Lecture #26

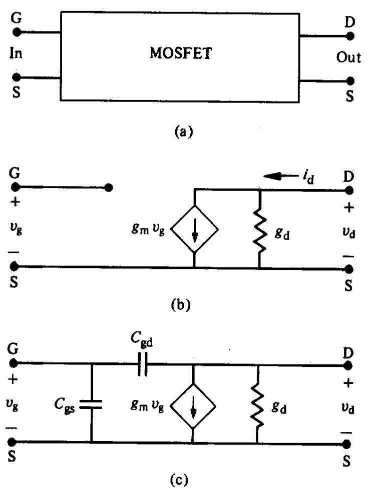
ANNOUNCEMENT

- The lowest HW grade will be dropped for each student

OUTLINE

- Small-signal MOSFET model
- MOSFET scaling
- Velocity saturation
- Short-channel MOSFETs

Small Signal Model

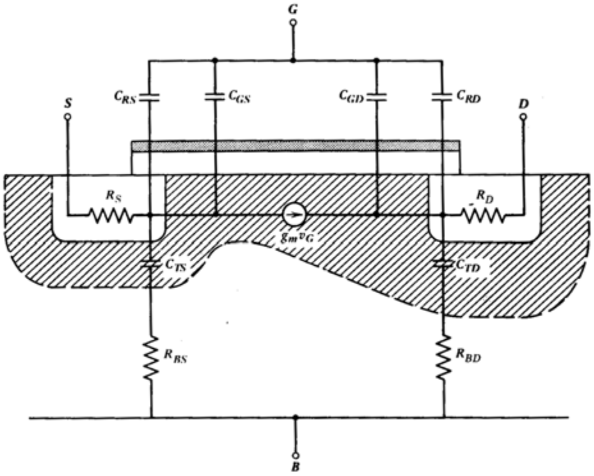


- Conductance parameters:

$$g_d = \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G=const} = \lambda I_{Dsat0}$$

$$g_m = \left.\frac{\partial I_D}{\partial V_G}\right|_{V_D=const} = \frac{W\mu_{eff}C_{oxe}}{mL}(V_{GS}-V_T)$$

Inclusion of Additional Parasitics



Cutoff Frequency

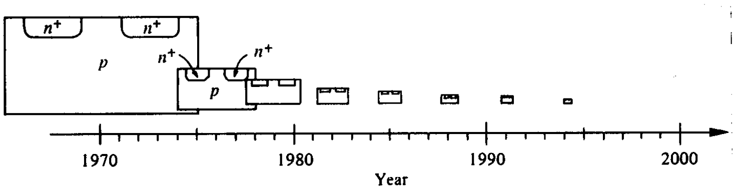
- $f_{\max}$ is the frequency where the MOSFET is no longer amplifying the input signal
  - Obtained by considering the small-signal model with the output terminals short-circuited, and finding the frequency where $|i_{out} / i_{in}| = 1$

$$f_{\max} = \frac{g_m}{2\pi C_{oxe}} = \frac{W\mu_{eff}}{2\pi m L}(V_{GS} - V_T) \propto \frac{1}{L}$$

→ Increased MOSFET operating frequencies are achieved by decreasing the channel length

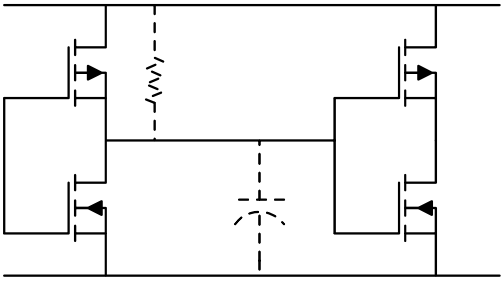
MOSFET Scaling

- MOSFETs have scaled in size over time
  - 1970’s: ~ 10 μm
  - Today: ~50 nm
- Reasons:
  - Speed
  - Density

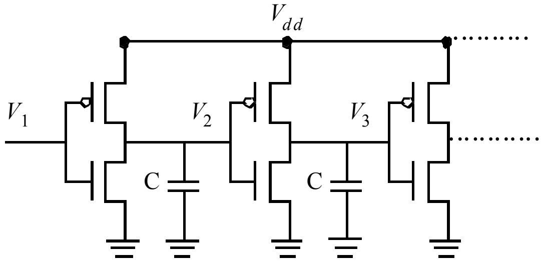


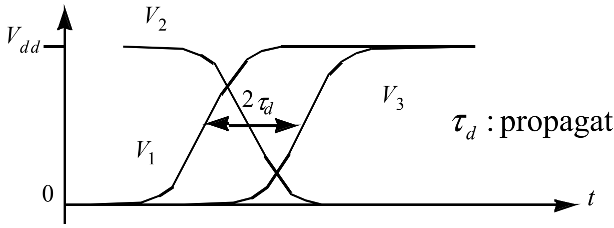
Benefit of Transistor Scaling

- I_{DS} ↑ as L ↓ (decreased effective “R”)
- Gate area ↓ as L ↓ (decreased load “C”)
- Therefore, RC ↓ (implies faster switch)



