Lecture #21

OUTLINE

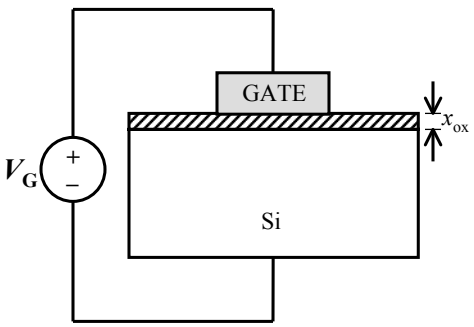
The MOS Capacitor

- Electrostatics

Reading: Course Reader

MOS Capacitor Structure

MOS capacitor (cross-sectional view)



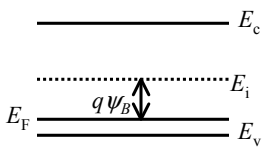
- Typical MOS capacitors and transistors in ICs today employ
  - heavily doped polycrystalline Si ("poly-Si") film as the gate-electrode material
    - n^+-type, for "n-channel" transistors (NMOS)
    - p^+-type, for "p-channel" transistors (PMOS)
  - SiO_2 as the gate dielectric
    - band gap = 9 eV
    - $\varepsilon_{r,SiO2} = 3.9$
  - Si as the semiconductor material
    - p-type, for "n-channel" transistors (NMOS)
    - n-type, for "p-channel" transistors (PMOS)

Bulk Semiconductor Potential ψ_B

$$q\psi_B \equiv E_i(bulk) - E_F$$

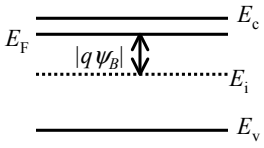
- p-type Si:

$$\psi_B = \frac{kT}{q}\ln(N_A / n_i) > 0$$

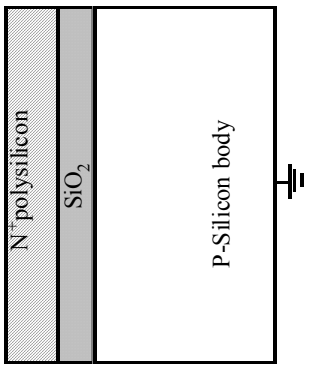


- n-type Si:

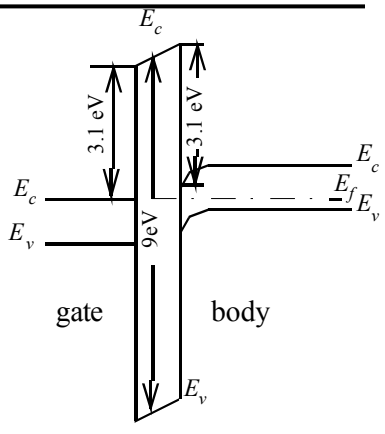
$$\psi_B = -\frac{kT}{q}\ln(N_D / n_i) < 0$$



MOS Equilibrium Energy-Band Diagram



(a)



(b)

How does one arrive at this energy-band diagram?

Guidelines for Drawing MOS Band Diagrams

- Fermi level E_F is flat (constant with distance x) in the Si
  - Since no current flows in the x direction, we can assume that equilibrium conditions prevail
- Band bending is linear in the oxide
  - No charge in the oxide => $d\mathcal{E}/dx = 0$ so $\mathcal{E}$ is constant
    => dE_c/dx is constant
- From Gauss' Law, we know that the electric field strength in the Si at the surface, $\mathcal{E}_{Si}$, is related to the electric field strength in the oxide, $\mathcal{E}_{ox}$:

$$\mathcal{E}_{ox} = \frac{\varepsilon_{Si}}{\varepsilon_{ox}}\ \mathcal{E}_{Si} \cong 3\,\mathcal{E}_{Si} \qquad so \quad \left.\frac{dE_c}{dx}\right|_{oxide} = 3 \times \left.\frac{dE_c}{dx}\right|_{Si\,(at\ the\ surface)}$$

MOS Band-Diagram Guidelines (cont.)

- The barrier height for conduction-band electron flow from the Si into SiO_2 is 3.1 eV
  - This is equal to the electron-affinity difference (χ_{Si} and χ_{SiO2})
- The barrier height for valence-band hole flow from the Si into SiO_2 is 4.8 eV
- The vertical distance between the Fermi level in the metal, E_{FM}, and the Fermi level in the Si, E_{FS}, is equal to the applied gate voltage:

$$qV_G = E_{FS} - E_{FM}$$

Voltage Drops in the MOS System

- In general,

$$V_G = V_{FB} + V_{ox} + \psi_s$$

where

$qV_{FB} = \phi_{MS} = \phi_M - \phi_S$

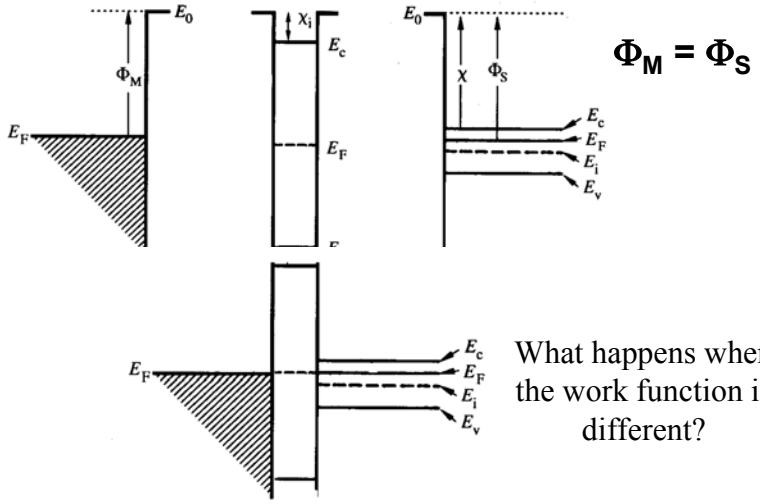
V_{ox} is the voltage dropped across the oxide
(V_{ox} = total amount of band bending in the oxide)

ψ_s is the voltage dropped in the silicon
(total amount of band bending in the silicon)

$$q\psi_s = E_i(bulk) - E_i(surface)$$

For example: When $V_G = V_{FB}$, $V_{ox} = \psi_s = 0$
i.e. there is no band bending

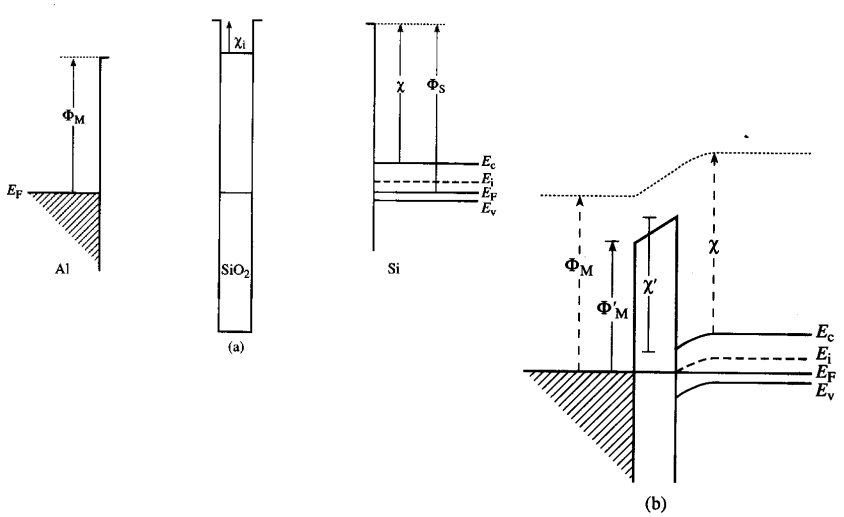
Special Case: Equal Work Functions



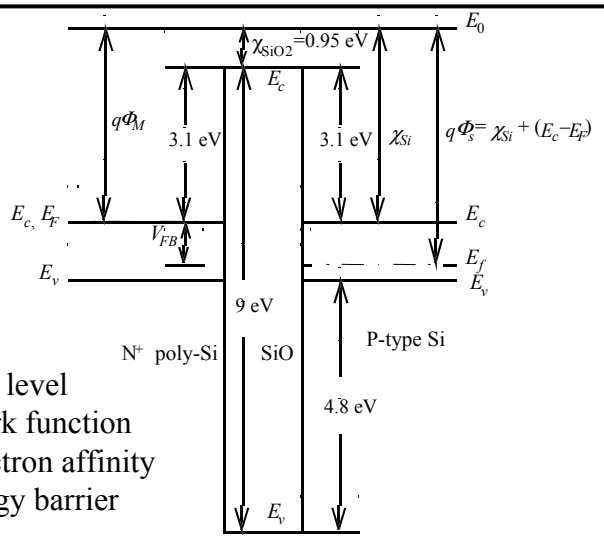
$\Phi_M = \Phi_S$

What happens when the work function is different?

