Lecture 19

High Frequency Analysis of FET Circuits

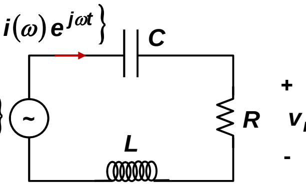
In this lecture you will learn:

- **High Frequency Analysis of FET Circuits**
- **Miller Effect and the Miller Capacitance**

Phasor Analysis: Complex Impedances

$$i(t) = \text{Re}\left\{ i(\omega)\, e^{j\omega t} \right\}$$

$$v(t) = \text{Re}\left\{ v(\omega)\, e^{j\omega t} \right\}$$



$$v_r(t) = \text{Re}\left\{ v_r(\omega)\, e^{j\omega t} \right\}$$

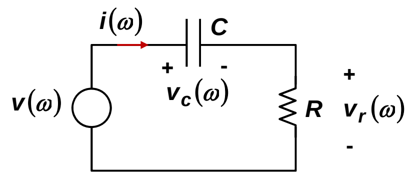
Capacitive Impedance/Admittance:

$$Z(\omega) = \frac{1}{Y(\omega)} = \frac{1}{j\omega C}$$

Inductance Impedance/Admittance:

$$Z(\omega) = \frac{1}{Y(\omega)} = j\omega L$$

Phasor Analysis: Calculations with Impedances

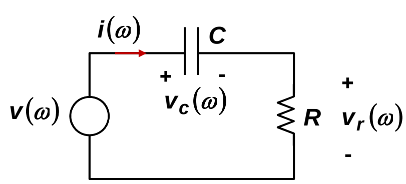


One can compute the voltage phasors using the impedances:

$$v_c(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} v(\omega) = \frac{1}{1 + j\omega RC} v(\omega)$$

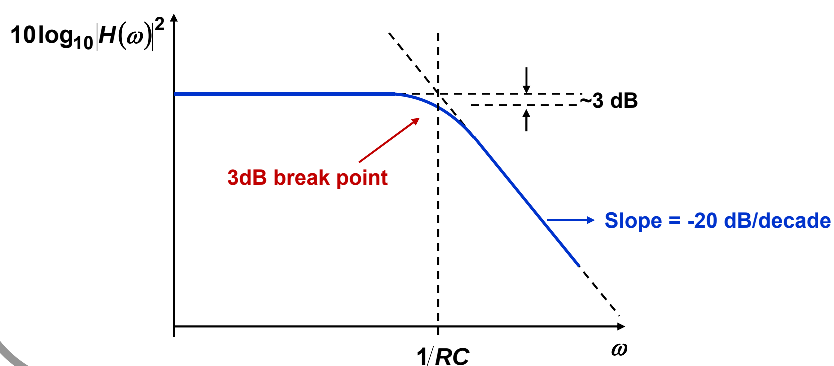
$$v_r(\omega) = \frac{R}{R + \frac{1}{j\omega C}} v(\omega) = \frac{j\omega RC}{1 + j\omega RC} v(\omega)$$

Phasor Analysis: Bode Plots

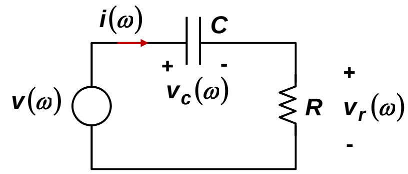


$$\frac{v_c(\omega)}{v(\omega)} = \frac{1}{1 + j\omega RC} = H(\omega)$$

A transfer function with a pole at frequency $1/RC$



Poles and Zeroes of Transfer Functions



$$\frac{v_c(\omega)}{v(\omega)} = \frac{1}{1+j\omega RC}$$ → **Pole** at the complex frequency $\frac{j}{RC}$

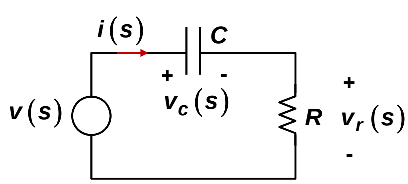
> With a little abuse of language we will say a pole at the frequency $\frac{1}{RC}$

$$\frac{v_r(\omega)}{v(\omega)} = \frac{j\omega RC}{1+j\omega RC}$$ → **Pole** at the complex frequency $\frac{j}{RC}$

Zero at the complex frequency 0

> With a little abuse of language we will say pole at the frequency $\frac{1}{RC}$
> and a zero at 0 frequency

Laplace Transform Calculations: Language Differences to Note



$s \Leftrightarrow j\omega$

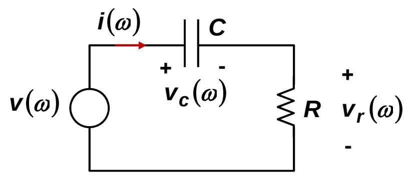
One can compute the voltages using Laplace transforms as well:

$$v_c(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} v(s) = \frac{1}{1+sRC} v(s)$$ → Pole at $-\frac{1}{RC}$

$$v_r(s) = \frac{R}{R + \frac{1}{sC}} v(s) = \frac{sRC}{1+sRC} v(s)$$ → Pole at $-\frac{1}{RC}$

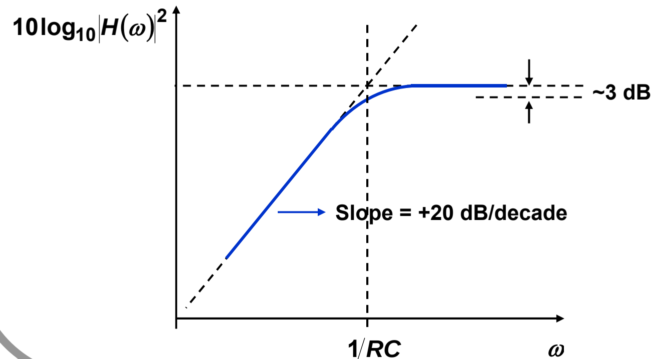
Zero at 0

Phasor Analysis: Bode Plots



$$\frac{v_r(\omega)}{v(\omega)} = \frac{j\omega RC}{1+j\omega RC} = H(\omega)$$

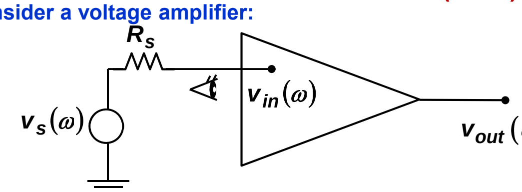
A transfer function with a <u>zero</u> at zero frequency and a <u>pole</u> at frequency $1/RC$



