit:

If $L_p \ll W_p$ then pretty much all the holes injected into the N-side recombine with electrons before they are able to cross the N-side

Short Base Limit:

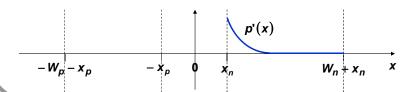
If $L_p >> W_n$ then pretty much all the holes injected into the N-side do not recombine with the electrons and are able to cross the N-side

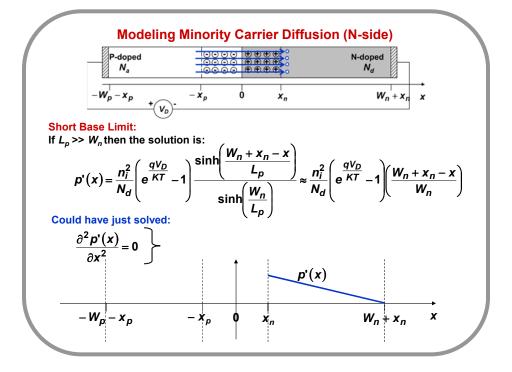
Modeling Minority Carrier Diffusion (N-side)

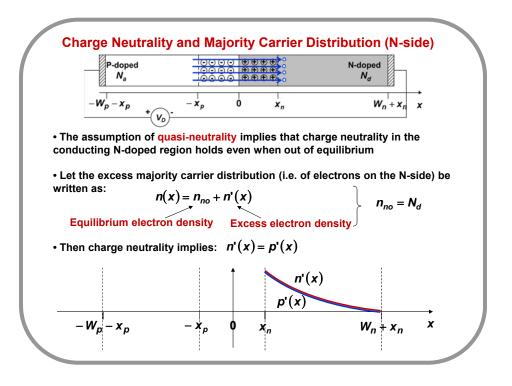


Long Base Limit: If $L_p << W_n$ then the solution is:

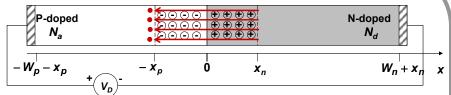
$$p'(x) = \frac{n_i^2}{N_d} \left(e^{\frac{qV_D}{KT}} - 1 \right) \frac{\sinh\left(\frac{W_n + x_n - x}{L_p}\right)}{\sinh\left(\frac{W_n}{L_p}\right)} \approx \frac{n_i^2}{N_d} \left(e^{\frac{qV_D}{KT}} - 1 \right) e^{-\frac{(x - x_n)}{L_p}}$$







Modeling Minority Carrier Diffusion (P-side)



Consider the P-side now:

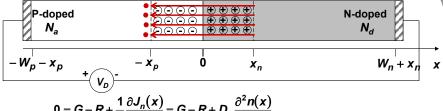
- The electrons diffuse from the N-side, cross the depletion region, and enter the P-side
- On the P-side, the electrons are the minority carriers
- The dynamics of electrons on the P-side in steady state are described by the Shockley equations:

$$\frac{\partial n(x)}{\partial t} = G - R + \frac{1}{q} \frac{\partial J_n(x)}{\partial x}$$

$$J_n(x) = q n(x) \mu_n E(x) + q D_n \frac{\partial n(x)}{\partial x} \approx q D_n \frac{\partial n(x)}{\partial x}$$

• The electric fields in the quasi-neutral regions are assumed to be small enough that they may be neglected in modeling minority carrier transport. Therefore, minority carriers flow by diffusion (not drift).