Circuit Example – CMOS Inverter





τ_d : propagation delay

$$\tau_d \equiv \frac{1}{2}(pull-down\ delay\ +\ pull-up\ delay)$$

$$pull-up\ delay \approx \frac{CV_{dd}}{2I_{dsatP}}$$

$$pull-down\ delay \approx \frac{CV_{dd}}{2I_{dsatN}}$$

$$\tau_d = \frac{CV_{dd}}{4}\left(\frac{1}{I_{dsatN}} + \frac{1}{I_{dsatP}}\right)$$

τ_d is reduced by increasing I_{Dsat}

$$R_N \text{ and } R_P = \frac{V_{dd}}{2I_{on}} = \frac{V_{dd}}{2I_{dsat}(|V_g| = V_{dd})}$$

Velocity Saturation

- velocity saturation has large and deleterious effect on the I_{Dsat} of MOSFETS

$$v = \frac{\mu \mathcal{E}}{1 + \frac{\mathcal{E}}{\mathcal{E}_{sat}}}$$

$$\mathcal{E} << \mathcal{E}_{sat} : v = \mu \mathcal{E}$$

$$\mathcal{E} >> \mathcal{E}_{sat} : v = \mu \mathcal{E}_{sat}$$

MOSFET *I*-*V* with Velocity Saturation

$$I_{DS} = \frac{\frac{W}{L} C_{oxe} \mu_{eff} (V_{GS} - V_T - \frac{m}{2} V_{DS}) V_{DS}}{1 + \frac{V_{DS}}{\mathcal{E}_{sat} L}}$$

$$\boxed{I_{DS} = \frac{long\text{ - }channel\ I_{DS}}{1 + V_{DS} / \mathcal{E}_{sat} L}}$$

Solving for $\frac{dI_{DS}}{dV_{DS}} = 0$,

$$V_{Dsat} = \frac{2(V_{GS} - V_T)}{1 + \sqrt{1 + 2(V_{GS} - V_T)/\mathcal{E}_{sat} L}}$$

A simpler and more accurate V_{Dsat} is:

$$\boxed{\frac{1}{V_{Dsat}} = \frac{m}{V_{GS} - V_T} + \frac{1}{\mathcal{E}_{sat} L}}$$

$$m = 1 + 3T_{oxe}/W_{dm}$$

$$\mathcal{E}_{sat} \equiv \frac{2v_{sat}}{\mu}$$

Drain Saturation Voltage V_{Dsat}

$$\frac{1}{V_{Dsat}} = \frac{m}{V_{GS} - V_{Tn}} + \frac{1}{\mathcal{E}_{sat} L}$$

- If $\mathcal{E}_{sat}L >> V_{GS}-V_{Tn}$ then the MOSFET is considered "long-channel". This condition can be satisfied when
  - L is large, or
  - V_{GS} is close to V_T

EXAMPLE: Drain Saturation Voltage

Question: *At V_{gs} = 1.8 V, what is the V_{Dsat} of an NFET with T_{oxe} = 3 nm, V_T = 0.25 V, and W_{dm} = 45 nm for (a) L =10 μm, (b) L = 1 um, (c) L = 0.1 μm, and (d) L = 0.05 μm*

Solution: *From V_{GS}, V_T, and T_{oxe}, μ_n is 200 $cm^2V^{-1}s^{-1}$.*

$$\mathcal{E}_{sat} = 2v_{sat}/\mu = 8 \times 10^4 \text{ V/cm}$$

$$m = 1 + 3T_{oxe}/W_{dm} = 1.2$$

$$V_{Dsat} = \left(\frac{m}{V_{GS} - V_T} + \frac{1}{\mathcal{E}_{sat} L} \right)^{-1}$$

$$V_{Dsat} = \left(\frac{m}{V_{GS} - V_T} + \frac{1}{\mathcal{E}_{sat} L} \right)^{-1}$$

(a) L = 10 μm, $V_{Dsat} = (1/1.3\text{V} + 1/80\text{V})^{-1} = 1.3$ V

(b) L = 1 μm, $V_{Dsat} = (1/1.3\text{V} + 1/8\text{V})^{-1} = 1.1$ V

(c) L = 0.1 μm, $V_{Dsat} = (1/1.3\text{V} + 1/.8\text{V})^{-1} = 0.5$ V

(d) L = 0.05 μm, $V_{Dsat} = (1/1.3\text{V} + 1/.4\text{V})^{-1} = 0.3$ V

I_{Dsat} with Velocity Saturation

Substituting V_{Dsat} for V_{DS} in I_{DS} equation gives:

$$\boxed{I_{Dsat} = \frac{W}{2mL} C_{oxe} \mu_{eff} \frac{(V_{GS} - V_T)^2}{1 + \frac{V_{GS} - V_T}{\mathcal{E}_{sat} L}} = \frac{long\text{-}channel\ I_{Dsat}}{1 + \frac{V_{GS} - V_T}{\mathcal{E}_{sat} L}}}$$

