Lecture #5

ANNOUNCEMENT

• Discussion Section 102 (Th 10-11AM) moved to **105 Latimer**

OUTLINE

– Mobility dependence on temperature
– Diffusion current
– Relationship between band diagrams & *V*, $\mathcal{E}$
– Non-uniformly doped semiconductor
– Einstein relationship
– Quasi-neutrality approximation

Read: Chapter 3.2

Mechanisms of Carrier Scattering

Dominant scattering mechanisms:

1. Phonon scattering (lattice scattering)
2. Impurity (dopant) ion scattering

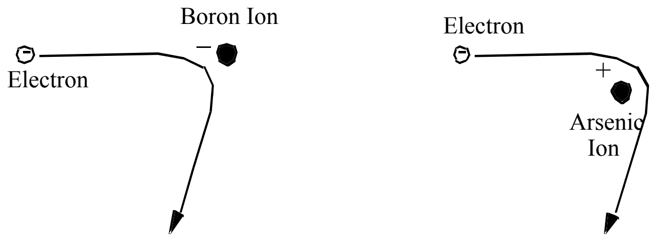
Phonon scattering mobility decreases when *T* increases:

$$\mu_{phonon} \propto \tau_{phonon} \propto \frac{1}{\text{phonon density} \times \text{carrier thermal velocity}} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2}$$

$$\mu = q\tau / m$$

$$v_{th} \propto \sqrt{T}$$

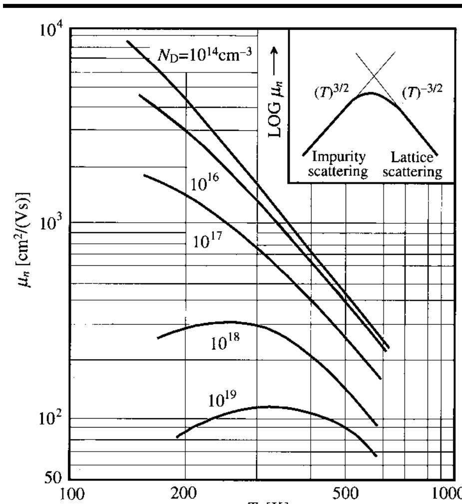
Impurity Ion Scattering



There is less change in the electron's direction of travel if the electron zips by the ion at a higher speed.

$$\mu_{impurity} \propto \frac{v_{th}^3}{N_A + N_D} \propto \frac{T^{3/2}}{N_A + N_D}$$

Temperature Effect on Mobility



$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}}$$

$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}}$$

Example: Temperature Dependence of ρ

Consider a Si sample doped with 10^{17}cm^{-3} As.
How will its resistivity change when the temperature is increased from T=300K to T=400K?

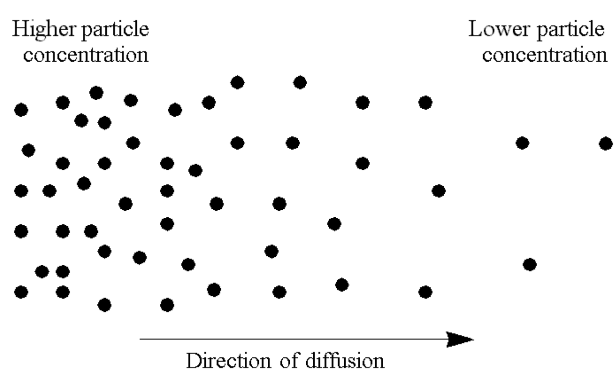
Solution:

The temperature dependent factor in σ (and therefore ρ) is μ_n. From the mobility *vs.* temperature curve for 10^{17}cm^{-3}, we find that μ_n decreases from 770 at 300K to 400 at 400K. As a result, ρ **increases** by

$$\frac{770}{400} = 1.93$$

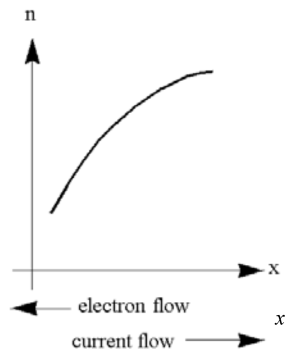
Diffusion

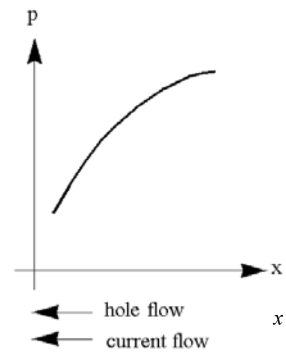
Particles diffuse from regions of higher concentration to regions of lower concentration region, due to random thermal motion.