Modeling Minority Carrier Diffusion (P-side)



$$0 = G - R + \frac{1}{q} \frac{\partial J_n(x)}{\partial x} = G - R + D_n \frac{\partial^2 n(x)}{\partial x^2}$$

· Let the total electron density on the P-side be written as:

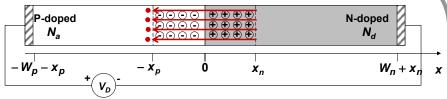
$$n(x) = n_{po} + n'(x)$$
Equilibrium electron density
$$\sum_{i=1}^{n} n_{po} = \frac{n_{i}^{2}}{N_{i}}$$

•Then the generation-recombination term becomes:

•And we get:

$$\frac{\partial^2 n'(x)}{\partial x^2} - \frac{n'(x)}{D_n \tau_n} = 0$$
 Diffusion equation for the excess electron density

Modeling Minority Carrier Diffusion (P-side)



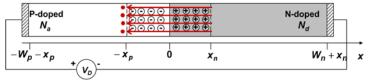
- We need to solve the second order diffusion equation: $\frac{\partial^2 n'(x)}{\partial x^2} \frac{n'(x)}{D_n \tau_n} = 0$ with the boundary condition: $n'(-x_p) = \frac{n_i^2}{N_a} \left(e^{\frac{qV_D}{KT}} 1 \right)$
- But it is a second order differential equation, so we need a second boundary condition!

The minority carrier lifetime τ_n is assumed to be zero at the metal contacts.

• Consequently, there cannot be any excess electron density at the left metal contact. This gives us the second boundary condition:

$$n'(-W_p-x_p)=0$$

Modeling Minority Carrier Diffusion (P-side)



We need to solve:

$$\frac{\partial^2 n'(x)}{\partial x^2} - \frac{n'(x)}{D_n \tau_n} = 0$$

$$\uparrow n'(-x_p) = \frac{n_i^2}{N_a} \left(e^{\frac{q v_D}{KT}} - 1 \right)$$

$$\uparrow n'(-W_p - x_p) = 0$$

Define a "minority carrier diffusion length" L_n for electrons as: $L_n = \sqrt{D_n \tau_n}$

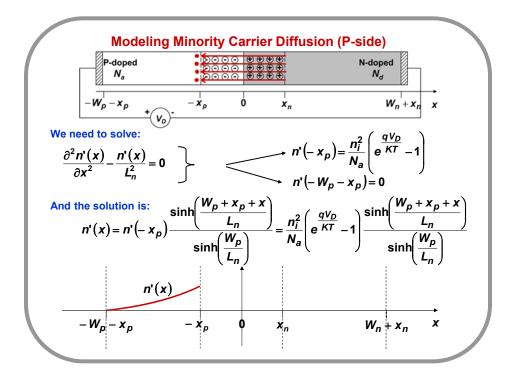
We need to solve:

refered to solve:

$$\frac{\partial^2 n'(x)}{\partial x^2} - \frac{n'(x)}{L_n^2} = 0$$

$$n'(-x_p) = \frac{n_i^2}{N_a} \left(\frac{qV_D}{KT} - 1 \right)$$

$$n'(-W_p - x_p) = 0$$



The Minority Carrier Diffusion Length (P-Side)

The minority carrier diffusion length L_n is the average length an electron injected into the P-side will diffuse before it finds a hole and recombines with it

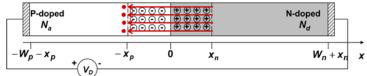
• Long Base Limit:

If $L_n << W_p$ then pretty much all the electrons injected into the P-side recombine with holes before they are able to cross the P-side

• Short Base Limit:

If $L_n>>W_p$ then pretty much all the electrons injected into the P-side do not recombine with the holes and are able to cross the P-side

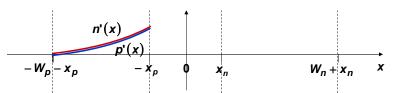
Charge Neutrality and Majority Carrier Distribution (P-side)



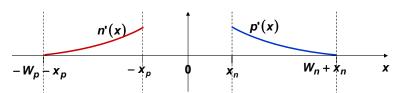
- The assumption of quasi-neutrality implies that charge neutrality pretty much holds even when out of equilibrium
- Let the excess majority carrier distribution (i.e. of holes on the P-side) be written

$$p(x) = p_{po} + p'(x)$$
 Equilibrium hole density Excess hole density

• Then charge neutrality implies: p'(x) = n'(x)



