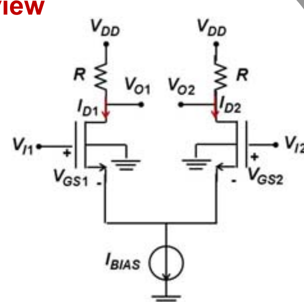
Lecture 17

Differential Amplifiers – II
Current Mirror Load and Single-Ended Output

In this lecture you will learn:

- Differential Amplifiers
- Use of Current Mirrors in Differential Amplifiers
- Small Signal and Large Signal Models with Current Mirrors

Differential Amplifier: Review



Difference-Mode Gain:

$$A_{vd} = \frac{v_{od}}{v_{id}} = -g_{m1}(r_{o1} \,||\, R)$$

Common-Mode Gain:

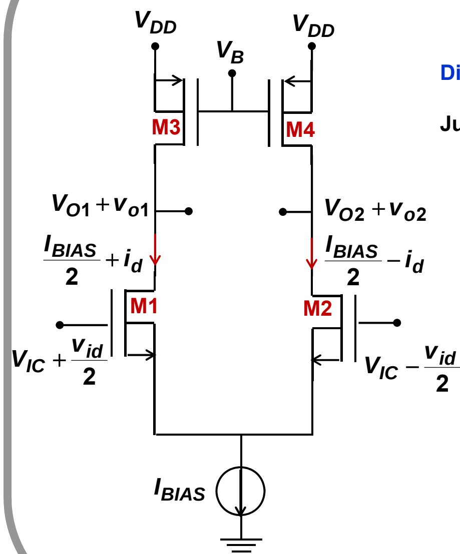
$$A_{vc} = \frac{v_{oc}}{v_{ic}} = -\frac{g_{m1}(r_{o1} \,||\, R)}{1 + \frac{2r_{oc}}{r_{o1}+R}\left[1+(g_{m1}+g_{mb1})r_{o1}\right]}$$

Common-Mode Rejection Ratio (CMRR):

$$CMRR = \frac{A_{vd}}{A_{vc}} = 1 + \frac{2r_{oc}}{r_{o1}+R}\left[1+(g_{m1}+g_{mb1})r_{o1}\right]$$
$$\sim g_{m1}(2r_{oc})$$

→ Large if r_{oc} is large

Differential Amplifier: Review

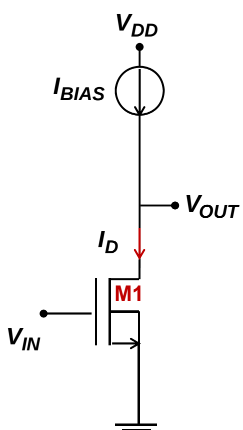


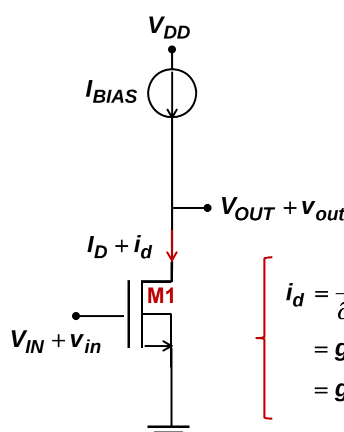
Difference-Mode Gain:

Just copy from the previous slide:

$$A_{vd}=\frac{v_{od}}{v_{id}}=-g_{mn}\left(r_{on}\,||\,r_{op}\right)$$

FET CS Amplifier Operation: Review





$$\begin{aligned} i_d &= \frac{\partial I_D}{\partial V_{GS}}v_{gs}+\frac{\partial I_D}{\partial V_{DS}}v_{ds} \\ &= g_m v_{gs}+g_o v_{ds} \\ &= g_m v_{in}+g_o v_{out} \end{aligned}$$

Assume a NFET CS amp with an ideal current source bias

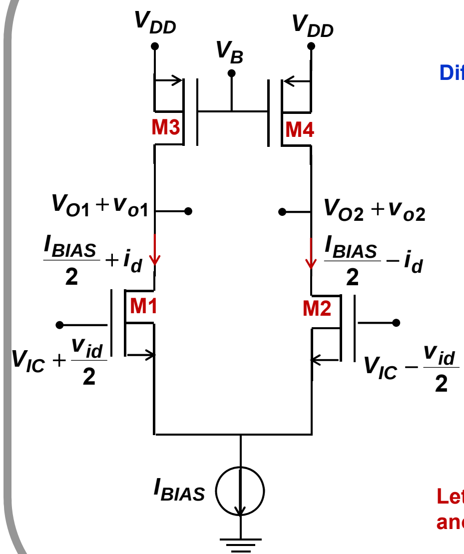
$$I_D = I_{BIAS}$$

Since: $i_d = 0$

$$\Rightarrow g_m v_{in}+g_o v_{out}=0$$

$$\Rightarrow \frac{v_{out}}{v_{in}}=-\frac{g_m}{g_o}=-g_m r_o$$

Differential Amplifier: Review

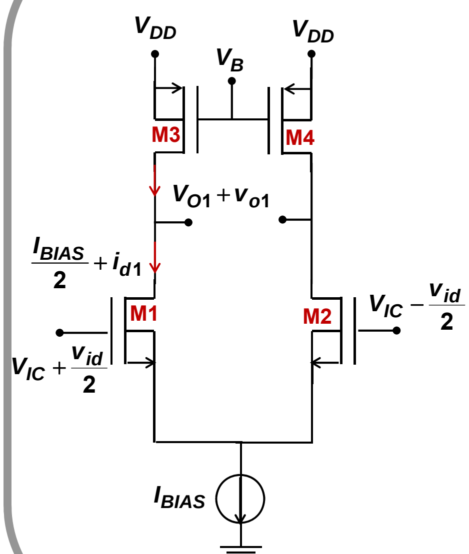


Difference-Mode Gain:

$$A_{vd}=\frac{v_{od}}{v_{id}}=-g_{mn}\left(r_{on}\,||\,r_{op}\right)$$

Lets get the above results AGAIN by another way …….