The Miller Effect and the Miller Capacitance
John A. Miller (1920)

Consider a voltage amplifier:

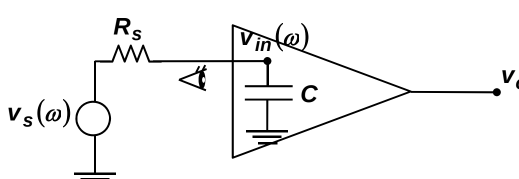
$|A| >> 1$
$A < 0$



$$v_{out}(\omega) = Av_{in}(\omega) = Av_s(\omega)$$

Open circuit voltage gain

Consider now a capacitor sitting at the input of an amplifier:



$$v_{out}(\omega) = Av_{in}(\omega)$$
$$= \frac{A}{1+j\omega R_s C} v_s(\omega)$$

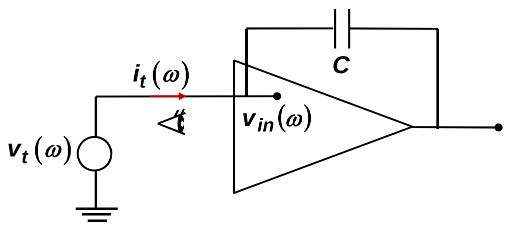
$$Z_{in}(\omega) = \frac{1}{j\omega C}$$

$$v_{in}(\omega) = v_s(\omega)\frac{Z_{in}(\omega)}{R_s + Z_{in}(\omega)} = \frac{v_s(\omega)}{1+j\omega R_s C}$$

Input voltage to the amplifier decreases, and so does the output voltage of the amplifier, at high frequencies (but not too bad…..)

The Miller Effect and the Miller Capacitance

Consider now a capacitor straddling the input and the output of an amplifier:



$v_{out}(\omega) = Av_{in}(\omega)$

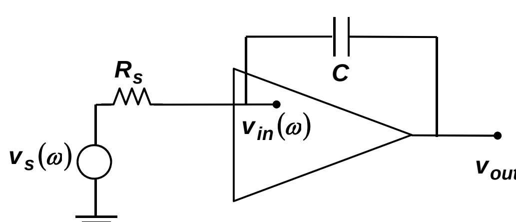
$|A| >> 1$
$A < 0$

$$i_t(\omega) = j\omega C\left[v_t(\omega) - v_{out}(\omega)\right] = j\omega C\left[v_t(\omega) - Av_t(\omega)\right]$$
$$= j\omega C(1-A)v_t(\omega)$$
$$\Rightarrow Z_{in}(\omega) = \frac{v_t(\omega)}{i_t(\omega)} = \frac{1}{j\omega C(1-A)}$$

The capacitance, as seen from the input end, is effectively very large!

The Miller Effect and the Miller Capacitance

Capacitor straddling the input and the output of an amplifier:



$|A| >> 1$
$A < 0$

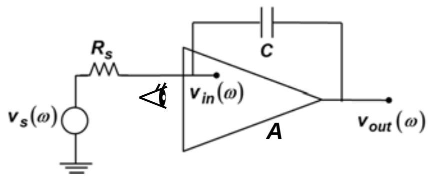
$$v_{out}(\omega) = Av_{in}(\omega)$$
$$= \frac{A}{1 + j\omega R_s C(1-A)} v_s(\omega)$$

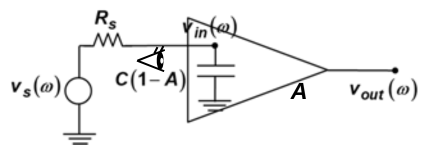
The pole has shifted to a much lower frequency!

$$Z_{in}(\omega) = \frac{1}{j\omega C(1-A)}$$
$$v_{in}(\omega) = v_s(\omega)\frac{Z_{in}(\omega)}{R_s + Z_{in}(\omega)} = \frac{v_s(\omega)}{1 + j\omega R_s C(1-A)}$$

The amplifier input voltage, and the output voltage, will now begin to drop-off at a much lower frequency!!
This is the Miller effect and the capacitance positioned this way is called the Miller capacitance

The Miller Effect and the Miller Capacitance: Summary





In both cases:

$$Z_{in}(\omega) = \frac{1}{j\omega C(1-A)}$$

$$v_{in}(\omega) = v_s(\omega)\frac{Z_{in}(\omega)}{R_s + Z_{in}(\omega)}$$

$$v_{out}(\omega) = Av_{in}(\omega)$$

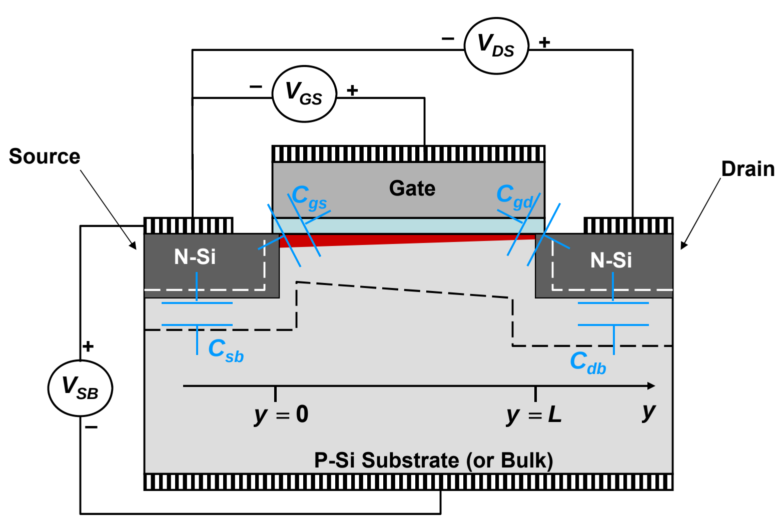
$$= \frac{A}{1 + j\omega R_s C(1-A)} v_s(\omega)$$

Lesson:

A capacitor *C* straddling the input-output terminals of an amplifier of gain *A* = -|*A*| behaves like a very large capacitor *C*(1-*A*) sitting between the input and the ground terminals.

