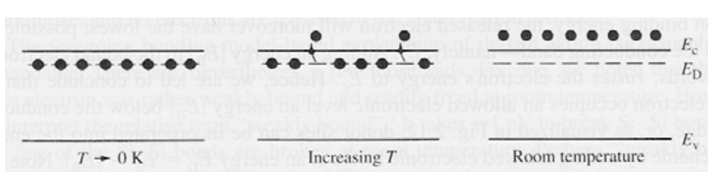
Lecture #3

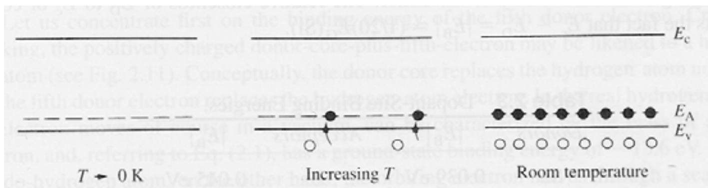
<u>OUTLINE</u>

- Thermal equilibrium
- Fermi-Dirac distribution
  – Boltzmann approximation
- Relationship between E_F and n, p
- Temperature dependence of E_F, n, p

Finish reading Chapter 2

Review: Energy Band Model and Doping





Important Constants

- Electronic charge, q
- Permittivity of free space, ε_o
- Boltzmann constant, k
- Planck constant, h
- Free electron mass, m_o
- Thermal voltage kT/q

Thermal Equilibrium

• No external forces applied:

– electric field = 0

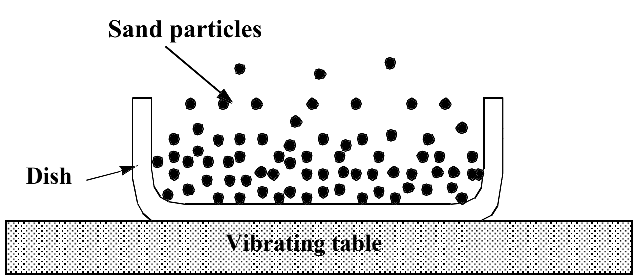
– magnetic field = 0

– mechanical stress = 0

• Thermal agitation –> electrons and holes exchange energy with the crystal lattice and each other

⇒ Every energy state in the conduction and valence bands has a certain probability of being occupied by an electron

Analogy for Thermal Equilibrium



• There is a certain probability for the electrons in the conduction band to occupy high-energy states under the agitation of thermal energy (vibrating atoms, etc.)

Fermi Function

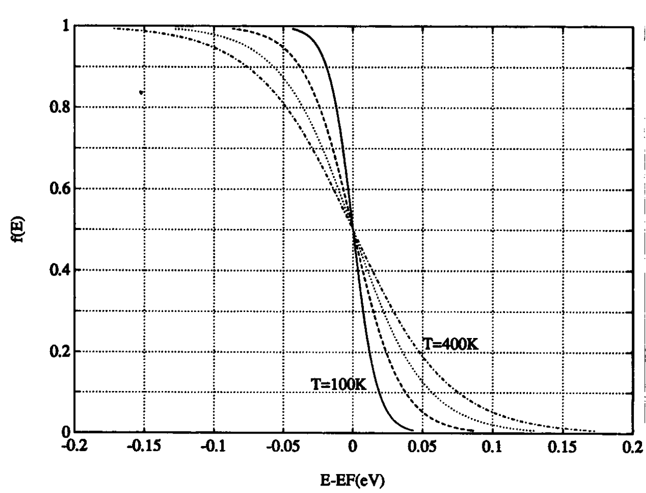
Probability that an available state at energy E is occupied:

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

E_F is called the ***Fermi energy*** or the ***Fermi level***

There is only one Fermi level in a system at equilibrium.