Differential Amplifier: Double-Ended Output

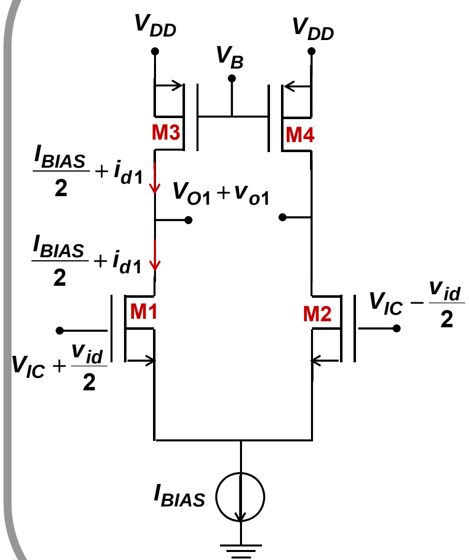


For M1:

$$i_{d1}=g_{mn}v_{gs1}+g_{on}v_{ds1}$$

$$=g_{mn}\frac{v_{id}}{2}+g_{on}v_{o1}$$

Differential Amplifier: Double-Ended Output



For M1:

$$i_{d1} = g_{mn}v_{gs1} + g_{on}v_{ds1}$$

$$= g_{mn}\frac{v_{id}}{2} + g_{on}v_{o1}$$

For M3:

$$i_{d1} = -i_{d3}$$

$$= -\left(g_{mp}v_{gs3} + g_{op}v_{ds3}\right)$$

$$= -\left(g_{mp}0 + g_{op}v_{o1}\right)$$

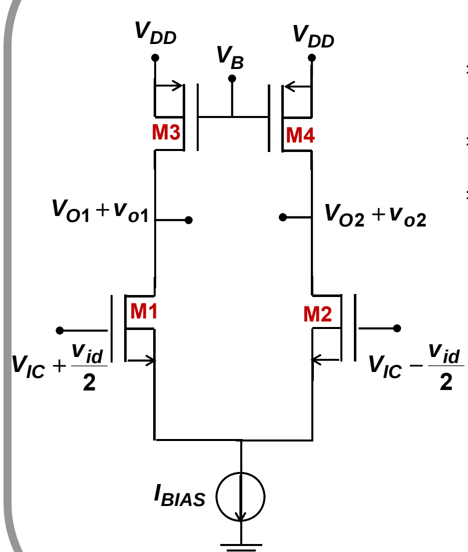
Equating:

$$\Rightarrow g_{mn}\frac{v_{id}}{2} + g_{on}v_{o1} = -g_{op}v_{o1}$$

$$\Rightarrow v_{o1} = -\left(\frac{v_{id}}{2}\right)\frac{g_{mn}}{g_{on}+g_{op}}$$

$$= -\left(\frac{v_{id}}{2}\right)g_{mn}\left(r_{on} \,||\, r_{op}\right)$$

Differential Amplifier: Review



$$\Rightarrow v_{o1} = -\left(\frac{v_{id}}{2}\right)g_{mn}\left(r_{on} \,||\, r_{op}\right)$$

$$\Rightarrow v_{o2} = -v_{o1} = \left(\frac{v_{id}}{2}\right)g_{mn}\left(r_{on} \,||\, r_{op}\right)$$

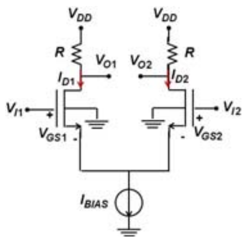
$$\Rightarrow v_{od} = v_{o1} - v_{o2}$$

$$= -\left(v_{id}\right)g_{mn}\left(r_{on} \,||\, r_{op}\right)$$

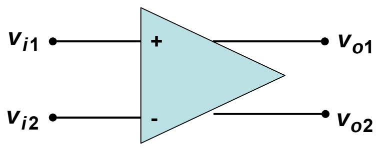
Difference-Mode Gain:

$$A_{vd} = \frac{v_{od}}{v_{id}} = -g_{mn}\left(r_{on} \,||\, r_{op}\right)$$

Differential Amplifier: Double-Ended Output



The FET differential amplifiers considered had a double-ended output



$$v_{o1} = \frac{A_{vd}}{2}(v_{i1} - v_{i2}) + A_{vc}\left(\frac{v_{i1} + v_{i2}}{2}\right)$$
$$= A_{vd}\frac{v_{id}}{2} + A_{vc}v_{ic}$$

$$v_{o2} = -\frac{A_{vd}}{2}(v_{i1} - v_{i2}) + A_{vc}\left(\frac{v_{i1} + v_{i2}}{2}\right)$$
$$= -A_{vd}\frac{v_{id}}{2} + A_{vc}v_{ic}$$

Difference-Mode Output:

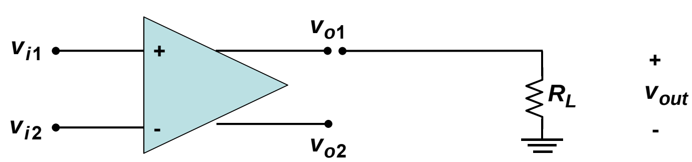
$$v_{od} = v_{o1} - v_{o2} = A_{vd}v_{id}$$

Common-Mode Output:

$$v_{oc} = \frac{v_{o1} + v_{o2}}{2} = A_{vc}v_{ic}$$

Differential Amplifier: Conversion to Single-Ended Output

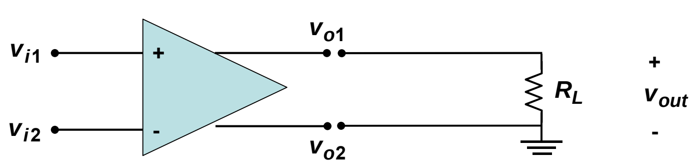
Suppose one tries to connect a load to one of the outputs:



$$v_{out} = v_{o1} = A_{vd}\frac{v_{id}}{2} + A_{vc}v_{ic} \approx A_{vd}\frac{v_{id}}{2}$$

→ We have lost half of the voltage
We can do better……….