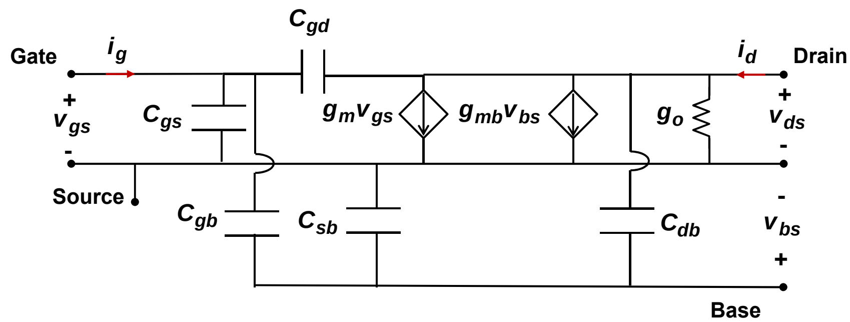
This capacitor reduces the gain bandwidth of the circuit

NFET: Capacitances



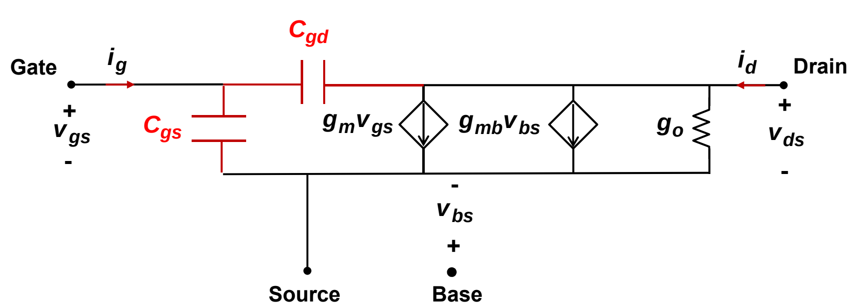
NFET: High Frequency Small Signal Model

FET high frequency model:



NFET: High Frequency Small Signal Model

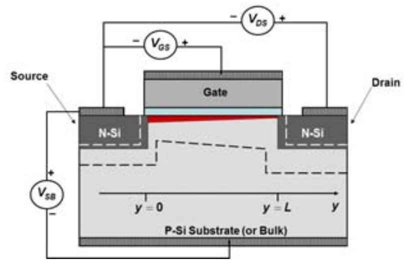
Our simplified high frequency model:



NFET Gate Charge

The total gate charge Q_{TG} (units: Coulombs) consists of the image charge due to:

1) The total inversion layer charge Q_{TN}
2) The total depletion layer charge Q_{TB}



$$Q_{TG} = -\left(Q_{TN} + Q_{TB}\right)$$

The inversion layer charge Q_N (units: Coulombs/unit area) is a function of position "y" in the device:

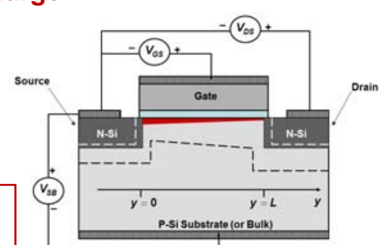
$$Q_N(y) = -C_{ox}\left(V_{GS} - V_{TN} - V_{CS}(y)\right)$$

The total inversion layer charge Q_{TN} (units: Coulombs) can be found by integration over the entire area of the FET:

$$Q_{TN} = W\int_0^L Q_N(y)\,dy$$
$$= -WC_{ox}\int_0^L \left(V_{GS} - V_{TN} - V_{CS}(y)\right)dy$$

NFET Gate Charge

$$Q_{TN} = -WC_{ox}\int_0^L \left(V_{GS} - V_{TN} - V_{CS}(y)\right)dy$$
$$= -WC_{ox}\int_0^{V_{DS}} \left(V_{GS} - V_{TN} - V_{CS}\right)\frac{dy}{dV_{CS}}\,dV_{CS}$$



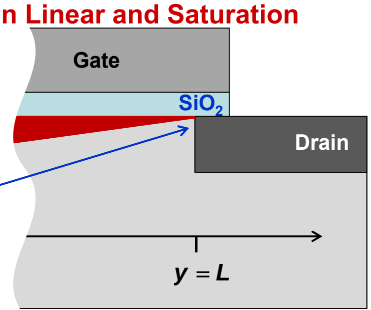
Recall that (Lecture 10):

$$I_D = WQ_N(y)\,\mu_n\,E_y(y)$$
$$= W\,\mu_n\,C_{ox}\left(V_{GS} - V_{TN} - V_{CS}(y)\right)\frac{dV_{CS}(y)}{dy}$$
$$\Rightarrow \frac{dy}{dV_{CS}} = \frac{W\,\mu_n\,C_{ox}\left(V_{GS} - V_{TN} - V_{CS}(y)\right)}{I_D}$$

$$Q_{TN} = -\frac{\mu_n}{I_D}\left(WC_{ox}\right)^2\int_0^{V_{DS}}\left(V_{GS} - V_{TN} - V_{CS}\right)^2 dV_{CS}$$
$$= \frac{\mu_n}{3I_D}\left(WC_{ox}\right)^2\left[\left(V_{GS} - V_{TN} - V_{DS}\right)^3 - \left(V_{GS} - V_{TN}\right)^3\right]$$

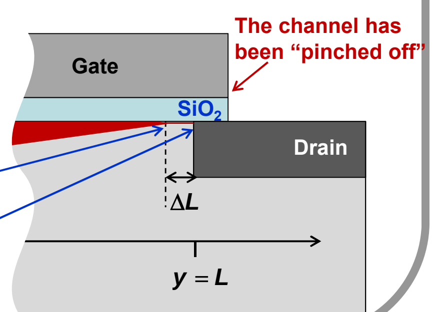
NFET: Inversion Layer Charge in Linear and Saturation

For $V_{DS} \leq V_{GS} - V_{TN}$: Linear Region:



Channel potential: $V_{CS}(y) = V_{DS}$

For $V_{DS} \geq V_{GS} - V_{TN}$: Saturation Region:



Channel potential: $V_{CS}(y) = V_{GS} - V_{TN}$

Channel potential: $V_{CS}(y) = V_{DS}$

NFET Gate Charge

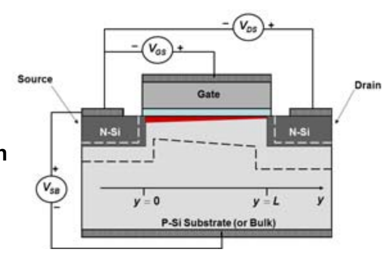
Inversion layer charge Q_N :

$$Q_{TN} = \frac{\mu_n}{3I_D}(WC_{ox})^2\left[(V_{GS} - V_{TN} - V_{DS})^3 - (V_{GS} - V_{TN})^3\right]$$