General Case: Different Work Functions



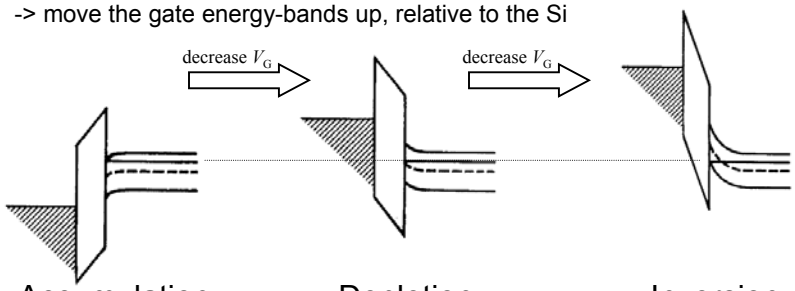
Flat-Band Condition



E_0 : Vacuum level
$E_0 - E_f$: Work function
$E_0 - E_c$: Electron affinity
Si/SiO_2 energy barrier

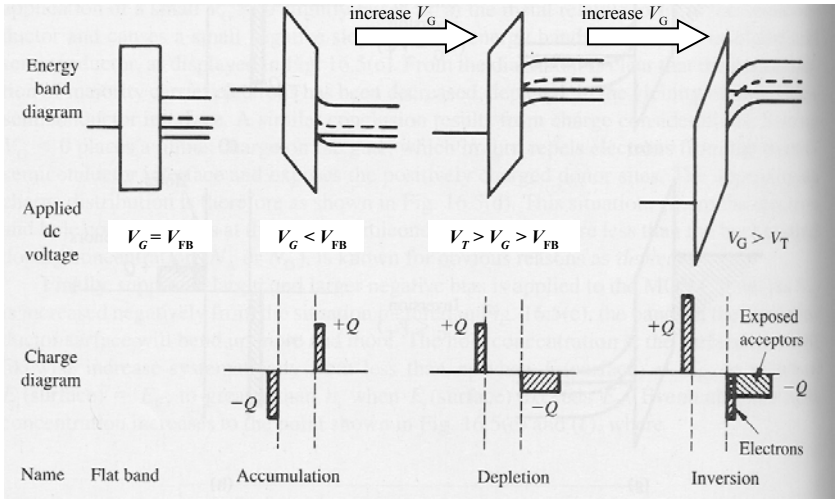
MOS Band Diagrams (n-type Si)

Decrease V_G (toward more negative values)
-> move the gate energy-bands up, relative to the Si



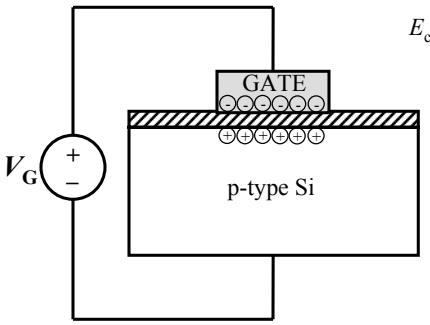
- Accumulation
  - $V_G > V_{FB}$
  - Electrons accumulate at surface
- Depletion
  - $V_G < V_{FB}$
  - Electrons repelled from surface
- Inversion
  - $V_G < V_T$
  - Surface becomes p-type

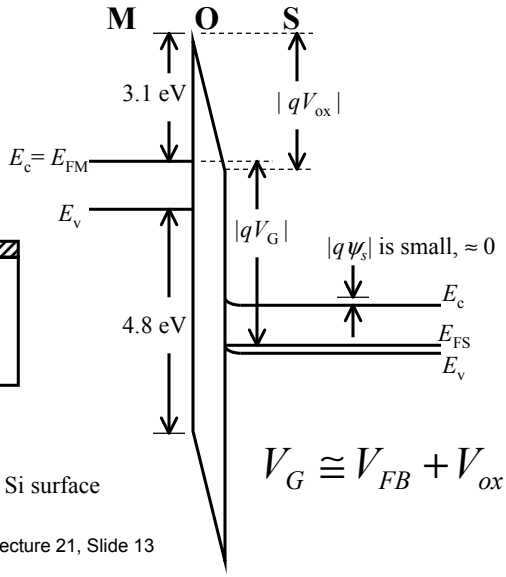
Biasing Conditions for p-type Si



Accumulation (n+ poly-Si gate, p-type Si)

$V_G < V_{FB}$





Mobile carriers (holes) accumulate at Si surface

$$V_G \cong V_{FB} + V_{ox}$$

Accumulation Layer Charge Density

$V_G < V_{FB}$

$$V_{ox} \cong V_G - V_{FB}$$

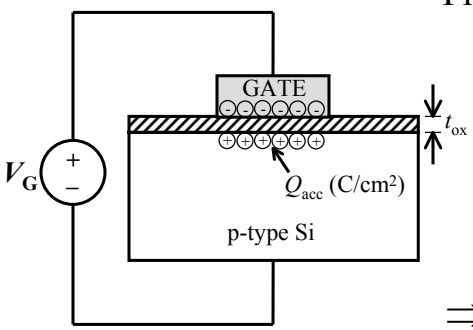
From Gauss' Law:

$$\mathcal{E}_{ox} = -Q_{acc} / \varepsilon_{SiO_2}$$

$$V_{ox} = \mathcal{E}_{ox} t_{ox} = -Q_{acc} / C_{ox}$$

where $C_{ox} \equiv \varepsilon_{SiO_2} / t_{ox}$

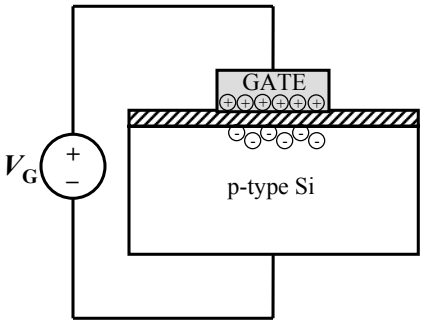
(units: F/cm²)