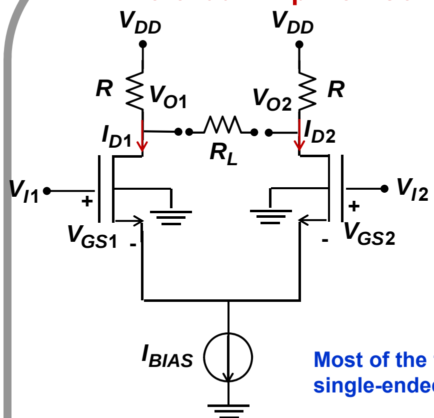
Try another scheme:



$$v_{out} = v_{od} = v_{o1} - v_{o2} = A_{vd}v_{id}$$

→ We have recovered the full signal but this scheme is not always practical

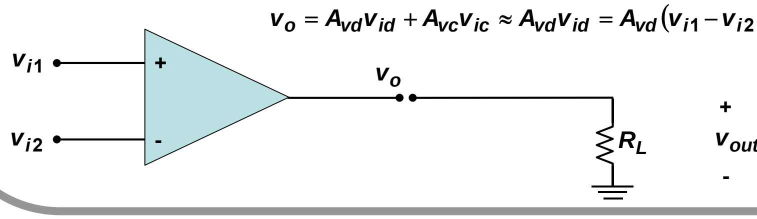
Differential Amplifier: Conversion to Single-Ended Output



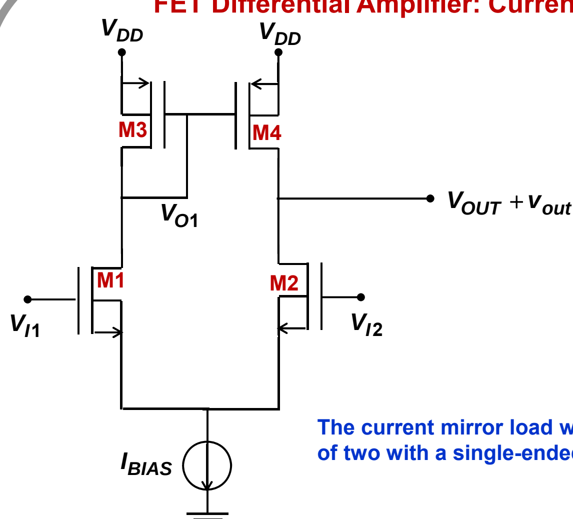
This is an awkward way to connect the load at the output ………!

Most of the time what one would really like to have is a single-ended output (without loosing the factor of 2):

$$v_o = A_{vd} v_{id} + A_{vc} v_{ic} \approx A_{vd} v_{id} = A_{vd}(v_{i1} - v_{i2}) = v_{out}$$

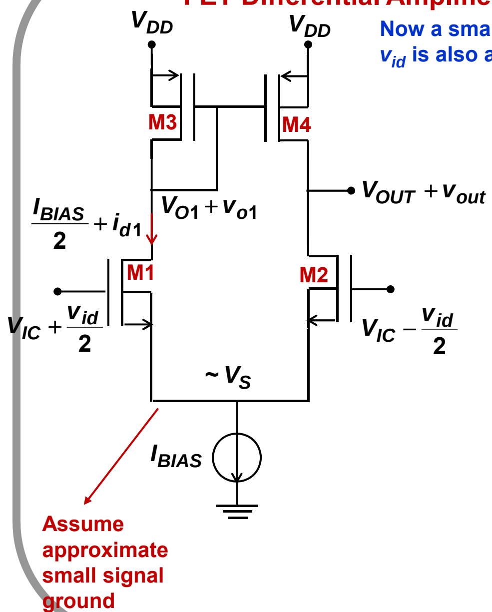


FET Differential Amplifier: Current Mirror Load



The current mirror load will let us recover the factor of two with a single-ended output!

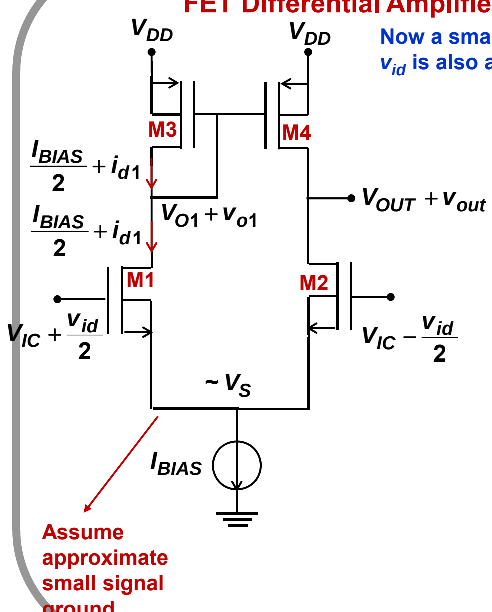
FET Differential Amplifier: Current Mirror Load



Now a small signal difference-mode input signal v_{id} is also applied

$$i_{d1} = g_{mn}v_{gs1} + g_{on}v_{ds1}$$
$$= g_{mn}\frac{v_{id}}{2} + g_{on}v_{o1}$$

