Lecture 5

PN Junctions in Thermal Equilibrium

In this lecture you will learn:

- Junctions of P and N doped semiconductors
- Electrostatics of PN junctions in thermal equilibrium
- Built-in junction potential, junction electric field, depletion regions, and all that…

Review: Potential of a Doped Semiconductor

What are the values of potentials in N-doped and P-doped semiconductors ??

N-doped Semiconductors (doping density is N_d):

The potential in n-doped semiconductors is denoted by: ϕ_n

$$n_o(x) \approx N_d$$

$$\Rightarrow N_d = n_i \, e^{\frac{q\,\phi_n(x)}{KT}}$$

$$\Rightarrow \phi_n = \frac{KT}{q} \log\left[\frac{N_d}{n_i}\right]$$

Example:

Suppose,

$$N_d = 10^{17}\,\text{cm}^{-3} \text{ and } n_i = 10^{10}\,\text{cm}^{-3}$$

$$\Rightarrow \phi_n = \frac{KT}{q} \log\left[\frac{N_d}{n_i}\right] = +\,0.41\,\text{Volts}$$

P-doped Semiconductors (doping density is N_a):

The potential in n-doped semiconductors is denoted by: ϕ_p

$$p_o(x) \approx N_a$$

$$\Rightarrow N_a = n_i \, e^{-\frac{q\,\phi_p(x)}{KT}}$$

$$\Rightarrow \phi_p = -\frac{KT}{q} \log\left[\frac{N_a}{n_i}\right]$$

Example:

Suppose,

$$N_a = 10^{17}\,\text{cm}^{-3} \text{ and } n_i = 10^{10}\,\text{cm}^{-3}$$

$$\Rightarrow \phi_p = -\frac{KT}{q} \log\left[\frac{N_a}{n_i}\right] = -\,0.41\,\text{Volts}$$

Can One Measure the Potential Difference between Doped Semiconductors Directly?

$$\phi_n = \frac{KT}{q}\log\left[\frac{N_d}{n_i}\right]$$

$$\phi_p = -\frac{KT}{q}\log\left[\frac{N_a}{n_i}\right]$$

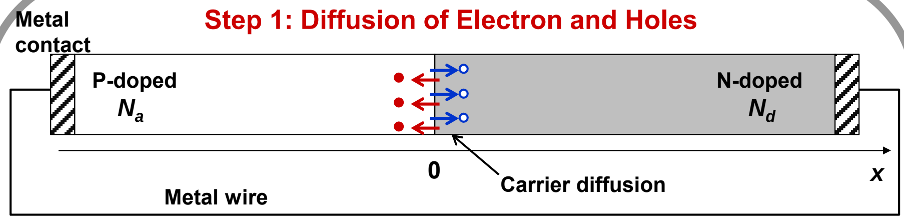
Answer: No!
For the explanation, you need to wait till the end of this handout…….

A Junction of P and N Doped Semiconductors

What do we know (before the junction is formed):

P-doped	N-doped
P-doped N_a	**N-doped** N_d
Doping: N_a	**Doping:** N_d
Carrier densities: $p_{po} = N_a$, $n_{po} = \frac{n_i^2}{N_a}$	**Carrier densities:** $n_{no} = N_d$, $p_{no} = \frac{n_i^2}{N_d}$
Potential: $\phi = \phi_p = -\frac{KT}{q}\log\left[\frac{N_a}{n_i}\right]$	**Potential:** $\phi = \phi_n = \frac{KT}{q}\log\left[\frac{N_d}{n_i}\right]$

Step 1: Diffusion of Electron and Holes



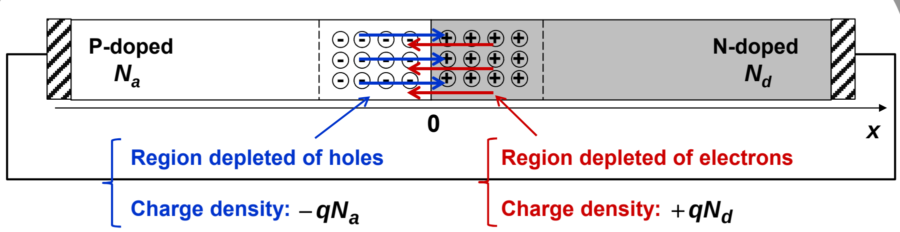
Holes are more in number on the P-side, so they will diffuse into the N-side from the P-side

