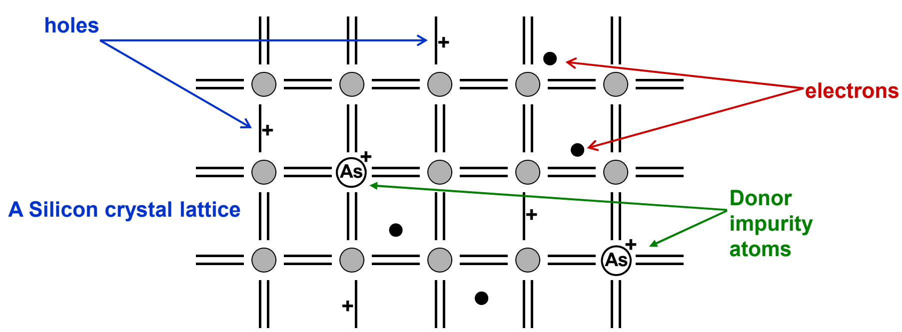
Lecture 3

Electron and Hole Transport in Semiconductors

In this lecture you will learn:

- How electrons and holes move in semiconductors
- Thermal motion of electrons and holes
- Electric current via drift
- Electric current via diffusion
- Semiconductor resistors

Review: Electrons and Holes in Semiconductors



- There are two types of mobile charges in semiconductors: electrons and holes

  In an intrinsic (or undoped) semiconductor electron density equals hole density

- Semiconductors can be doped in two ways:

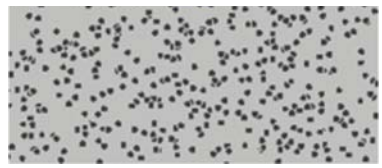
  N-doping: to increase the electron density
  P-doping: to increase the hole density

Thermal Motion of Electrons and Holes

In <u>thermal equilibrium</u> carriers (i.e. electrons or holes) are not standing still but are moving around in the crystal lattice because of their thermal energy

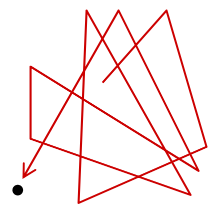
The root-mean-square velocity of electrons can be found by equating their kinetic energy to the thermal energy:

$$\frac{1}{2} m_n v_{th}^2 = \frac{1}{2} KT$$
$$\Rightarrow v_{th} = \sqrt{\frac{KT}{m_n}}$$



In pure Silicon at room temperature:

$$v_{th} \sim 10^7 \text{ cm/s}$$



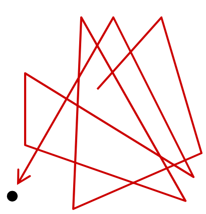
Brownian Motion

Thermal Motion of Electrons and Holes

In <u>thermal equilibrium</u> carriers (i.e. electrons or holes) are not standing still but are moving around in the crystal lattice and undergoing collisions with:

- vibrating Silicon atoms
- with other electrons and holes
- with dopant atoms (donors or acceptors) and other impurity atoms

Mean time between collisions = τ_c



Brownian Motion

In between two successive collisions electrons (or holes) move with an average velocity which is called the thermal velocity = v_{th}

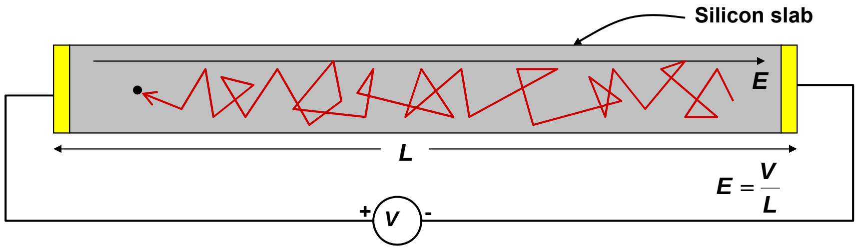
In pure Silicon, $\tau_c \approx 0.1 \times 10^{-12} \text{ s} = 0.1 \text{ ps}$

$$v_{th} \approx 10^7 \text{ cm/s}$$

Mean distance traveled between collisions is called the mean free path $= \lambda = v_{th}\, \tau_c$

In pure Silicon, $\lambda = 10^7 \times 0.1 \times 10^{-12} = 10^{-6} \text{ cm} = 0.01 \mu\text{m}$