FET Differential Amplifier: Current Mirror Load



Now a small signal difference-mode input signal v_{id} is also applied

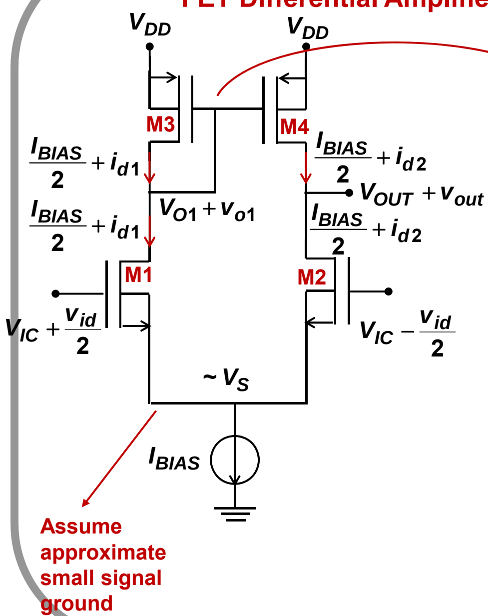
$$i_{d1} = g_{mn}v_{gs1} + g_{on}v_{ds1}$$
$$= g_{mn}\frac{v_{id}}{2} + g_{on}v_{o1}$$

$$i_{d1} = -i_{d3}$$
$$= -\left(g_{mp}v_{gs3} + g_{op}v_{ds3}\right)$$
$$= -\left(g_{mp}v_{o1} + g_{op}v_{o1}\right)$$
$$\approx -g_{mp}v_{o1}$$

Equating:

$$\Rightarrow g_{mn}\frac{v_{id}}{2} + g_{on}v_{o1} = -g_{mp}v_{o1}$$
$$\Rightarrow v_{o1} = -\left(\frac{v_{id}}{2}\right)\frac{g_{mn}}{g_{on}+g_{mp}}$$
$$\approx -\left(\frac{v_{id}}{2}\right)\frac{g_{mn}}{g_{mp}}$$

FET Differential Amplifier: Current Mirror Load



We have:

$$v_{o1} \approx -\left(\frac{v_{id}}{2}\right)\frac{g_{mn}}{g_{mp}}$$

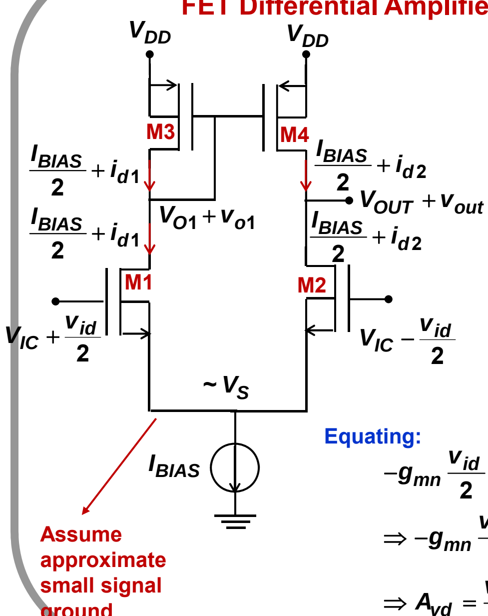
What we have now is the following situation:

M4 is trying to push down extra current because its V_{GS4} has become more negative

And M2 is trying to draw less current because its V_{GS2} has become less positive

The output node voltage V_{OUT} must increase by a lot so that the current pushed down by M4 is decreased and the current pulled in by M2 is increased until these currents are both equal

FET Differential Amplifier: Current Mirror Load



So far we have:

$$v_{o1} \approx -\left(\frac{v_{id}}{2}\right)\frac{g_{mn}}{g_{mp}}$$

Lets look at the right arm now:

$$i_{d2} = g_{mn}v_{gs2} + g_{on}v_{ds2}$$

$$= -g_{mn}\frac{v_{id}}{2} + g_{on}v_{out}$$

$$i_{d2} = -i_{d4}$$

$$= -\left(g_{mp}v_{gs4} + g_{op}v_{ds4}\right)$$

$$= -\left(g_{mp}v_{o1} + g_{op}v_{out}\right)$$

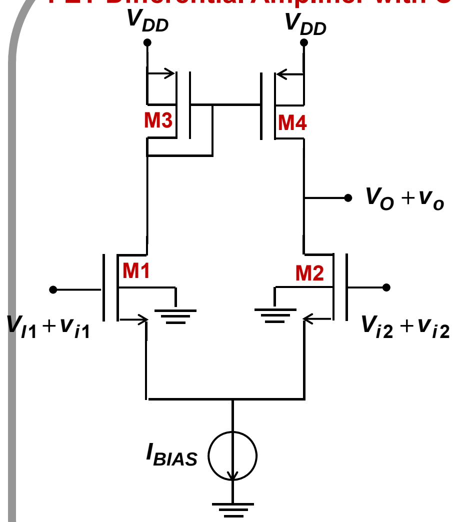
Equating:

$$-g_{mn}\frac{v_{id}}{2} + g_{on}v_{out} = -g_{mp}v_{o1} - g_{op}v_{out}$$

$$\Rightarrow -g_{mn}\frac{v_{id}}{2} + g_{on}v_{out} = g_{mn}\frac{v_{id}}{2} - g_{op}v_{out}$$

$$\Rightarrow A_{vd} = \frac{v_{out}}{v_{id}} = \frac{g_{mn}}{g_{on} + g_{op}} = g_{mn}\left(r_{on} \,||\, r_{op}\right)$$

FET Differential Amplifier with Current Mirror: Small Signal Analysis



Need to make and analyze a small circuit model and calculate:

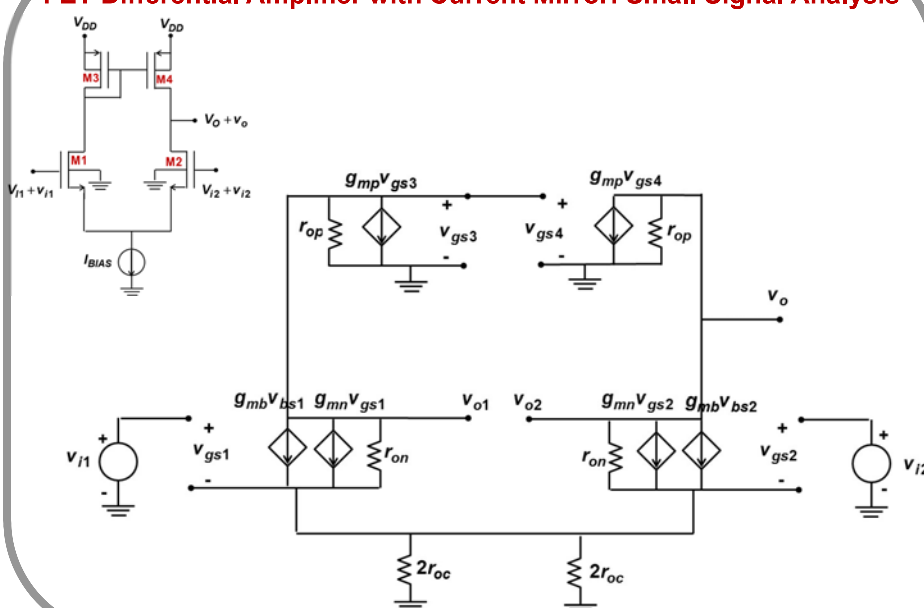
$$A_{vd} = \frac{v_o}{v_{id}}$$ Difference-Mode Gain

(calculated under a pure difference-mode input)

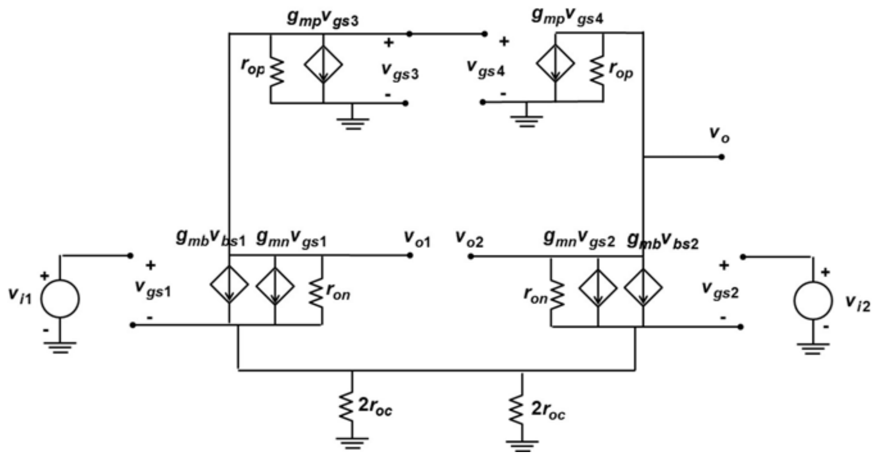
$$A_{vc} = \frac{v_o}{v_{ic}}$$ Common-Mode Gain

(calculated under a pure common-mode input)

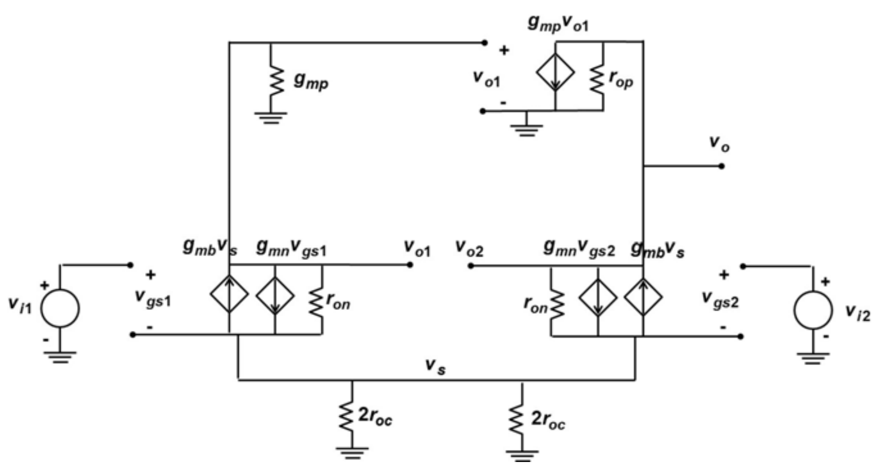
FET Differential Amplifier with Current Mirror: Small Signal Analysis



FET Differential Amplifier with Current Mirror: Small Signal Analysis



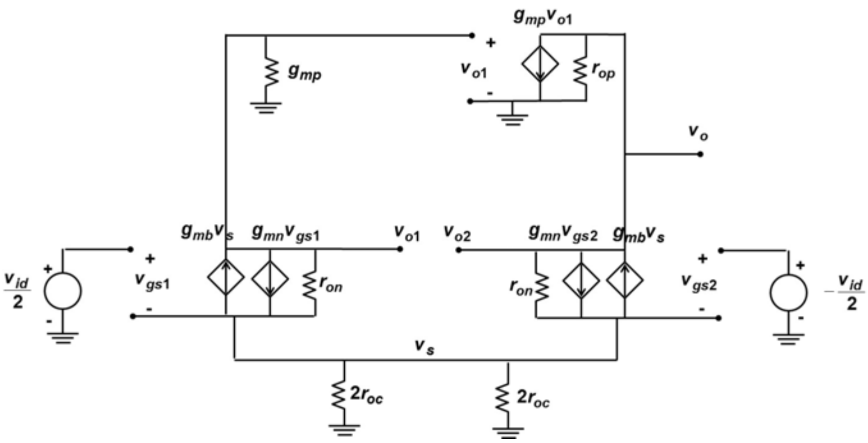
FET Differential Amplifier with Current Mirror: Small Signal Analysis



Small signal circuit model

Note the lack of symmetry!!