Diffusion Current

$$J_{\mathrm{N,diff}} = qD_N \frac{dn}{dx} \qquad J_{\mathrm{P,diff}} = -qD_P \frac{dp}{dx}$$





D is the ***diffusion constant***, or ***diffusivity***.

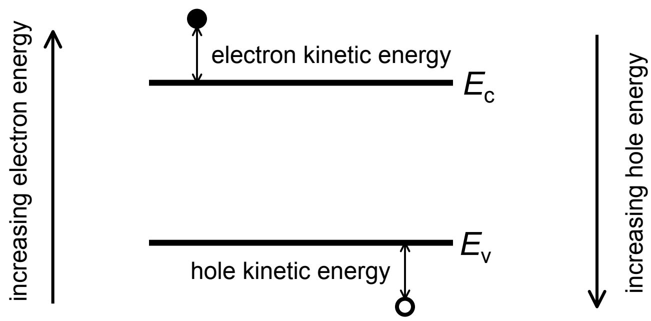
Total Current

$$J = J_{\mathrm{N}} + J_{\mathrm{P}}$$

$$J_{\mathrm{N}} = J_{\mathrm{N,drift}} + J_{\mathrm{N,diff}} = qn\mu_n \mathcal{E} + qD_N \frac{dn}{dx}$$

$$J_{\mathrm{P}} = J_{\mathrm{P,drift}} + J_{\mathrm{P,diff}} = qp\mu_p \mathcal{E} - qD_P \frac{dp}{dx}$$

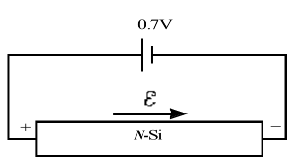
Band Diagram: Potential *vs.* Kinetic Energy

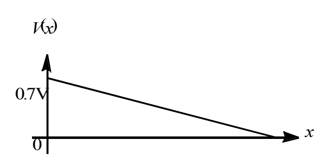


E_c represents the electron potential energy:

$$\text{P.E.} = E_c - E_{\text{reference}}$$

Electrostatic Potential *V*



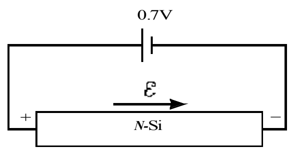


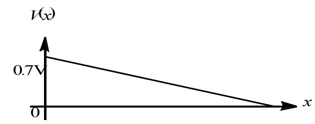
- Potential energy of a –*q* charged particle is related to the electrostatic potential *V*(*x*):

$$\text{P.E.} = -qV$$

$$V = \frac{1}{q}(E_{\text{reference}} - E_c)$$

Electric Field $\mathcal{E}$



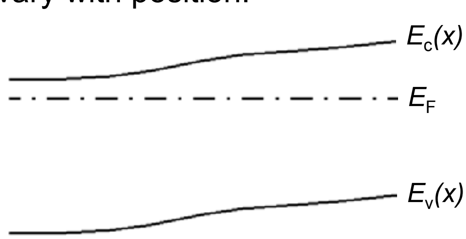


$$\mathcal{E} = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx}$$

- Variation of E_c with position is called ***"band bending."***

Non-Uniformly-Doped Semiconductor

- The position of E_F relative to the band edges is determined by the carrier concentrations, which is determined by the dopant concentrations.
- **In equilibrium, E_F is constant**; therefore, the band energies vary with position:



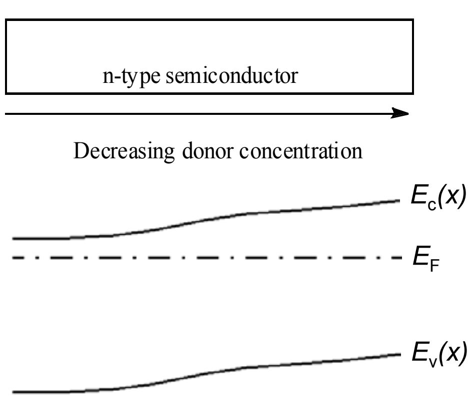
- In equilibrium, there is no net flow of electrons or holes

$$J_N = 0 \quad \text{and} \quad J_P = 0$$

➔ The drift and diffusion current components must balance each other exactly. (A built-in electric field exists, such that the drift current exactly cancels out the diffusion current due to the concentration gradient.)