Minority Carrier Current Flow



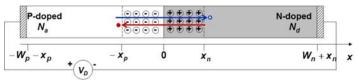
Electron current on the P-side:
$$J_{n}(x) \approx J_{n}^{diff}(x) = q D_{n} \frac{\partial n'(x)}{\partial x}$$

$$J_{n}(x) = q n_{i}^{2} \frac{D_{n}}{N_{a} L_{n}} \left(e^{\frac{q V_{D}}{KT}} - 1\right) \frac{\cosh\left(\frac{W_{p} + x_{p} + x}{L_{n}}\right)}{\sinh\left(\frac{W_{p}}{L_{n}}\right)}$$

$$J_{p}(x) = q n_{i}^{2} \frac{D_{p}}{N_{d} L_{p}} \left(e^{\frac{q V_{D}}{KT}} - 1\right) \frac{\cosh\left(\frac{W_{n} + x_{n} - x}{L_{p}}\right)}{\sinh\left(\frac{W_{n}}{L_{p}}\right)}$$



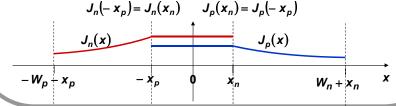
Total Current Flow



• The total current in steady state is the sum of electron and hole currents and is independent of position:

$$J_T = J_n(x) + J_n(x)$$

- So we can compute the total current in forward bias if we know the total electron current (drift and diffusion components) and the total hole current at any one location in the device wherever that location might be.
- Assumption: the minority carrier diffusion currents inside the depletion region are constant (valid if there is no recombination in the depletion region), i.e.:

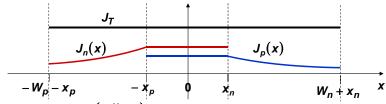


Total Current Flow

• Now using:

$$J_T = J_n(x) + J_p(x)$$

with "x" anywhere inside the depletion region, we can calculate the total current



$$J_n(-x_p) = q n_i^2 \frac{D_n}{N_a L_n} \left(e^{\frac{q V_D}{KT}} - 1 \right) \coth \left(\frac{W_p}{L_n} \right)$$

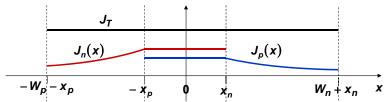
$$J_p(x_n) = q n_i^2 \frac{D_p}{N_d L_p} \left(e^{\frac{q V_D}{KT}} - 1 \right) \coth \left(\frac{W_n}{L_p} \right)$$

$$J_T = J_n(x_n) + J_p(x_n) = J_n(-x_p) + J_p(x_n)$$

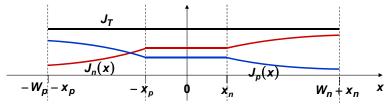
$$J_T = q n_i^2 \left(\frac{D_n}{N_a L_n} \coth \left(\frac{W_p}{L_n} \right) + \frac{D_p}{N_d L_p} \coth \left(\frac{W_n}{L_p} \right) \right) \left(e^{\frac{q V_D}{KT}} - 1 \right)$$

Majority Carrier Current Flow

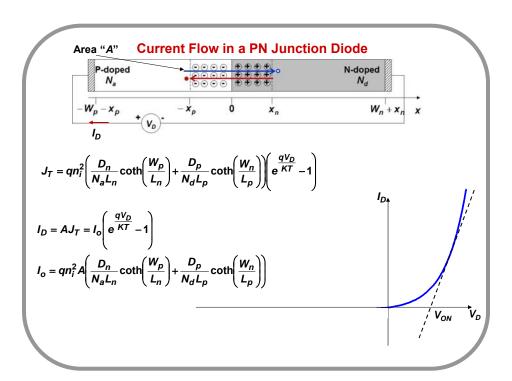
- So far we have assumed that minority carrier current is entirely due to diffusion
- · With the above assumption, we were able to calculate the total current