Small Signal Analysis: Difference-Mode Input

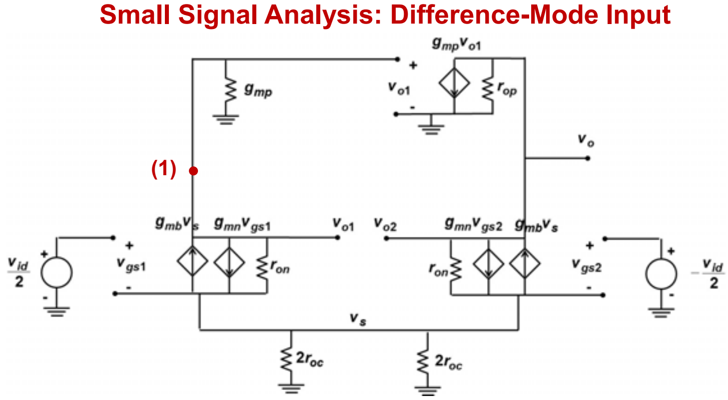


There are three unknown internal voltage variables: v_{o1} v_{o2} v_s

We therefore need three equations

Try KCL at three different nodes…………

Small Signal Analysis: Difference-Mode Input



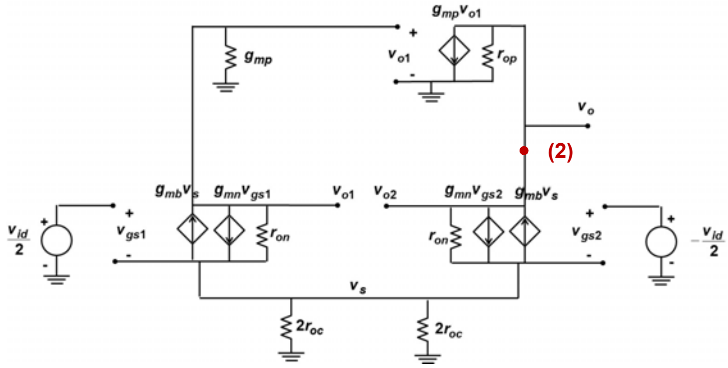
(1) $g_{mn}\left(\frac{v_{id}}{2} - v_s\right) + g_{on}\left(v_{o1} - v_s\right) - g_{mb}v_s + g_{mp}v_{o1} = 0$

small

$\Rightarrow g_{mn}\left(\frac{v_{id}}{2} - v_s\right) - \left(g_{on} + g_{mb}\right)v_s + \left(\cancel{g_{on}} + g_{mp}\right)v_{o1} = 0$

$\Rightarrow g_{mn}\left(\frac{v_{id}}{2} - v_s\right) - \left(g_{on} + g_{mb}\right)v_s + g_{mp}v_{o1} = 0$

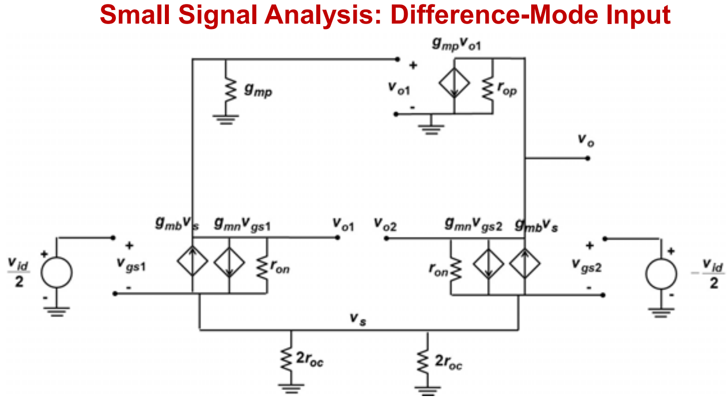
Small Signal Analysis: Difference-Mode Input



(2) $g_{mn}\left(-\dfrac{v_{id}}{2}-v_s\right)+g_{on}\left(v_{o2}-v_s\right)-g_{mb}v_s+g_{mp}v_{o1}+g_{op}v_{o2}=0$

$$\Rightarrow g_{mn}\left(-\frac{v_{id}}{2}-v_s\right)+\left(g_{on}+g_{op}\right)v_{o2}-\left(g_{on}+g_{mb}\right)v_s+g_{mp}v_{o1}=0$$

Small Signal Analysis: Difference-Mode Input



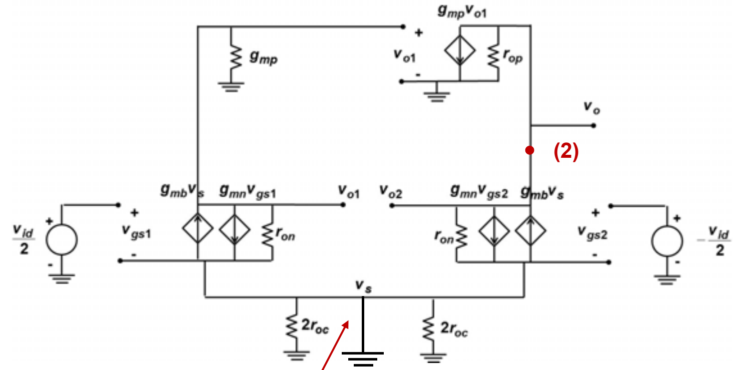
Now subtract (2) from (1):

$$\Rightarrow g_{mn}v_{id}-\left(g_{on}+g_{op}\right)v_{o2}=0$$

$$\Rightarrow v_o=v_{o2}=\frac{g_{mn}}{g_{on}+g_{op}}v_{id}=g_{mn}\left(r_{on}\,||\,r_{op}\right)v_{id}$$

$$\Rightarrow A_{vd}=\frac{v_o}{v_{id}}\approx g_{mn}\left(r_{on}\,||\,r_{op}\right)$$