Very short channel case: $\mathcal{E}_{sat} L << V_{GS} - V_T$

$$\begin{aligned} I_{Dsat} &= \frac{W}{2m} C_{oxe} \mu_n \mathcal{E}_{sat} (V_{GS} - V_T) \\ &= W v_{sat} C_{oxe} (V_{GS} - V_T) / m \end{aligned}$$

- I_{Dsat} is proportional to $V_{GS} – V_T$ rather than $(V_{GS} – V_T)^2$

Summary: NMOSFET *I-V*

- Linear region:

$$I_{DS} = \frac{\frac{W}{L} C_{oxe} \mu_{eff} (V_{GS} - V_{Tn} - \frac{m}{2} V_{DS}) V_{DS}}{1 + \frac{V_{DS}}{\mathcal{E}_{sat} L}}$$

$$\mathcal{E}_{sat} = 2 v_{sat} / \mu_{eff}$$

$$v_{sat} = \begin{cases} 8 \times 10^6 \text{ cm/s for electrons} \\ 6 \times 10^6 \text{ cm/s for holes} \end{cases}$$

- Saturation region:

$$I_{DS} = I_{Dsat} = \frac{\frac{W}{2mL} C_{oxe} \mu_{eff} (V_{GS} - V_{Tn})^2}{1 + \frac{(V_{GS} - V_{Tn})}{\mathcal{E}_{sat} L}}$$

Very-Short-Channel MOSFETs

- If $\mathcal{E}_{sat}L << V_{GS}-V_{Tn}$:

$$V_{Dsat} \cong \mathcal{E}_{sat}L < \frac{(V_{GS}-V_{Tn})}{m}$$

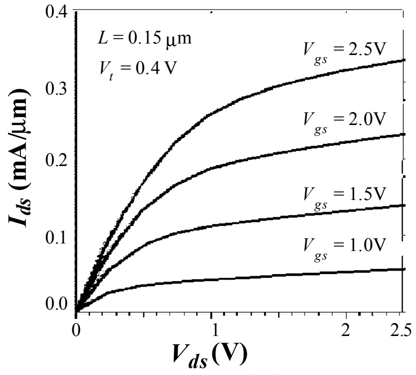
$$I_{Dsat} = \frac{W}{2m}C_{oxe}\mu_{eff}\mathcal{E}_{sat}(V_{GS}-V_{Tn})$$

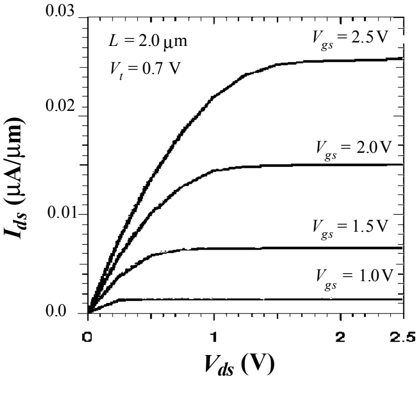
$$= \frac{W}{m}C_{oxe}v_{sat}(V_{GS}-V_{Tn})$$

$\Rightarrow$ I_{Dsat} is not sensitive to L

- **To increase I_{Dsat}** (for faster circuit operation), we must increase $C_{oxe}(V_{GS}-V_{Tn})$, *i.e.* **reduce T_{oxe} and V_{Tn}**

Short- *vs.* Long-Channel MOSFET





Short-channel MOSFET:

- I_{Dsat} is proportional to $V_{GS}-V_{Tn}$ rather than $(V_{GS}-V_{Tn})^2$
- V_{Dsat} is lower than for long-channel MOSFET
- Channel-length modulation is apparent

Velocity Overshoot

- When L is comparable to or less than the mean free path, some of the electrons travel through the channel without experiencing a single scattering event
  - $\rightarrow$ projectile-like motion ("**ballistic transport**")

$\Rightarrow$ The average velocity of carriers exceeds v_{sat}

e.g. 35% for L = 0.12 μm NMOSFET

$\Rightarrow$ Effectively, v_{sat} and $\mathcal{E}_{sat}$ increase when L is very small

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PMOSFET *I-V* with Velocity Saturation

- Linear region:

$$I_{DS} = \frac{-\frac{W}{L} C_{oxe} \mu_{eff} (V_{GS} + V_{Tp} - \frac{m}{2} V_{DS}) V_{DS}}{1 + \frac{|V_{DS}|}{\mathcal{E}_{sat} L}}$$

- Saturation region:

$$I_{DS} = I_{Dsat} = \frac{-\frac{W}{2mL} C_{oxe} \mu_{eff} (V_{GS} - V_{Tp})^2}{1 + \frac{|V_{GS} - V_{Tp}|}{\mathcal{E}_{sat} L}}$$