Expression only works in the linear region

In saturation ($V_{DS} > V_{GS} - V_{TN}$) but the inversion charge remains fixed at the value when $V_{DS} = V_{GS} - V_{TN}$ because of pinch-off:

$$Q_{TN} = -\frac{2}{3}WLC_{ox}(V_{GS} - V_{TN})$$



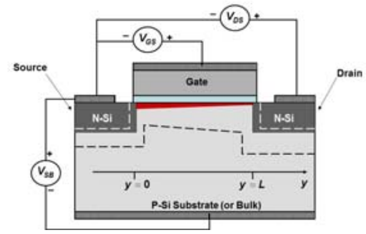
Therefore, in saturation (since the depletion region thickness and charge do not change with the gate voltage), we get:

$$Q_{TG} = -(Q_{TN} + Q_{TB})$$

$$C_{gs} = \left.\frac{\partial Q_{TG}}{\partial V_{GS}}\right|_{V_{GD},V_{GB}} = -\left.\frac{\partial Q_{TN}}{\partial V_{GS}}\right|_{V_{GD},V_{GB}} = \frac{2}{3}WLC_{ox}$$

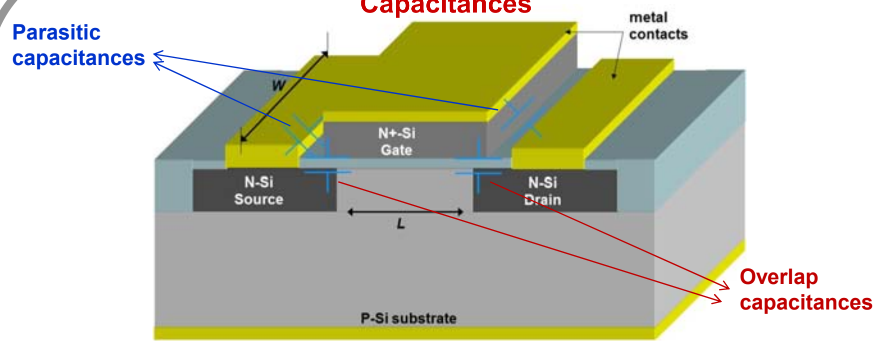
NFET Capacitances in Saturation: A Simple Model

In **saturation**, because of pinch off, the inversion layer charge Q_{TN} is not affected by the drain voltage:



$$C_{gd} = \left.\frac{\partial Q_{TG}}{\partial V_{GD}}\right|_{V_{GS},V_{GB}} = -\left.\frac{\partial Q_{TG}}{\partial V_{DS}}\right|_{V_{GS},V_{GB}} = \left.\frac{\partial Q_{TN}}{\partial V_{DS}}\right|_{V_{GS},V_{GB}} = 0$$

Capacitances



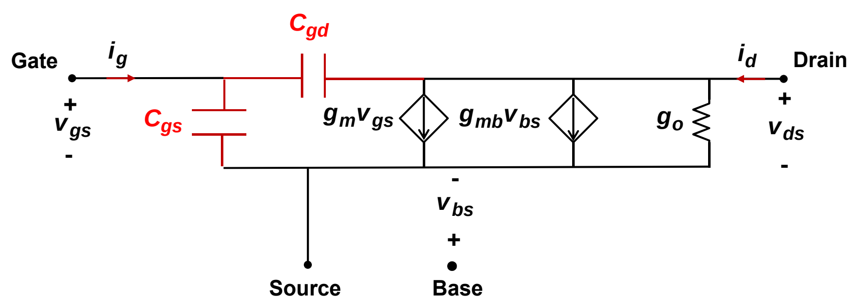
In Saturation:

$$C_{gs} = \left.\frac{\partial Q_G}{\partial V_{GS}}\right|_{V_{GS},V_{SB}} = \frac{2}{3}WLC_{ox} + WC_{ov} + WC_p > \frac{2}{3}WLC_{ox}$$

$$C_{gd} = \left.\frac{\partial Q_G}{\partial V_{GD}}\right|_{V_{GS},V_{SB}} = 0 + WC_{ov} + WC_p > 0$$

NFET: High Frequency Small Signal Model for Saturation Region

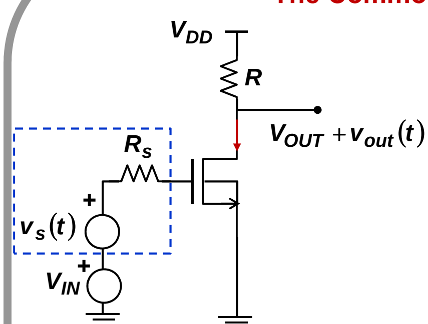
Our simplified high frequency model:



$$C_{gs} = \frac{2}{3} WLC_{ox} + WC_{ov} + WC_p$$

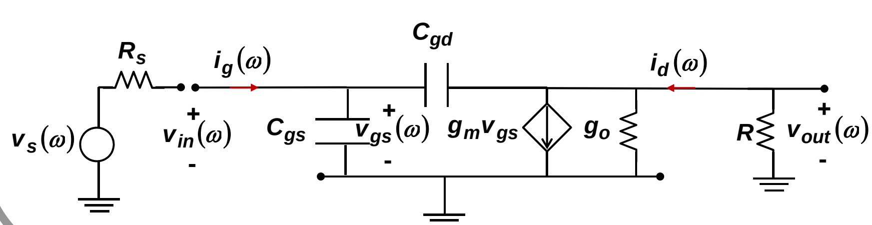
$$C_{gd} = WC_{ov} + WC_p$$

The Common Source Amplifier

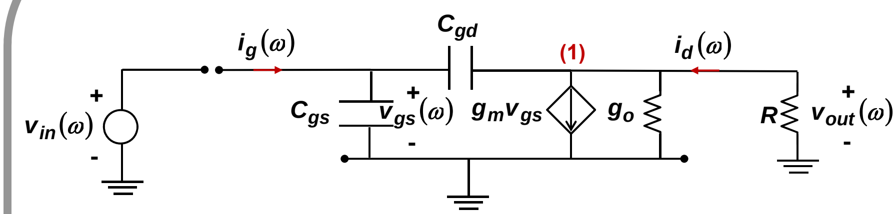


$$v_s(t) = \mathrm{Re}\{ v_s(\omega) e^{j\omega t} \}$$

A high frequency small signal model can be built as follows:



The Common Source Amplifier: Open Circuit Voltage Gain



Need to find:

$$A_v(\omega)=\frac{v_{out}(\omega)}{v_{in}(\omega)} \quad \text{and} \quad H(\omega)=\frac{v_{out}(\omega)}{v_s(\omega)}$$

KCL at (1) gives:

$$i_d(\omega)=g_o v_{out}(\omega)+g_m v_{in}(\omega)+[v_{out}(\omega)-v_{in}(\omega)]j\omega C_{gd}$$

Also:

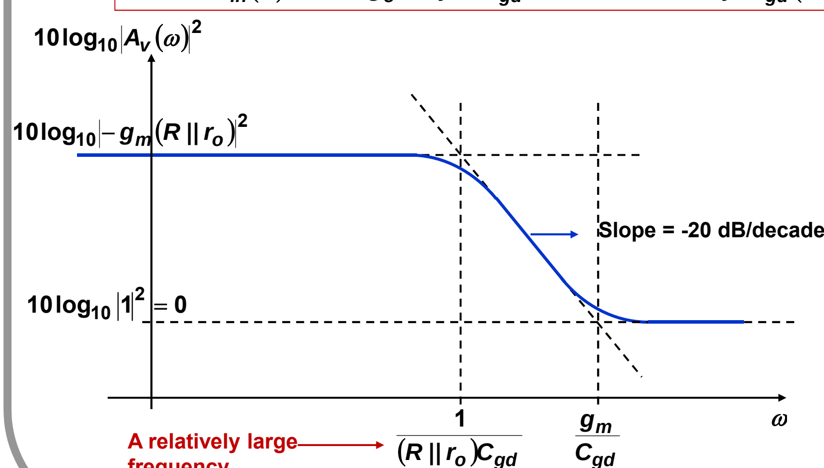
$$v_{out}(\omega)=-i_d(\omega)R$$

The above two give:

$$A_v(\omega)=\frac{v_{out}(\omega)}{v_{in}(\omega)}=-\frac{g_m R-j\omega R C_{gd}}{1+g_o R+j\omega R C_{gd}}$$

The Common Source Amplifier: Open Circuit Voltage Gain

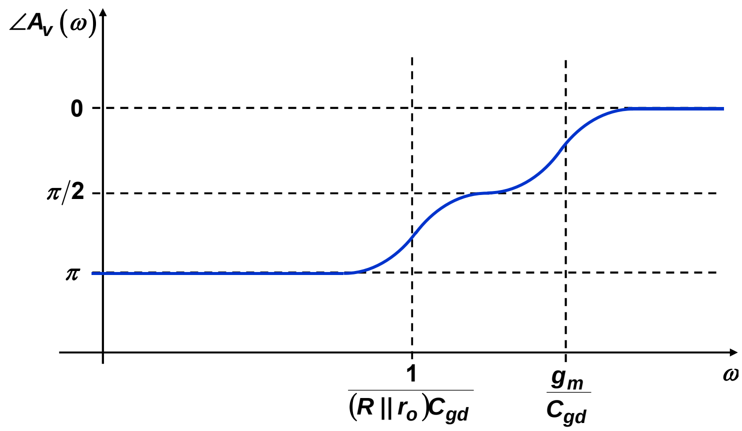
$$\boxed{A_v(\omega)=\frac{v_{out}(\omega)}{v_{in}(\omega)}=-\frac{g_m R-j\omega R C_{gd}}{1+g_o R+j\omega R C_{gd}}=-g_m(R\,||\,r_o)\frac{1-j\omega\frac{C_{gd}}{g_m}}{1+j\omega C_{gd}(R\,||\,r_o)}}$$



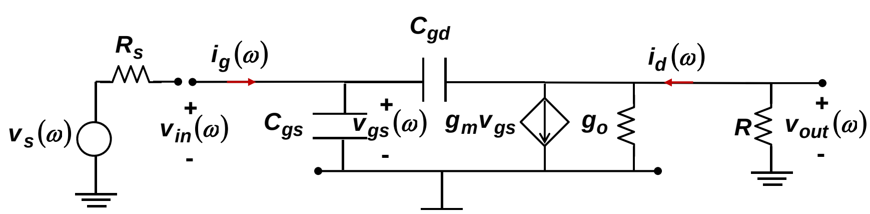
These poles might not limit the frequency performance…….PTO

The Common Source Amplifier: Open Circuit Voltage Gain

$$A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = -\frac{g_m R - j\omega R C_{gd}}{1 + g_o R + j\omega R C_{gd}} = -g_m (R \,||\, r_o) \frac{1 - j\omega \dfrac{C_{gd}}{g_m}}{1 + j\omega C_{gd}(R \,||\, r_o)}$$



The Common Source Amplifier: Input-Output Response



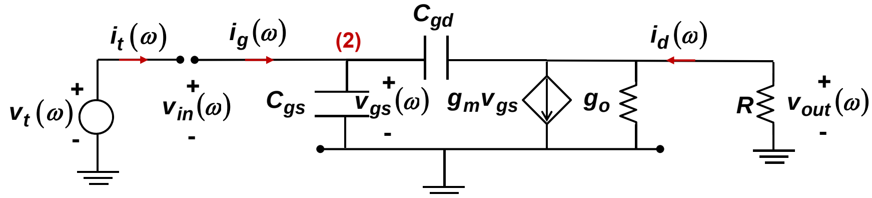
So far we have found the open circuit voltage gain:

$$A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = -\frac{g_m R - j\omega R C_{gd}}{1 + g_o R + j\omega R C_{gd}} = -g_m (R \,||\, r_o) \frac{1 - j\omega \dfrac{C_{gd}}{g_m}}{1 + j\omega C_{gd}(R \,||\, r_o)}$$

What about:

$$\frac{v_{out}(\omega)}{v_s(\omega)} = ??$$

The Common Source Amplifier: Input Impedance



KCL at (2) gives:

$$\frac{v_{out}(\omega)}{v_{in}(\omega)} = A_v(\omega)$$

$$i_t(\omega) = j\omega C_{gs} v_t(\omega) + j\omega C_{gd}\left[v_t(\omega) - v_{out}(\omega)\right]$$