Small Signal Analysis: Difference-Mode Input



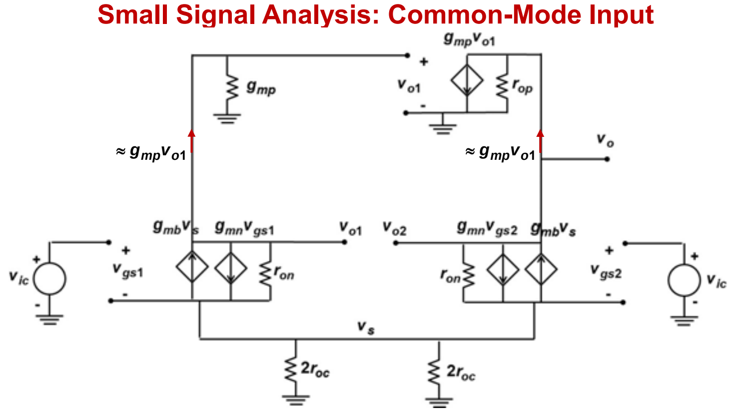
One can also get the correct results by assuming a-priori that the in the difference-mode the voltage v_s is approximately zero (i.e. the node is a small signal ground)

If v_s is assumed to be approximately zero then:

$$v_{o1} \approx -\frac{g_{mn}}{g_{mp}}\frac{v_{id}}{2} \Rightarrow v_o = v_{o2} \approx \frac{\left(-g_{mp}v_{o1} + g_{mn}\frac{v_{id}}{2}\right)}{\left(g_{on} + g_{op}\right)} = \frac{g_{mn}v_{id}}{\left(g_{on} + g_{op}\right)}$$

$$= g_{mn}\left(r_{on} \,||\, r_{op}\right)v_{id}$$

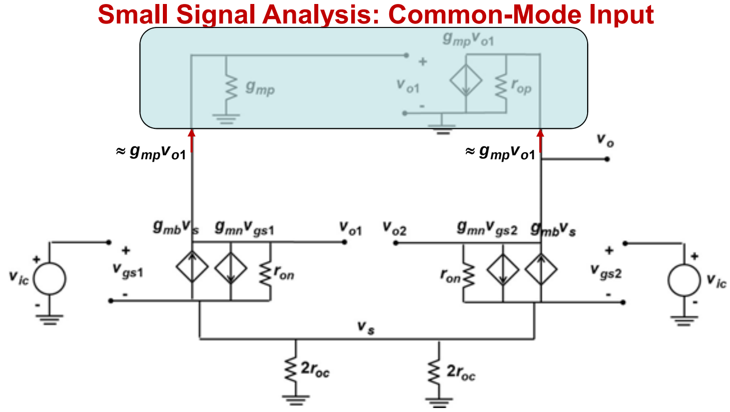
Small Signal Analysis: Common-Mode Input



One good way to think about the amplifier in common-mode operation:

If the output resistance of M4 is assumed to be very large, then the current mirror, as the name suggests will ensure that the drain currents of M1 and M2 are identical (both equal to $-g_{mp}v_{o1}$) – as you can see above as well.

Small Signal Analysis: Common-Mode Input

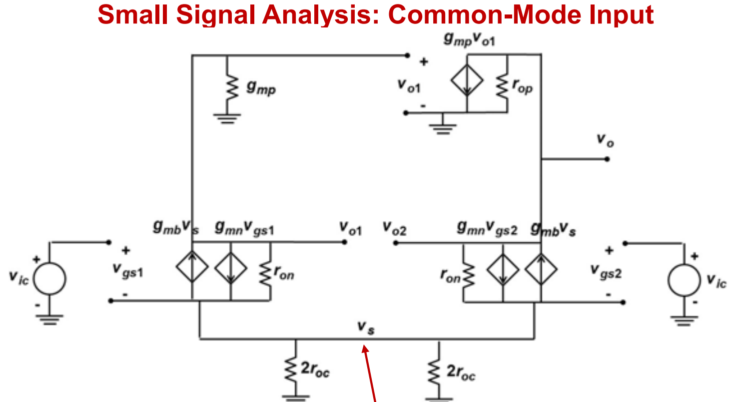


One good way to think about the amplifier in common-mode operation:

If the output resistance of M4 is assumed to be very large, then the current mirror, as the name suggests will ensure that the drain currents of M1 and M2 are identical (both equal to $-g_{mp}v_{o1}$) – as you can see above as well.

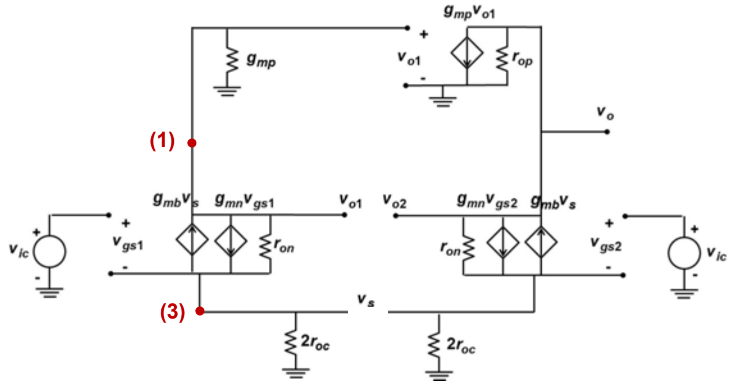
Since the input voltages, v_{ic}, on the two sides are also the same, the circuit (at least the bottom portion) has complete left-right symmetry in common-mode operation

Small Signal Analysis: Common-Mode Input



So one can assume that the in the common-mode, even with the current mirror, no current flows in the bottom most horizontal wire (i.e. it is a small signal open)