--- On reaching the N-side, the hole will find an electron and recombine with it

Electrons are more in number on the N-side, so they will diffuse into the P-side from the N-side

--- On reaching the P-side, the electron will find a hole and recombine with it

Step 2: Creation of "Depletion" Regions



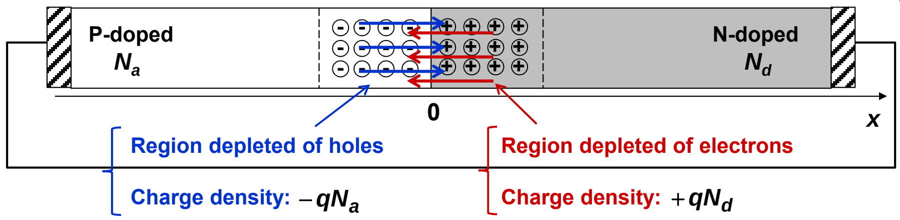
Both the N-doped and P-doped materials were charge neutral before the junction was formed

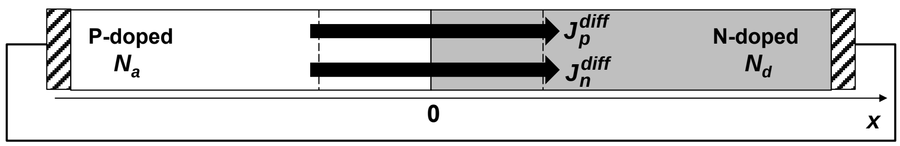
As the holes diffuse from the P-side into the N-side, they leave behind negatively charged acceptor atoms in a region near the interface on the P-side which becomes "depleted" of holes

As the electrons diffuse from the N-side into the P-side, they leave behind positively charged donor atoms in a region near the interface on the N-side which becomes "depleted" of electrons

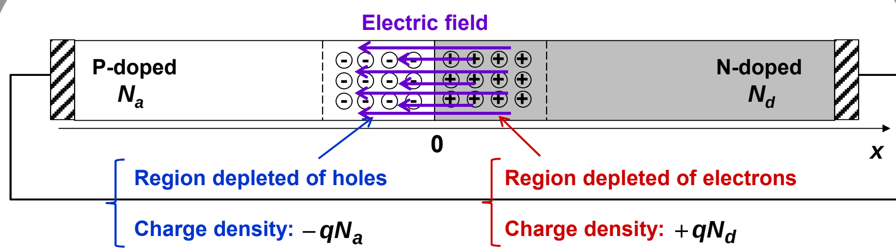
As the carrier keep diffusing, the widths of the depletion regions keep increasing ……..

Diffusion Currents





Generation of Electric Field

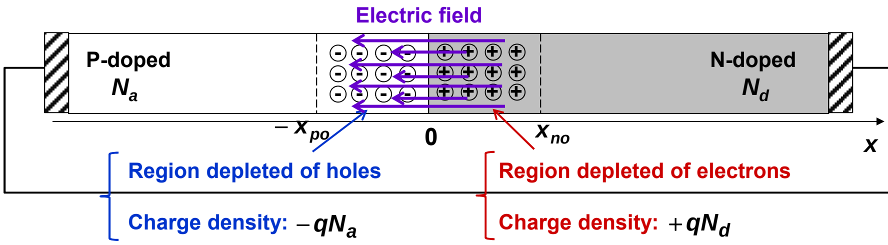


The charge densities in the depletion regions generate an electric field

As the depletion regions grow in thickness, the magnitude of the electric field also increases

Question: How do we figure out how big this electric field is?........