Effect of Temperature on *f*(*E*)

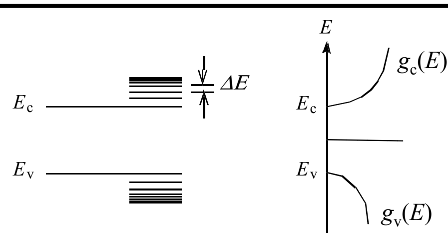


Boltzmann Approximation

$$\text{If } E - E_F > 3kT,\ \ f(E) \approx e^{-(E-E_F)/kT}$$

$$\text{If } E_F - E > 3kT,\ \ f(E) \approx 1 - e^{E-E_F/kT}$$

Density of States



$g(E)dE$ = number of states per cm^3 in the energy range between E and $E+dE$

Near the band edges:

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3} \quad E \geq E_c$$

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3} \quad E \leq E_v$$

Equilibrium Distribution of Carriers

- Obtain $n(E)$ by multiplying $g_c(E)$ and $f(E)$
- Obtain $p(E)$ by multiplying $g_v(E)$ and $1-f(E)$

Equilibrium Carrier Concentrations

- Integrate $n(E)$ over all the energies in the conduction band to obtain n

$$n = \int_{E_c}^{\text{top of conduction band}} g_c(E) f(E) dE$$

- By using the Boltzmann approximation, and extending the integration limit to ∞, we obtain

$$n = N_c e^{-(E_c - E_F)/kT} \quad \text{where} \quad N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

- Integrate $p(E)$ over all the energies in the valence band to obtain p

$$p = \int_{\text{bottom of valence band}}^{E_v} g_v(E)\left[1 - f(E)\right]dE$$

- By using the Boltzmann approximation, and extending the integration limit to -∞, we obtain

$$p = N_v e^{-(E_F - E_v)/kT} \quad \text{where} \quad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

Intrinsic Carrier Concentration n_i

Multiply $n = N_c e^{-(E_c - E_F)/kT}$ and $p = N_v e^{-(E_F - E_v)/kT}$

$$np = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

Recall that $np = n_i^2$

$$\boxed{n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}}$$

Intrinsic Fermi Level

- To find E_F for an intrinsic semiconductor ($n = p = n_i$):

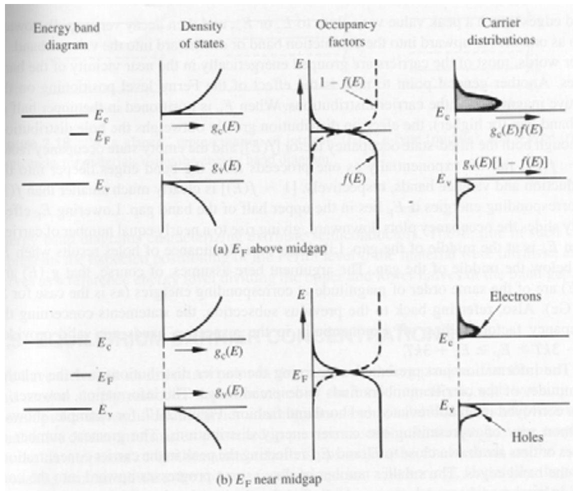
$$n = N_c e^{(E_i - E_C)/kT} = N_v e^{(E_V - E_i)/kT} = p$$

$$E_i = \frac{E_C + E_V}{2} + \frac{kT}{2}\ln\left(\frac{N_V}{N_C}\right)$$

$$E_i = \frac{E_C + E_V}{2} + \frac{3kT}{4}\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$n(n_i, E_F)$ and $p(n_i, E_F)$