Drift: Motion of Electrons Under an Applied Electric Field

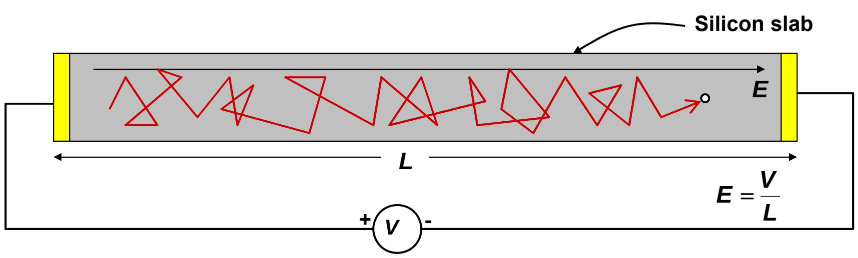


- Force on an electron because of the electric field = $F_n = -qE$
- The electron moves in the direction opposite to the applied field with a constant drift velocity equal to v_{dn}
- The electron drift velocity v_{dn} is proportional to the electric field strength

$$v_{dn} \propto -E \quad \Rightarrow \quad v_{dn} = -\mu_n E$$

- The constant μ_n is called the electron mobility. It has units: $\frac{\text{cm}^2}{\text{V - s}}$
- In pure Silicon, $\mu_n \approx 1500 \ \frac{\text{cm}^2}{\text{V - s}}$

Drift: Motion of Holes Under an Applied Electric Field



- Force on a hole because of the electric field = $F_p = qE$
- The hole moves in the direction of the applied field with a constant drift velocity equal to v_{dp}
- The hole drift velocity v_{dp} is proportional to the electric field strength

$$v_{dp} \propto E \quad \Rightarrow \quad v_{dp} = \mu_p E$$

- The constant μ_p is called the hole mobility. It has units: $\frac{\text{cm}^2}{\text{V - s}}$
- In pure Silicon, $\mu_p \approx 500 \ \frac{\text{cm}^2}{\text{V - s}}$

Derivation of Expressions for Mobility

Electrons:

Force on an electron because of the electric field $= F_n = -qE$

Acceleration of the electron $= a = \frac{F_n}{m_n} = -\frac{q\,E}{m_n}$

Since the mean time between collisions is τ_c, the acceleration lasts only for a time period of τ_c before a collision completely destroys electron's velocity

So in time τ_c electron's velocity reaches a value $= a\,\tau_c = -\frac{q\,\tau_c}{m_n}E$

This is the average drift velocity of the electron, i.e. $v_{dn} = -\frac{q\,\tau_c}{m_n}E$

Comparing with $v_{dn} = -\mu_n\,E$ we get, $\mu_n = \frac{q\,\tau_c}{m_n}$

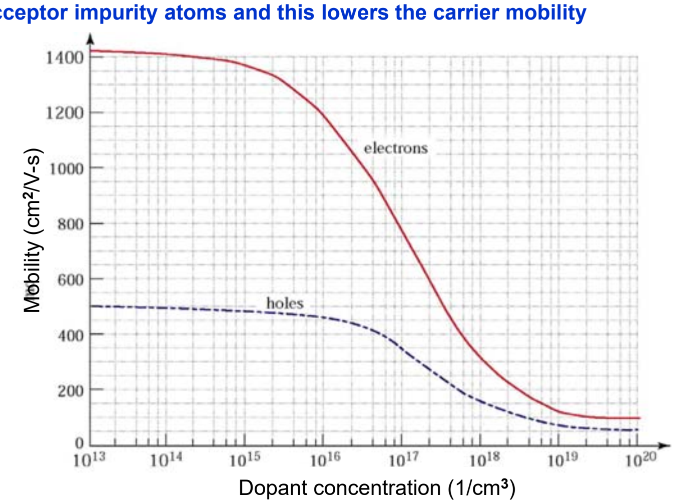
Holes:

Similarly for holes one gets, $\mu_p = \frac{q\,\tau_c}{m_p}$

Special note: Masses of electrons and holes (m_n and m_p) in Solids are not the same as the mass of electrons in free space which equals 9.1×10^{-31} kg