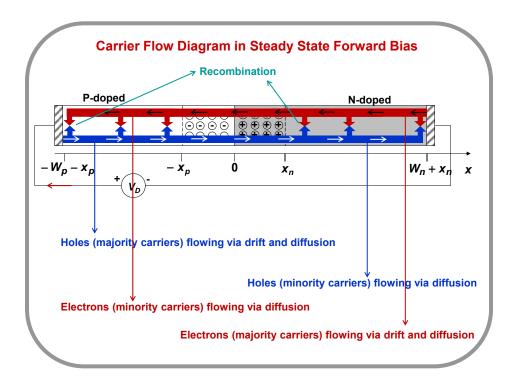


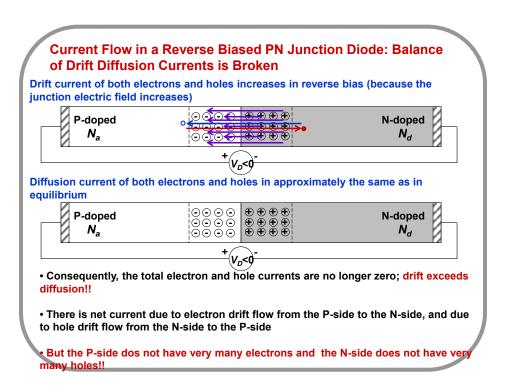
- Now we want to see how the majority carriers contribute to the total current
- Since we already know the total current everywhere, and the minority carrier current everywhere, the difference must be the majority carrier current

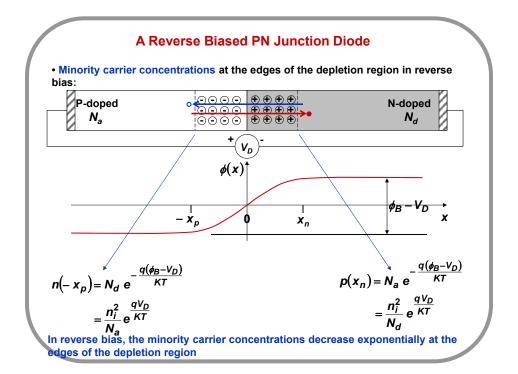


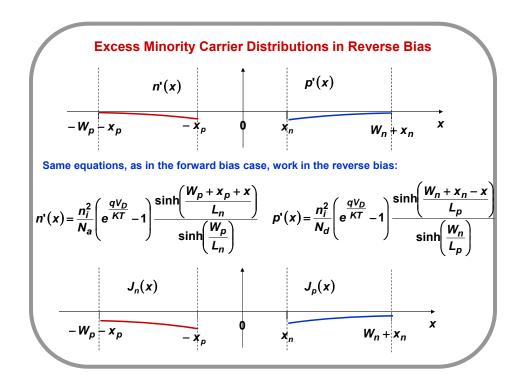
The majority carriers flow by both drift and diffusion











Total Current Flow in Reverse Bias

• Now using:
$$J_T = J_n(x) + J_p(x)$$
with "x" anywhere inside the depletion region, we can calculate the total current
$$J_p(x)$$

$$-W_p - x_p \quad J_T \quad -x_p \quad 0 \quad x_n \quad W_n + x_n$$

$$J_n(-x_p) = q n_i^2 \frac{D_n}{N_a L_n} \left(e^{\frac{q V_D}{KT}} - 1 \right) \coth\left(\frac{W_p}{L_n} \right)$$

$$J_p(x_n) = q n_i^2 \frac{D_p}{N_d L_p} \left(e^{\frac{q V_D}{KT}} - 1 \right) \coth\left(\frac{W_n}{L_p} \right)$$

$$J_T = J_n(x_n) + J_p(x_n) = J_n(-x_p) + J_p(x_n)$$

$$J_T = q n_i^2 \left(\frac{D_n}{N_a L_n} \coth\left(\frac{W_p}{L_n} \right) + \frac{D_p}{N_d L_p} \coth\left(\frac{W_n}{L_p} \right) \right) \left(e^{\frac{q V_D}{KT}} - 1 \right)$$