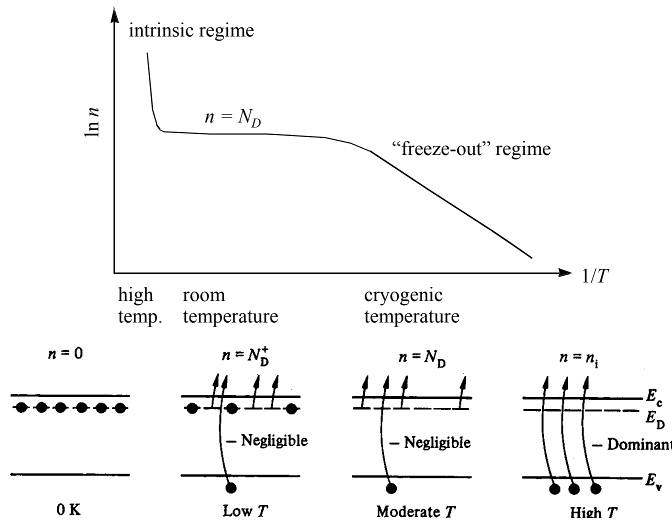
Shifting the Fermi Level



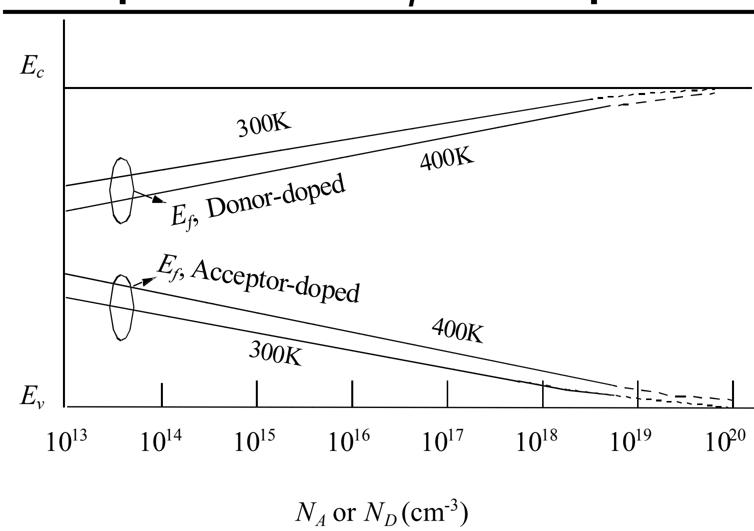
Example: Energy-band diagram

Question: Where is E_F for n = 1x10^{17} cm^{-3}?

Carrier Concentration vs. Temperature



Dependence of E_F on Temperature

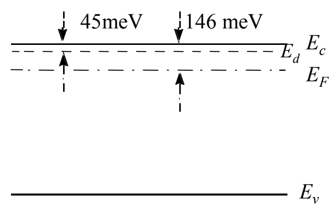


Dopant Ionization

Q: N_d = 10^{17} cm^{-3}. What fraction of the donors are not ionized?

Solution: *First assume that all the donors* ***are*** *ionized.*

$$n = N_D = 10^{17}\text{cm}^{-3} \Rightarrow E_f = E_c - 146\text{meV}$$



$$\text{Probability of non-ionization} \approx \frac{1}{1+e^{(E_d-E_F)/kT}} = \frac{1}{1+e^{((146-45)\text{meV})/26\text{meV}}} = 0.02$$

Therefore, it is reasonable to assume complete ionization, i.e., $n = N_D$

Summary

Thermal equilibrium:

Balance between internal processes with no external stimulus (no electric field, no light, *etc.*)

=> Electron-hole pair (EHP) generation rate = EHP recombination rate

- ***Fermi function***: probability that a state at energy *E* is filled with an electron under equilibrium conditions: $$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$
  - ***Boltzmann approximation***:

    For high *E, i.e. E* - E_F > 3*kT*: $f(E) \cong e^{-(E-E_F)/kT}$

    For low *E, i.e.* E_F – *E* > 3*kT*: $1 - f(E) \cong e^{-(E_F-E)/kT}$

$$n = N_c e^{-(E_c - E_F)/kT} = n_i e^{(E_F - E_i)/kT}$$

$$p = N_v e^{-(E_F - E_v)/kT} = n_i e^{(E_i - E_F)/kT}$$