Mobility Vs Doping

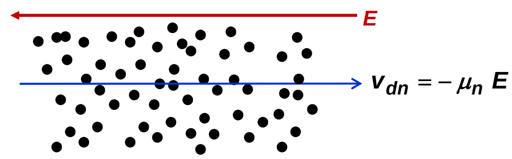
More doping (n-type of p-type) means more frequent collisions with charged donor and acceptor impurity atoms and this lowers the carrier mobility



Note: Doping in the above figure can either be n-type or p-type

Drift Current Density of Electrons

Consider electrons moving under an applied electric field:



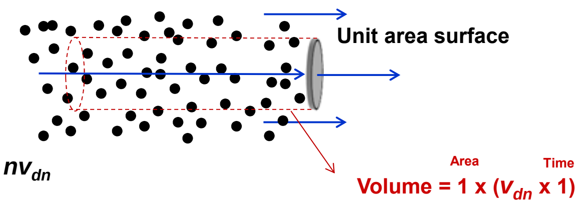
Flux Density:

Flux density is the number of particles crossing a unit area surface per second

It has units cm^{-2}-s^{-1}

Density: n

Velocity: v_{dn}



Flux density: nv_{dn}

Drift Current Density of Electrons

Electrons Drift Current Density:

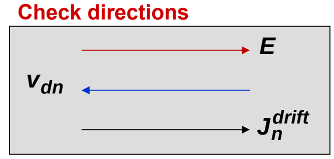
Electron flux density from drift $= n\, v_{dn}$

Electron drift current density J_n^{drift} is,

$$J_n^{drift} = -q \times (\text{electron flux density})$$
$$= -q\, n\, v_{dn} = q\, n\, \mu_n\, E$$

J_n^{drift} has units: $\dfrac{\text{Coulombs}}{\text{cm}^2 \text{- s}} = \dfrac{\text{Amps}}{\text{cm}^2}$

Check directions



$\boxed{v_{dn} = -\mu_n E}$

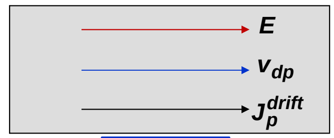
$\boxed{J_n^{drift} = q\, n\, \mu_n\, E}$

Drift Current Density of Holes

Holes Drift Current Density:
The hole drift current density is J_p^{drift} ,

$$J_p^{drift} = +q \times (\text{hole flux density})$$
$$= +q\ p\ v_{dp} = q\ p\ \mu_p\ E$$

Check directions



$$\boxed{v_{dp} = \mu_p\ E}$$
$$\boxed{J_p^{drift} = q\ p\ \mu_p\ E}$$

J_p^{drift} has units: $\dfrac{\text{Coulombs}}{\text{cm}^2\text{ - s}} = \dfrac{\text{Amps}}{\text{cm}^2}$

Conductivity and Resistivity

Total Drift Current Density:
The total drift current density J^{drift} is the sum of J_n^{drift} and J_p^{drift}

$$J^{drift} = J_n^{drift} + J_p^{drift}$$
$$= q\left(n\ \mu_n + p\ \mu_p\right)E$$
$$= \sigma\ E$$

The quantity σ is the conductivity of the semiconductor:

$$\sigma = q\left(n\ \mu_n + p\ \mu_p\right)$$

Conductivity describes how much current flows when an electric field is applied. Another related quantity is the resistivity ρ which is the inverse of the conductivity,

$$\rho = \frac{1}{\sigma}$$

Units of conductivity are: Ohm^{-1}-cm^{-1} or Ω^{-1}-cm^{-1} or S-cm^{-1}
Units of resistivity are: Ohm-cm or Ω-cm or S^{-1}-cm

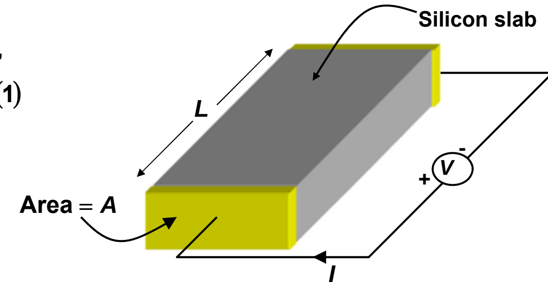
Example: A Semiconductor Resistor

For a resistor we know that,

$$I = \frac{V}{R} \qquad \text{(1)}$$

We also know that,

$$I = J^{drift} A = \sigma E A = \sigma \frac{V}{L} A = \frac{\sigma A}{L} V \qquad \text{(2)}$$



$$(1) + (2) \Rightarrow R = \frac{L}{\sigma A} = \frac{\rho L}{A} \quad \Big\} \text{ where } \sigma = \frac{1}{\rho} = q\left(n\,\mu_n + p\,\mu_p\right)$$