Generation of Electric Field



Assume that the thickness of the depletion region on the P-side is: x_{po}

Assume that the thickness of the depletion region on the N-side is: x_{no}

Gauss's Law on the P-side:

$$\frac{dE}{dx} = -\frac{qN_a}{\varepsilon_s} \qquad \text{Boundary condition } \{E(-x_{po}) = 0$$

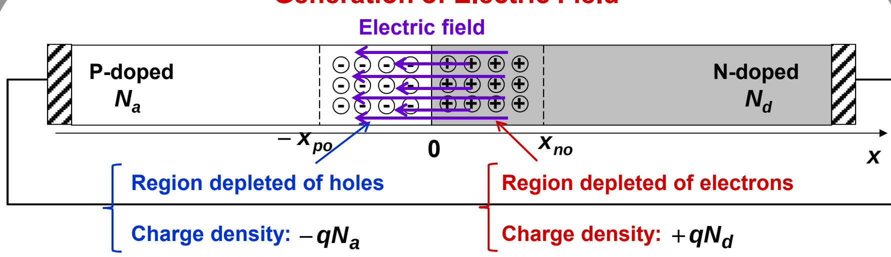
$$\Rightarrow E(x) = -\frac{qNa}{\varepsilon_s}(x + x_{po})$$

Gauss's Law on the N-side:

$$\frac{dE}{dx} = +\frac{qN_d}{\varepsilon_s} \qquad \text{Boundary condition } \{E(x_{no}) = 0$$

$$\Rightarrow E(x) = -\frac{qN_d}{\varepsilon_s}(x_{no} - x)$$

Generation of Electric Field

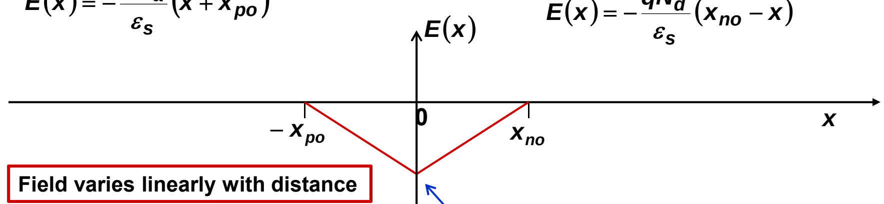


Field on the P-side:

$$E(x) = -\frac{qN_a}{\varepsilon_s}(x + x_{po})$$

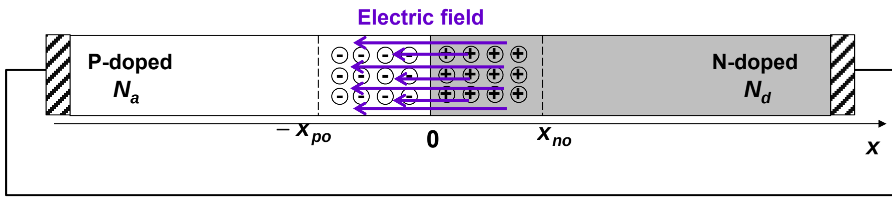
Field on the N-side:

$$E(x) = -\frac{qN_d}{\varepsilon_s}(x_{no} - x)$$

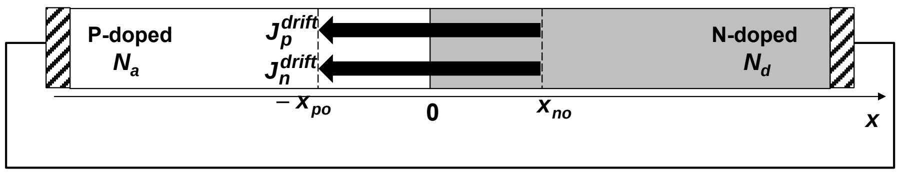


The field must be continuous at the junction: $qN_a x_{po} = qN_d x_{no}$

Drift Currents



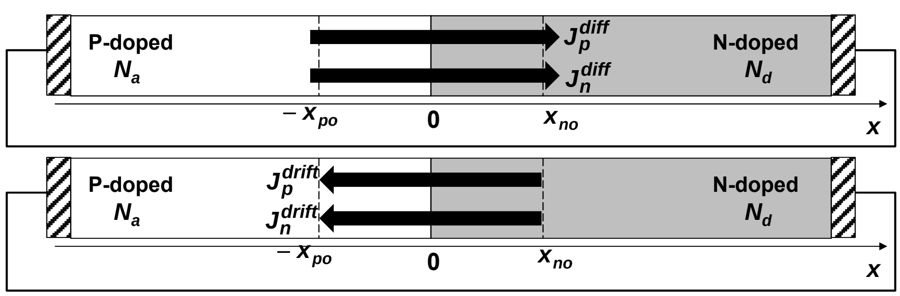
As the carriers diffuse, and the depletion regions grow in thickness, and the electric field increases, **the drift currents due to the electric field also become non-zero**…………