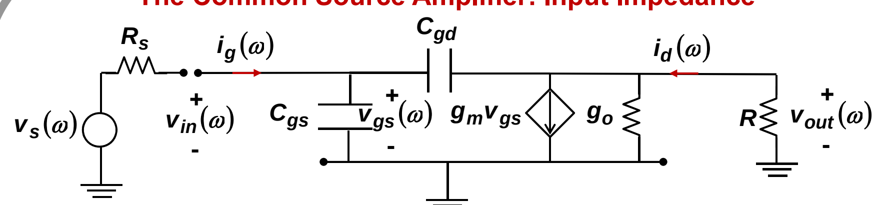
$$= \left(j\omega C_{gs} + j\omega C_{gd}\left[1 - A_v(\omega)\right]\right) v_t(\omega)$$

$$\frac{v_t(\omega)}{i_t(\omega)} = Z_{in}(\omega) = \frac{1}{j\omega C_{gs} + j\omega C_{gd}\left[1 - A_v(\omega)\right]}$$

The input impedance is:

$$Z_{in}(\omega) \approx \frac{1}{j\omega\left[C_{gs} + C_{gd}(1 - A_v(\omega))\right]}$$

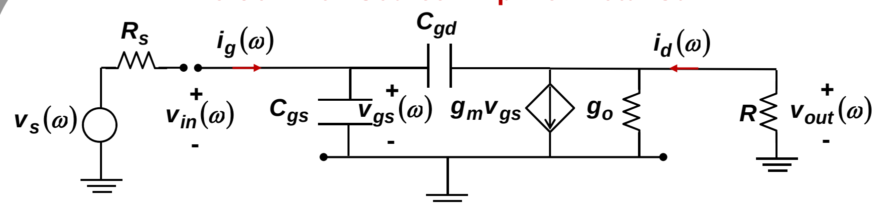
The Common Source Amplifier: Input Impedance



$$\frac{v_{in}(\omega)}{v_s(\omega)} \approx \underbrace{\frac{Z_{in}(\omega)}{R_s + Z_{in}(\omega)}}_{\text{Input voltage divider}} = \frac{1}{1 + j\omega\underbrace{\left[C_{gs} + C_{gd}(1 - A_v(\omega))\right]}_{\text{Miller Effect!!}}R_s}$$

Looking in from the input terminal the capacitance C_{gd} seems to be magnified by a factor proportional to the open circuit voltage gain of the amplifier!!

The Common Source Amplifier: Total Gain



Finally:

$$H(\omega)=\frac{v_{out}(\omega)}{v_s(\omega)}=\frac{v_{out}(\omega)}{v_{in}(\omega)}\frac{v_{in}(\omega)}{v_s(\omega)}=A_v(\omega)\frac{Z_{in}(\omega)}{R_s+Z_{in}(\omega)}$$

$$=\frac{A_v(\omega)}{1+j\omega\left[C_{gs}+C_{gd}(1-A_v(\omega))\right]R_s}$$

Where:

$$A_v(\omega)=\frac{v_{out}(\omega)}{v_{in}(\omega)}=-g_m(R\,||\,r_o)\left[\frac{1-j\omega\dfrac{C_{gd}}{g_m}}{1+j\omega C_{gd}(R\,||\,r_o)}\right]$$

The Common Source Amplifier: Poles and Zeros of the Total Gain

$$H(\omega)=\frac{A_v(\omega)}{1+j\omega\left[C_{gs}+C_{gd}(1-A_v(\omega))\right]R_s}=\frac{H(0)(1-j\omega\tau_3)}{(1+j\omega\tau_1)(1+j\omega\tau_2)}\quad\left\{H(0)\approx-g_m(R\,||\,r_o)\right.$$

A little tedious algebra can show that the transfer function above has two poles and a zero

$$\frac{1}{\tau_3}=\frac{g_m}{C_{gd}}$$

1) Assuming $g_m R_S >> 1$, these poles are:

$$\frac{1}{\tau_1}\approx\frac{g_m}{C_{gs}}+\frac{1}{C_{gd}(R\,||\,r_o)}\qquad\qquad\frac{1}{\tau_2}\approx\frac{1}{\left[C_{gs}+C_{gd}(g_m(R\,||\,r_o))\right]R_s}$$

This pole will likely determine the smallest frequency at which the total gain rolls over

2) Assuming $g_m R_S << 1$, these poles are:

$$\frac{1}{\tau_1}\approx\frac{1}{R_s C_{gs}}\qquad\qquad\frac{1}{\tau_2}\approx\frac{1}{C_{gd}(R\,||\,r_o)}$$

This pole will likely determine the smallest frequency at which the total gain rolls over

The Common Source Amplifier: The Miller Approximation

$$\boxed{H(\omega)=\frac{v_{out}(\omega)}{v_s(\omega)}=\frac{A_v(\omega)}{1+j\omega\left[C_{gs}+C_{gd}\left(1-A_v(\omega)\right)\right]\,R_s}}$$

If the poles in $A_v(\omega)$ are at high enough frequencies such that the lowest poles of the full transfer function are determined by other poles (which would be the case if $g_m R_S >> 1$) then one may approximate the open circuit gain $A_v(\omega)$ by its low frequency value:

$$A_v(\omega)=\frac{v_{out}(\omega)}{v_{in}(\omega)}=-\frac{g_m R-j\omega RC_{gd}}{1+g_o R+j\omega RC_{gd}}\approx A_v(\omega=0)=-g_m\left(R\,||\,r_o\right)$$

Miller approximation

And then:

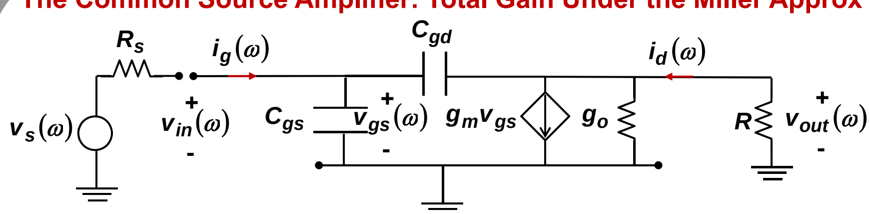
$$H(\omega)=\frac{v_{out}(\omega)}{v_s(\omega)}=\frac{A_v(\omega)}{1+j\omega\left[C_{gs}+C_{gd}\left(1-A_v(\omega)\right)\right]\,R_s}$$

$$\approx\frac{-g_m\left(R\,||\,r_o\right)}{1+j\omega\left[C_{gs}+C_{gd}\underbrace{\left(1+g_m\left(R\,||\,r_o\right)\right)}\right]\,R_s}$$