Lessons:

- **Knowing electron and hole densities and mobilities, one can calculate the electrical resistance of semiconductors**

- **n-doping or p-doping can be used to change the conductivity of semiconductors by orders of magnitudes**

Diffusion

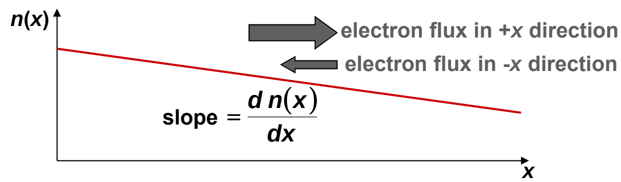
Diffusion of ink in a glass beaker

Why does diffusion happen?

Diffusion and Diffusivity

There is another mechanism by which current flows in semiconductors …….

• Suppose the electron density inside a semiconductor is not uniform in space, as shown below



• Since the electrons move about randomly in all directions (Brownian motion), as time goes on more electrons will move from regions of higher electron density to regions of lower electron density than the electrons that move from regions of lower electron density to regions of higher electron density

• Net electron flux density in $+x$ direction $\propto -\dfrac{d\,n(x)}{dx}$

$$= -D_n \frac{d\,n(x)}{dx}$$

• The constant D_n is called the diffusivity of electrons (units: cm^2-s^{-1})

Diffusion Current Density

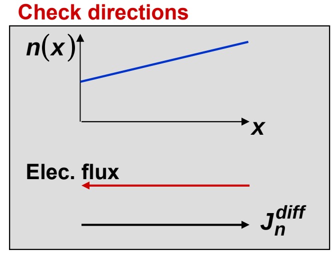
Electrons Diffusion Current Density:

Electron flux density from diffusion $= -D_n \dfrac{d\,n(x)}{dx}$

Electron diffusion current density J_n^{diff} is,

$$J_n^{diff} = -q \times (\text{electron flux density})$$
$$= q\,D_n \frac{d\,n(x)}{dx}$$

Check directions



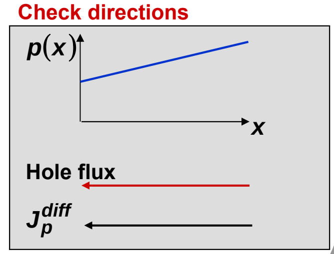
Holes Diffusion Current Density:

Hole flux density from diffusion $= -D_p \dfrac{d\,p(x)}{dx}$

Hole diffusion current density J_p^{diff} is,

$$J_p^{diff} = +q \times (\text{hole flux})$$
$$= -q\,D_p \frac{d\,p(x)}{dx}$$

Check directions



J_n^{diff} and J_p^{diff} has units $\dfrac{\text{Coulombs}}{\text{cm}^2\text{ - s}} = \dfrac{\text{Amps}}{\text{cm}^2}$

Einstein Relations

Einstein worked on other things besides the theory of relativity……..

- We introduced two material constants related to carrier transport:
  1) Mobility
  2) Diffusivity

- Both are connected with the transport of carriers (electrons or holes)

- It turns out that their values are related by the Einstein relationships

Einstein Relation for Electrons:

$$\frac{D_n}{\mu_n} = \frac{K\,T}{q}$$

Einstein Relation for Holes:

$$\frac{D_p}{\mu_p} = \frac{K\,T}{q}$$

> **Example:**
>
> In pure Silicon, $\mu_n \approx 1500\ \text{cm}^2/\text{V-s}$
>
> $\mu_p \approx 500\ \text{cm}^2/\text{V-s}$
>
> This implies, $D_n \approx 37.5\ \text{cm}^2/\text{s}$
>
> $D_p \approx 12.5\ \text{cm}^2/\text{s}$