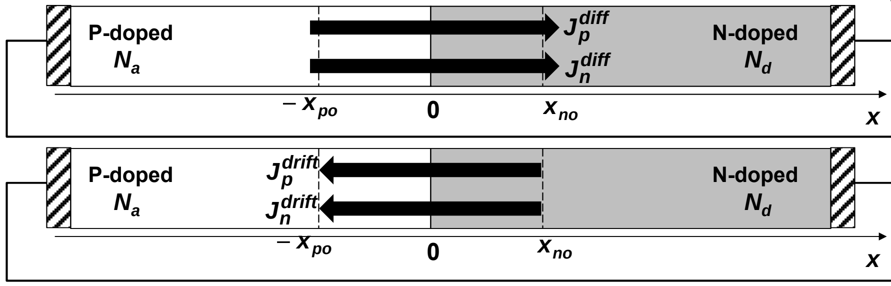
Step 3: Establishment of Equilibrium

As the carriers diffuse, and the depletion regions grow in thickness, and the electric field increases, **the drift currents due to the electric field also increase** …………and as the depletion region width increases, **the diffusion currents decrease**………

A stage is reached when the carrier diffusion currents are exactly balanced by the (oppositely directed) drift currents:



Step 3: Establishment of Equilibrium



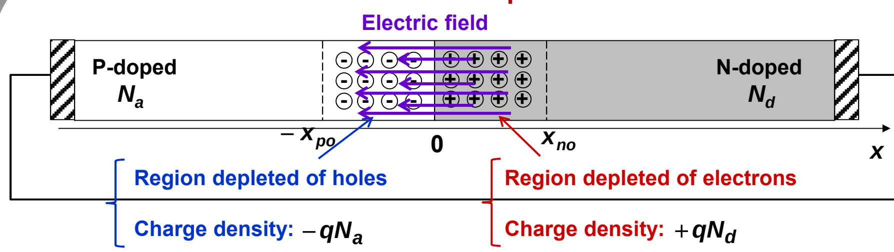
In thermal equilibrium:

$$J_p(x) = J_p^{drift}(x) + J_p^{diff}(x) = 0 \qquad J_n(x) = J_n^{drift}(x) + J_n^{diff}(x) = 0$$

---- The electron diffusion current is balanced by the equal and opposite electron drift current
---- The hole diffusion current is balanced by the equal and opposite hole drift current

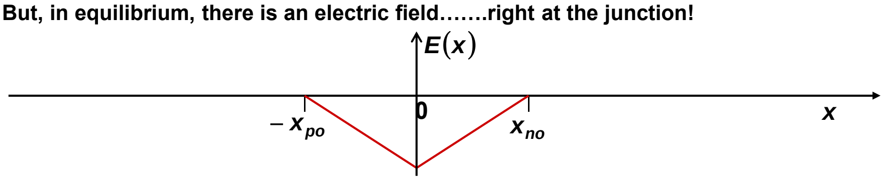
So the net currents of both the electrons as well as the holes go to zero …. and equilibrium is established!!

A PN Junction in Equilibrium



In equilibrium, there is no NET current (of either electrons or the holes)

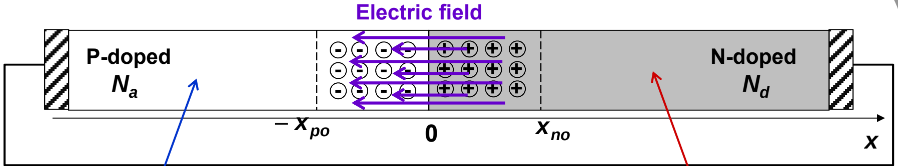
But, in equilibrium, there is an electric field…….right at the junction!