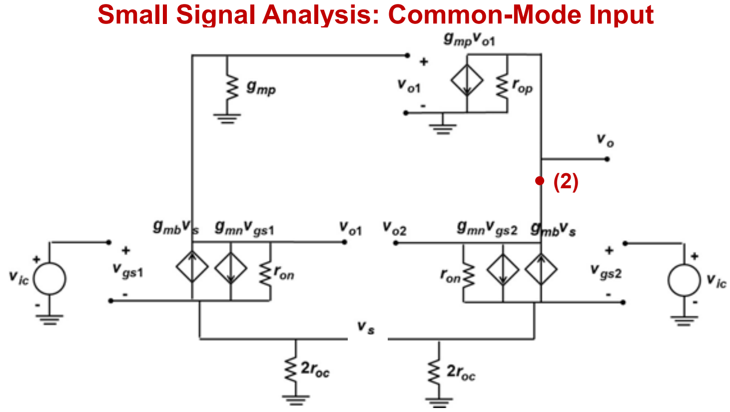
Small Signal Analysis: Common-Mode Input



Doing KCL at (1) and (3) gives:

$$v_{o1} \approx -\frac{\frac{g_{mn}}{g_{mp}} r_{on}}{r_{on} + \frac{1}{g_{mp}} + 2r_{oc} + (g_{mn} + g_{mb}) r_{on} (2r_{oc})} v_{ic}$$

Small Signal Analysis: Common-Mode Input

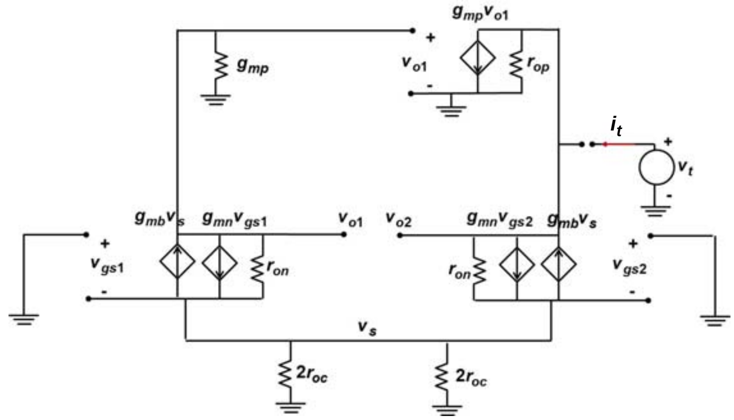


Using the left-right symmetry gives:

$$v_o = v_{o2} \approx v_{o1}$$

$$A_{vc} = \frac{v_o}{v_{ic}} \approx -\frac{\frac{g_{mn}}{g_{mp}} r_{on}}{r_{on} + \frac{1}{g_{mp}} + 2r_{oc} + (g_{mn} + g_{mb}) r_{on} (2r_{oc})}$$

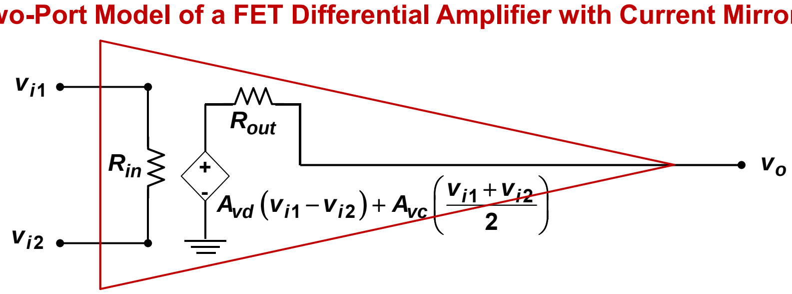
Small Signal Analysis: Output Resistance

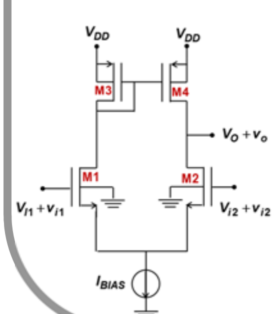


$$R_{out} \approx r_{on} \,||\, r_{op}$$

Left for homework

Two-Port Model of a FET Differential Amplifier with Current Mirror





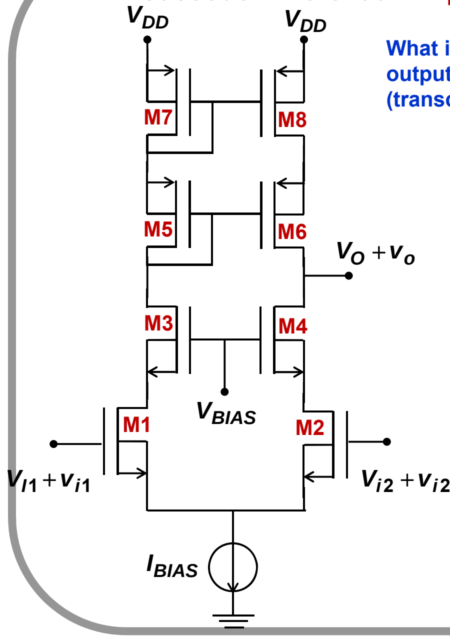
$$A_{vd} \approx g_{mn}(r_{on} \,||\, r_{op})$$

$$A_{vc} = \frac{v_o}{v_{ic}} \approx -\frac{\frac{g_{mn}}{g_{mp}} r_{on}}{r_{on} + \frac{1}{g_{mp}} + 2r_{oc} + (g_{mn}+g_{mb})r_{on}(2r_{oc})}$$

$$R_{out} \approx r_{on} \,||\, r_{op}$$

$$R_{in} = \infty$$

FET Cascode Differential Amplifier with Cascode Current Mirror



What if one needs a large gain and a large output resistance from a differential (transconductance) amplifier?

$$R_{out} \approx (g_{mn} r_{on} r_{on}) \,||\, (g_{mp} r_{op} r_{op})$$

$$A_{vd} \approx g_{mn} \left[(g_{mn} r_{on} r_{on}) \,||\, (g_{mp} r_{op} r_{op}) \right]$$