$$J_N = qn\mu_n \mathcal{E} + qD_N \frac{dn}{dx} = 0$$

Consider a piece of a non-uniformly doped semiconductor:



$$n = N_c e^{-(E_c - E_F)/kT}$$

$$\frac{dn}{dx} = -\frac{N_c}{kT} e^{-(E_c - E_F)/kT} \frac{dE_c}{dx}$$

$$= -\frac{n}{kT} \frac{dE_c}{dx}$$

$$= -\frac{n}{kT} q \mathcal{E}$$

Einstein Relationship between *D* and μ

Under equilibrium conditions, $J_N = 0$ and $J_P = 0$

$$J_N = qn\mu_n\mathcal{E} + qD_N\frac{dn}{dx} = 0$$

$$0 = qn\mu_n\mathcal{E} - qn\frac{qD_N}{kT}\mathcal{E} \quad \rightarrow \quad \boxed{D_N = \frac{kT}{q}\mu_n}$$

Similarly, $\boxed{D_P = \frac{kT}{q}\mu_p}$

<u>Note</u>: The Einstein relationship is valid for a non-degenerate semiconductor, even under non-equilibrium conditions

Example: Diffusion Constant

What is the hole diffusion constant in a sample of silicon with μ_p = 410 cm^2 / V s ?

<u>Solution</u>:

$$D_P = \left(\frac{kT}{q}\right)\mu_p = (26\,\text{mV})\cdot 410\,\text{cm}^2\text{V}^{-1}\text{s}^{-1} = 11\,\text{cm}^2/\text{s}$$

Remember: ***kT/q* = 26 mV** at room temperature.

Potential Difference due to *n(x)*, *p(x)*

- The ratio of carrier densities (*n*, *p*) at two points depends exponentially on the potential difference between these points:

$$E_F - E_{i1} = kT\ln\left(\frac{n_1}{n_i}\right) \quad \Rightarrow \quad E_{i1} = E_F - kT\ln\left(\frac{n_1}{n_i}\right)$$

$$\text{Similarly, } E_{i2} = E_F - kT\ln\left(\frac{n_2}{n_i}\right)$$

$$\text{Therefore} \quad E_{i1} - E_{i2} = kT\left[\ln\left(\frac{n_2}{n_i}\right) - \ln\left(\frac{n_1}{n_i}\right)\right] = kT\ln\left(\frac{n_2}{n_1}\right)$$

$$V_2 - V_1 = \frac{1}{q}\left(E_{i1} - E_{i2}\right) = \frac{kT}{q}\ln\left(\frac{n_2}{n_1}\right)$$

Quasi-Neutrality Approximation

- If the dopant concentration profile varies gradually with position, then the majority-carrier concentration distribution does not differ much from the dopant concentration distribution.

Summary

- **Carrier mobility varies with temperature**
  - decreases w/ increasing *T* if lattice scattering dominant
  - decreases w/ decreasing *T* if impurity scattering dominant

- **Electron/hole concentration gradient → diffusion**

$$J_{\text{N,diff}} = qD_N \frac{dn}{dx} \qquad J_{\text{P,diff}} = -qD_P \frac{dp}{dx}$$

- **Current flowing in a semiconductor is comprised of drift & diffusion components for electrons & holes**

$$J = J_{\text{N,drift}} + J_{\text{N,diff}} + J_{\text{P,drift}} + J_{\text{P,diff}}$$

  - In equilibrium, $J_N = J_{N,drift} + J_{N,diff} = 0$

- **The characteristic constants of drift and diffusion are related:**

$$\frac{D}{\mu} = \frac{kT}{q}$$

- **In thermal equilibrium, E_F is constant**

- **E_c represents the electron potential energy**

  Variation in $E_c(x)$ → variation in electric potential *V*

  Electric field $\mathcal{E} = \frac{dE_c}{dx} = \frac{dE_v}{dx}$

- **$E - E_c$ = electron kinetic energy**