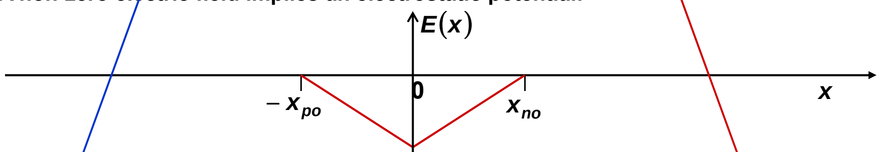
A non-zero electric field implies a spatially varying electrostatic potential:

$$E(x) = -\frac{d\phi(x)}{dx}$$

A PN Junction in Equilibrium



A non-zero electric field implies an electrostatic potential:



$$E(x) = -\frac{d\phi(x)}{dx}$$

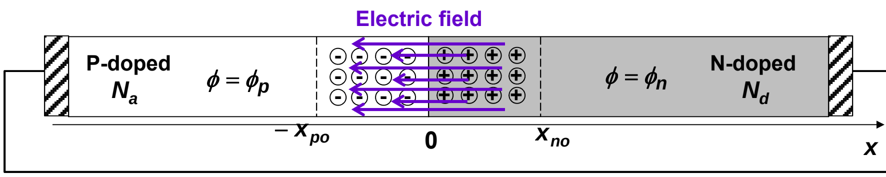
Potential on the P-side:

$$\phi = \phi_p = -\frac{KT}{q}\log\left[\frac{N_a}{n_i}\right]$$

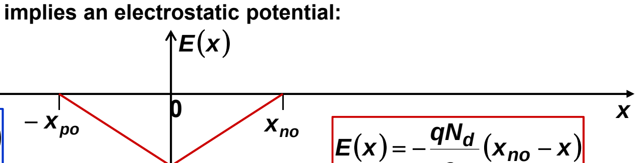
Potential on the N-side:

$$\phi = \phi_n = \frac{KT}{q}\log\left[\frac{N_d}{n_i}\right]$$

A PN Junction in Equilibrium: Electrostatic Potential



A non-zero electric field implies an electrostatic potential:



$$E(x) = -\frac{qN_a}{\varepsilon_s}\left(x + x_{po}\right)$$

$$E(x) = -\frac{qN_d}{\varepsilon_s}\left(x_{no} - x\right)$$

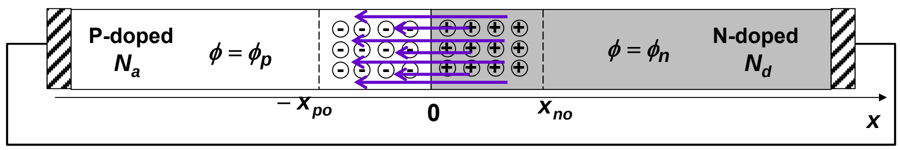
$$\frac{d\phi(x)}{dx} = -E(x)$$

$$\Rightarrow \phi(x) = \phi_p + \frac{qN_a}{2\varepsilon_s}\left(x + x_{po}\right)^2$$

$$\frac{d\phi(x)}{dx} = -E(x)$$

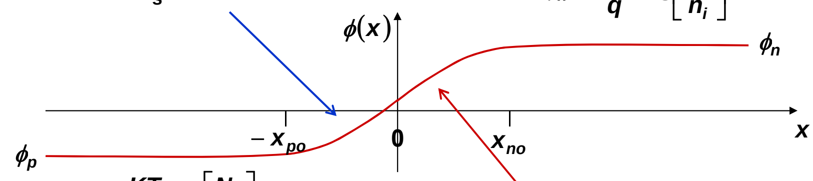
$$\Rightarrow \phi(x) = \phi_n - \frac{qN_d}{2\varepsilon_s}\left(x_{no} - x\right)^2$$