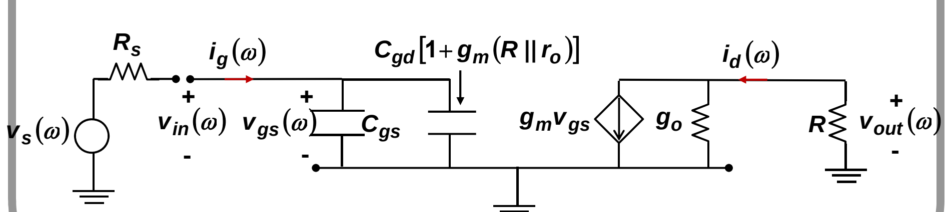
Looking in from the input terminal the capacitance C_{gd} seems to be magnified by a factor proportional to the low frequency open circuit voltage gain of the amplifier!!

And now the single pole is at: $$\frac{1}{\tau}\approx\frac{1}{\left[C_{gs}+C_{gd}\left(1+g_m\left(R\,||\,r_o\right)\right)\right]R_s}$$

The Common Source Amplifier: Total Gain Under the Miller Approx

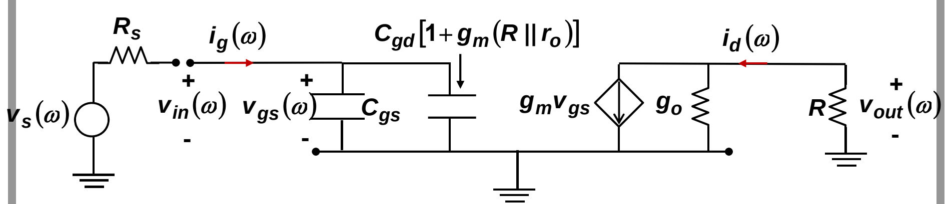


Miller approximation is equivalent to saying that the following circuit will describe the input-output response at not too high frequencies………



The Common Source Amplifier: Input Impedance

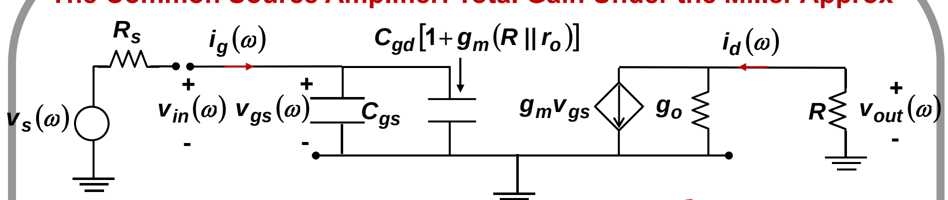
Miller approximation is equivalent to saying that the following circuit will describe the input-output response at not too high frequencies………

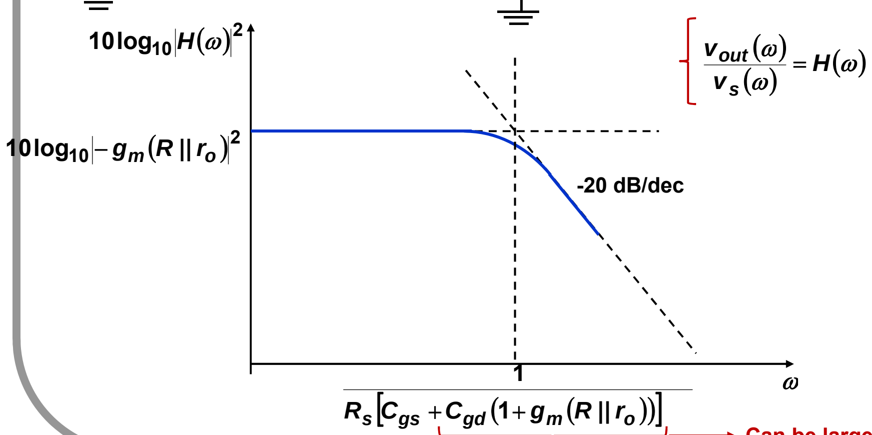


Looking in from the input terminal the capacitance C_{gd} seems to be magnified by a factor proportional to the low frequency open circuit voltage gain of the amplifier!!

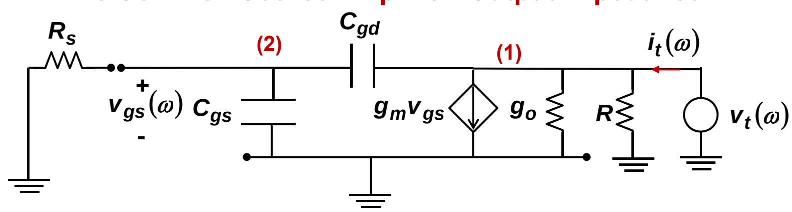
$$Z_{in}(\omega) \approx \frac{1}{j\omega\left[C_{gs} + \underbrace{C_{gd}\left(1+g_m(R \| r_o)\right)}\right]}$$

The Common Source Amplifier: Total Gain Under the Miller Approx





The Common Source Amplifier: Output Impedance



Need to find:

$$Z_{out}(\omega)$$

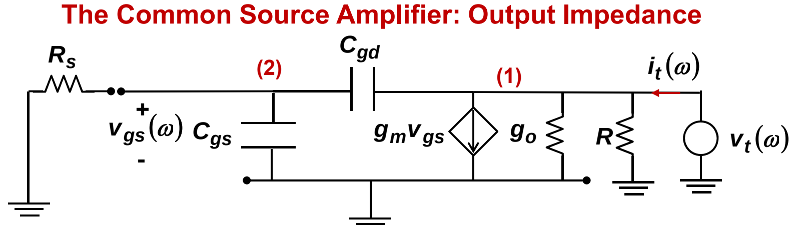
KCL at (2) gives:

$$[v_t(\omega) - v_{gs}(\omega)]j\omega C_{gd} = j\omega C_{gs} v_{gs} + \frac{v_{gs}}{R_s}$$