- K is the Boltzman constant and its value is: $1.38\text{x}10^{-23}$ $\text{Joules}/{}^\circ\text{K}$
- $\frac{K\,T}{q}$ has a value equal to 0.0258 Volts at room temperature (at 300 ^{o}K)

Total Electron and Hole Current Densities

Total electron and hole current densities is the sum of drift and diffusive components

Electrons:

$$J_n(x) = J_n^{drift}(x) + J_n^{diff}(x)$$
$$= q\,n(x)\,\mu_n\,E(x) + q\,D_n\,\frac{d\,n(x)}{dx}$$

Holes:

$$J_p(x) = J_p^{drift}(x) + J_p^{diff}(x)$$
$$= q\,p(x)\,\mu_p\,E(x) - q\,D_p\,\frac{d\,p(x)}{dx}$$

Electric currents are driven by electric fields and also by carrier density gradients

Thermal Equilibrium - I

There cannot be any net electron current or net hole current in thermal equilibrium ……… what does this imply ??

Consider electrons first:

$$J_n(x) = J_n^{drift}(x) + J_n^{diff}(x) = 0$$

$$\Rightarrow q\, n_o(x)\, \mu_n E(x) + q\, D_n \frac{d\, n_o(x)}{dx} = 0 \qquad (1)$$

(1) can also be written as: $\frac{d \log[n_o(x)]}{dx} = -\frac{q}{KT} E(x)$

Since the electric field is minus the gradient of the potential: $E(x) = -\frac{d\phi(x)}{dx}$

We have: $\frac{d \log[n_o(x)]}{dx} = \frac{q}{KT}\frac{d\phi(x)}{dx}$

The solution of the above differential equation is: $n_o(x) = \text{constant} \times e^{\frac{q\phi(x)}{KT}}$

But what is that "constant" in the above equation ???

Thermal Equilibrium - II

We have: $n_o(x) = \text{constant} \times e^{\frac{q\phi(x)}{KT}}$

Note: one can only measure potential differences and not the absolute values of potentials

Convention: The potential of pure intrinsic Silicon is used as the reference value and assumed to be equal to zero.

So for intrinsic Silicon, $n_o(x) = \text{constant} \times e^{\frac{q\phi(x)}{KT}} = \text{constant}$

But we already know that in intrinsic Silicon, $n_o(x) = n_i$

So it must be that, $\text{constant} = n_i$

And we get the final answer, $\boxed{n_o(x) = n_i\, e^{\frac{q\phi(x)}{KT}}}$

Consider Holes Now:

One can repeat the above analysis for holes and obtain: $\boxed{p_o(x) = n_i\, e^{-\frac{q\phi(x)}{KT}}}$

Check: $n_o(x)\, p_o(x) = n_i^2$

Potential of Doped Semiconductors

What are the values of potentials in N-doped and P-doped semiconductors ??

N-doped Semiconductors (doping density is N_d):

The potential in n-doped semiconductors is denoted by: ϕ_n

$$n_o(x) \approx N_d$$

$$\Rightarrow N_d = n_i \, e^{\frac{q\phi_n(x)}{KT}}$$

$$\Rightarrow \phi_n = \frac{KT}{q} \log\left[\frac{N_d}{n_i}\right]$$

Example:

Suppose,

$$N_d = 10^{17} \text{cm}^{-3} \text{ and } n_i = 10^{10} \text{cm}^{-3}$$

$$\Rightarrow \phi_n = \frac{KT}{q} \log\left[\frac{N_d}{n_i}\right] = +0.4 \text{ Volts}$$

P-doped Semiconductors (doping density is N_a):

The potential in p-doped semiconductors is denoted by: ϕ_p

$$p_o(x) \approx N_a$$

$$\Rightarrow N_a = n_i \, e^{-\frac{q\phi_p(x)}{KT}}$$

$$\Rightarrow \phi_p = -\frac{KT}{q} \log\left[\frac{N_a}{n_i}\right]$$

Example:

Suppose,

$$N_a = 10^{17} \text{cm}^{-3} \text{ and } n_i = 10^{10} \text{cm}^{-3}$$

$$\Rightarrow \phi_p = -\frac{KT}{q} \log\left[\frac{N_a}{n_i}\right] = -0.4 \text{ Volts}$$