A PN Junction in Equilibrium: Electrostatic Potential



$$\phi(x) = \phi_p + \frac{qN_a}{2\varepsilon_s}(x + x_{po})^2$$

$$\phi_n = \frac{KT}{q}\log\left[\frac{N_d}{n_i}\right]$$

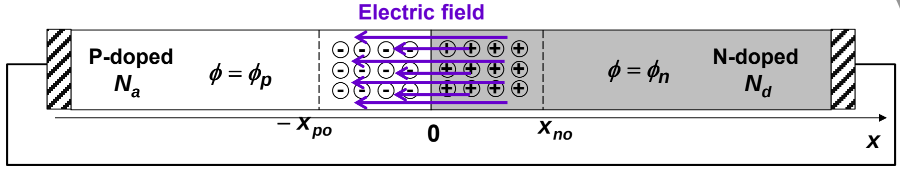


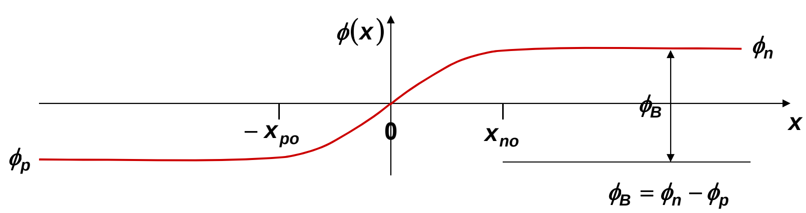
$$\phi_p = -\frac{KT}{q}\log\left[\frac{N_a}{n_i}\right]$$

$$\phi(x) = \phi_n - \frac{qN_d}{2\varepsilon_s}(x_{no} - x)^2$$

Potential varies quadratically with distance

The Built-In Potential





Built-In Junction Potential:

$$\phi_B = \phi_n - \phi_p$$

$$= \frac{KT}{q}\log\left[\frac{N_d N_a}{n_i^2}\right]$$

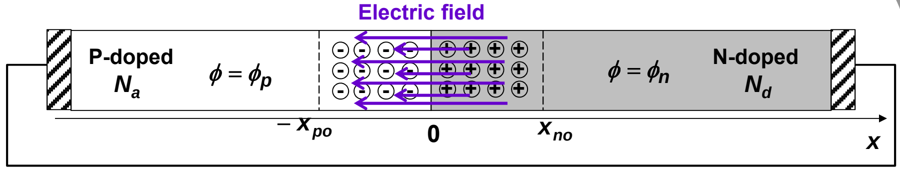
Example:

$N_d = 10^{17}\,1/\text{cm}^3 \quad N_a = 10^{17}\,1/\text{cm}^3$

$$\phi_B = \phi_n - \phi_p$$

$$= \frac{KT}{q}\log\left[\frac{N_d N_a}{n_i^2}\right] = 0.83 \text{ Volts}$$

A PN Junction in Equilibrium: Depletion Region Widths



The electric field and the potential must be continuous at the junction

Field and Potential on the P-side:

$$E(x) = -\frac{qN_a}{\varepsilon_s}\left(x + x_{po}\right)$$

$$\phi(x) = \phi_p + \frac{qN_a}{2\varepsilon_s}\left(x + x_{po}\right)^2$$

Field and Potential on the N-side:

$$E(x) = -\frac{qN_d}{\varepsilon_s}\left(x_{no} - x\right)$$

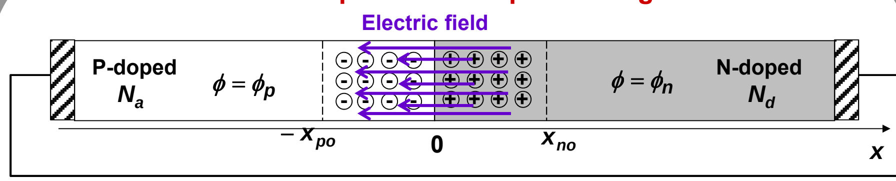
$$\phi(x) = \phi_n - \frac{qN_d}{2\varepsilon_s}\left(x_{no} - x\right)^2$$

$$qN_a x_{po} = qN_d x_{no}$$

$$\phi_p + \frac{qN_a}{2\varepsilon_s}\left(x_{po}\right)^2 = \phi_n - \frac{qN_d}{2\varepsilon_s}\left(x_{no}\right)^2$$

These two equations can be solved to find x_{po} and x_{no}

A PN Junction in Equilibrium: Depletion Region Widths



The result is:

$$x_{po} = \sqrt{\frac{2\,\varepsilon_s\,\phi_B}{qN_a}\left(\frac{N_d}{N_a + N_d}\right)} \qquad x_{no} = \sqrt{\frac{2\,\varepsilon_s\,\phi_B}{qN_d}\left(\frac{N_a}{N_a + N_d}\right)}$$

Observations:

i) If $N_d >> N_a$:

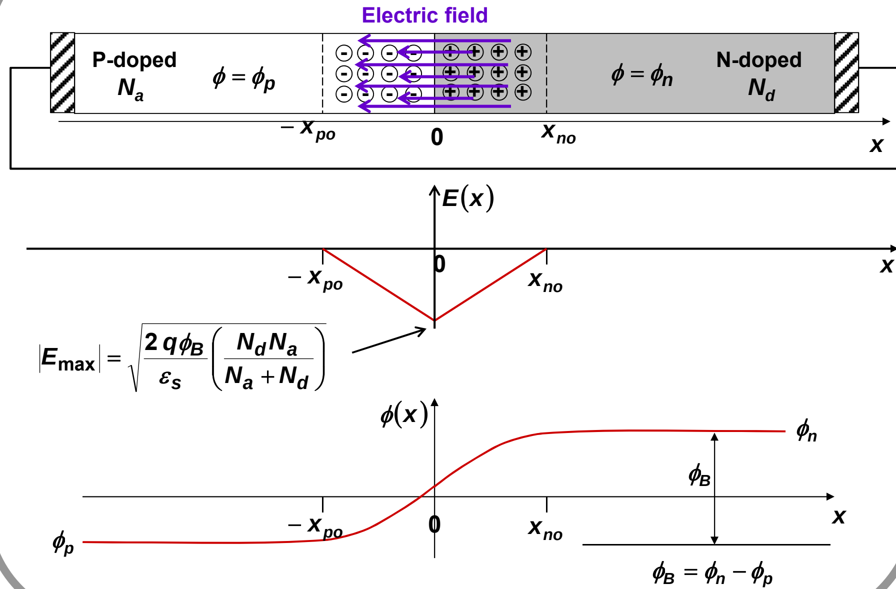
$$x_{po} \approx \sqrt{\frac{2\,\varepsilon_s\,\phi_B}{qN_a}} >> x_{no} \approx \sqrt{\frac{2\,\varepsilon_s\,\phi_B}{qN_d}\left(\frac{N_a}{N_d}\right)}$$

ii) If $N_a >> N_d$:

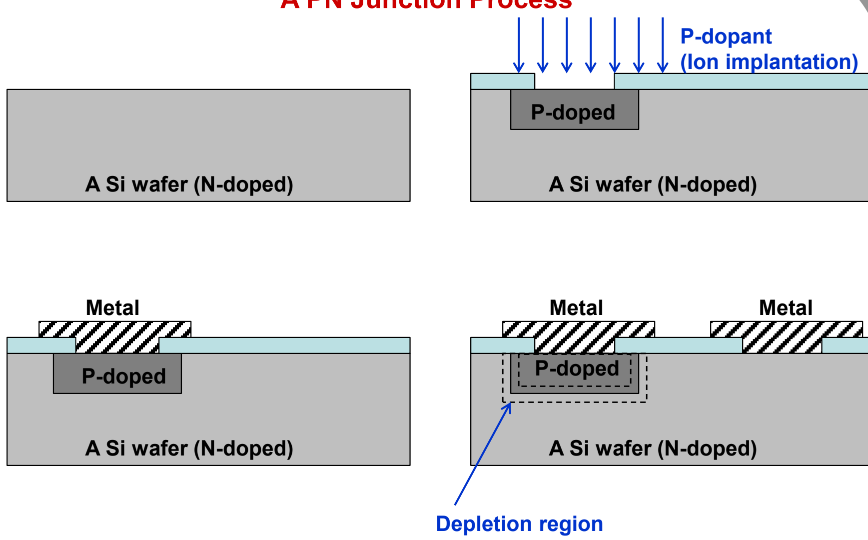
$$x_{po} \approx \sqrt{\frac{2\,\varepsilon_s\,\phi_B}{qN_a}\left(\frac{N_d}{N_a}\right)} << x_{no} \approx \sqrt{\frac{2\,\varepsilon_s\,\phi_B}{qN_d}}$$

In an asymmetrically doped junction, the depletion region on the lightly doped side is larger

A PN Junction in Equilibrium: A Summary



A PN Junction Process



Can One Measure the Potential Difference between Doped Semiconductors Directly?

Metals also have a potential: ϕ_M

There is contact potential at the metal-semiconductor interface
Contact potential is just the built-in potential at the metal-semiconductor interface

After accounting for the contact-potential, the potential difference measured by the voltmeter is zero!