This gives:

$$v_{gs}(\omega) = \frac{j\omega C_{gd} R_s}{j\omega(C_{gs} + C_{gd})R_s + 1} v_t(\omega)$$

The Common Source Amplifier: Output Impedance



$$v_{gs}(\omega) = \frac{j\omega C_{gd} R_s}{j\omega(C_{gs} + C_{gd})R_s + 1} v_t(\omega)$$

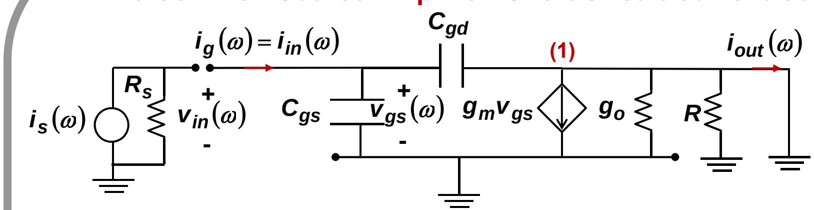
KCL at (1) gives:

$$i_t(\omega) = \frac{v_t(\omega)}{(R \,||\, r_o)} + g_m v_{gs}(\omega) + [v_t(\omega) - v_{gs}(\omega)]j\omega C_{gd}$$

$$\Rightarrow \frac{1}{Z_{out}(\omega)} = \frac{1}{(R \,||\, r_o)} + \frac{(g_m + j\omega C_{gs})j\omega C_{gd} R_s}{j\omega(C_{gs} + C_{gd})R_s + 1}$$

Not a bad approximation even at moderately high frequencies

The Common Source Amplifier: Short Circuit Current Gain



Need to find:

$$\frac{i_{out}(\omega)}{i_{in}(\omega)}$$

$$Z_{in}(\omega) \approx \frac{1}{j\omega\left[C_{gs} + C_{gd}\right]}$$

Start from KCL at (1):

$$(0 - v_{in}(\omega))j\omega C_{gd} + g_m v_{in}(\omega) + i_{out}(\omega) = 0$$

$$\Rightarrow \frac{i_{out}(\omega)}{v_{in}(\omega)} = -\left(g_m - j\omega C_{gd}\right)$$

$$\Rightarrow \frac{i_{out}(\omega)}{i_{in}(\omega)} = \frac{i_{out}(\omega)}{v_{in}(\omega)}\frac{v_{in}(\omega)}{i_{in}(\omega)} = -\left(g_m - j\omega C_{gd}\right)Z_{in}(\omega) \approx -g_m Z_{in}(\omega)$$

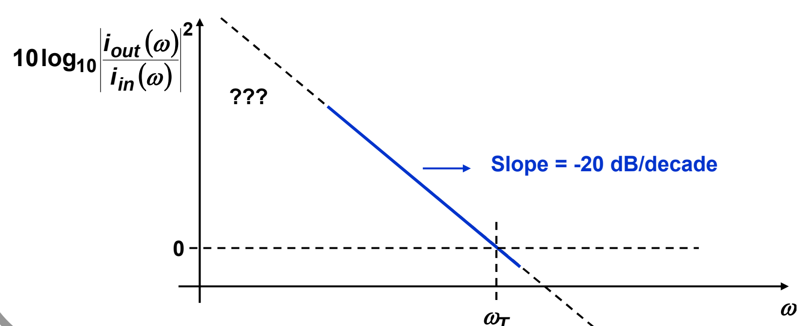
$$= -\frac{g_m}{j\omega\left(C_{gs} + C_{gd}\right)}$$

Short Circuit Current Gain and the Transition Frequency (f_T or ω_T)

For most transistors, the short circuit current gain falls of with frequency with a -20 dB/dec slope (at high enough frequencies)

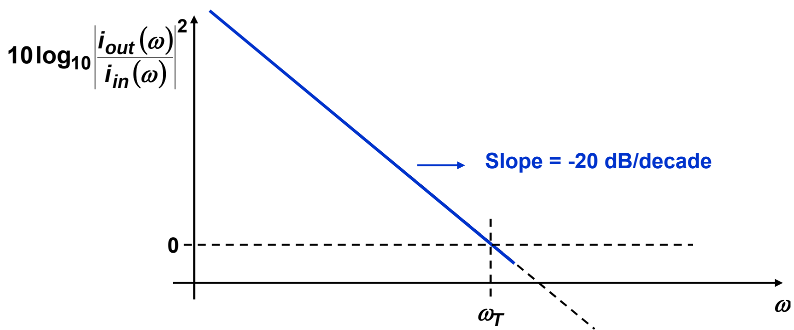
The frequency at which the short circuit current gain equals unity is called the transition frequency (f_T or ω_T)

The transition frequency expresses the maximum useful frequency of operation of the transistor



The Common Source Amplifier: The Transition Frequency

$$\frac{i_{out}(\omega)}{i_{in}(\omega)} = -\frac{g_m}{j\omega(C_{gs}+C_{gd})}$$



The transition frequency is:

$$\omega_T = \frac{g_m}{C_{gs}+C_{gd}} \approx \frac{g_m}{C_{gs}}$$

The Common Source Amplifier: The Transition Frequency

The FET transition frequency is:

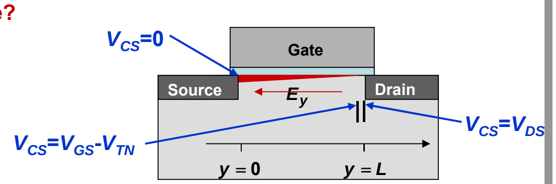
$$\omega_T \approx \frac{g_m}{C_{gs}}$$

This is the highest frequency at which the transistor is still useful

Q: What is its physical significance?

$$g_m = \frac{W}{L}\mu_n C_{ox}(V_{GS}-V_{TN})$$

$$C_{gs} = \frac{2}{3}WLC_{ox}$$



Therefore the transition frequency is:

$$\omega_T = \frac{g_m}{C_{gs}} = \frac{3}{2}\mu_n\frac{(V_{GS}-V_{TN})}{L^2}$$

Electron transit time through the FET

$$\omega_T \propto \mu_n\frac{(V_{GS}-V_{TN})}{L^2} = \frac{\mu_n\frac{(V_{GS}-V_{TN})}{L}}{L} = \frac{\mu_n|E_y|}{L} = \frac{v_n}{L} = \frac{1}{\tau_t}$$

Therefore, the electron transit time sets the maximum frequency of operation of the FET!!