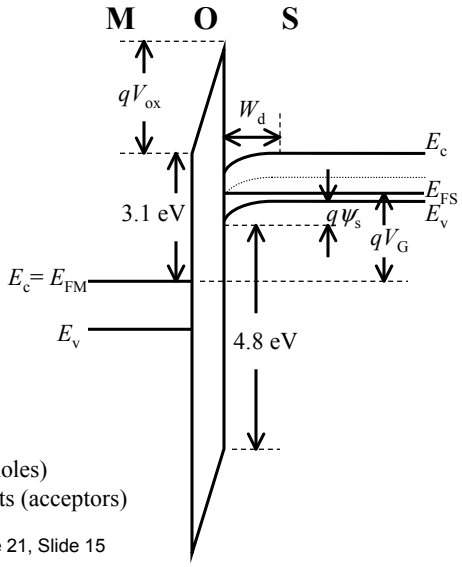


$$\Rightarrow \boxed{Q_{acc} = -C_{ox}(V_G - V_{FB})} > 0$$

Depletion (n+ poly-Si gate, p-type Si)

$V_T > V_G > V_{FB}$





Si surface is depleted of mobile carriers (holes)
=> Surface charge is due to ionized dopants (acceptors)

Depletion Width W_d (p-type Si)

- Depletion Approximation:
  The surface of the Si is depleted of mobile carriers to a depth W_d.
- The charge density within the depletion region is

$$\rho \cong -qN_A \qquad (0 \le x \le W_d)$$

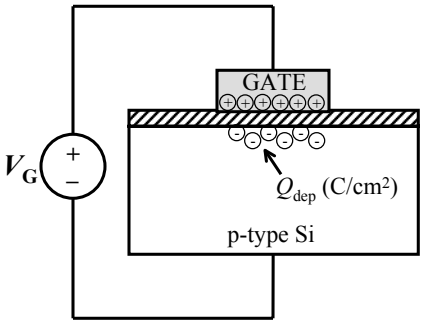
- Poisson's equation: $\dfrac{d\,\mathcal{E}}{dx} = \dfrac{\rho}{\varepsilon_{Si}} \cong -\dfrac{qN_A}{\varepsilon_{Si}} \qquad (0 \le x \le W_d)$

- Integrate twice, to obtain ψ_S:

$$\psi_s = \frac{qN_A}{2\varepsilon_{Si}} W_d^{\,2} \quad \Rightarrow \boxed{W_d = \sqrt{\frac{2\varepsilon_{Si}\psi_s}{qN_A}}}$$

To find ψ_s for a given V_G, we need to consider the voltage drops in the MOS system…

Voltage Drops in Depletion (p-type Si)



From Gauss' Law:

$$\mathcal{E}_{ox} = -Q_{dep} / \varepsilon_{SiO_2}$$

$$V_{ox} = \mathcal{E}_{ox} t_{ox} = -Q_{dep} / C_{ox}$$

Q_{dep} is the integrated charge density in the Si:

$$Q_{dep} = -qN_A W_d = -\sqrt{2qN_A \varepsilon_{Si} \psi_s}$$

$$V_G = V_{FB} + \psi_s + V_{ox} = V_{FB} + \psi_s + \frac{\sqrt{2qN_A \varepsilon_{si} \psi_s}}{C_{ox}}$$

Surface Potential in Depletion (p-type Si)

$$V_G = V_{FB} + \psi_s + \frac{\sqrt{2qN_A \varepsilon_{si} \psi_s}}{C_{ox}}$$

- Solving for ψ_S, we have

$$\sqrt{\psi_s} = \frac{\sqrt{qN_A \varepsilon_{si}}}{\sqrt{2} C_{ox}} \left[\sqrt{1 + \frac{2C_{ox}^2 (V_G - V_{FB})}{qN_A \varepsilon_{si}}} - 1 \right]$$

$$\psi_s = \frac{qN_A \varepsilon_{si}}{2C_{ox}^2} \left[\sqrt{1 + \frac{2C_{ox}^2 (V_G - V_{FB})}{qN_A \varepsilon_{si}}} - 1 \right]^2$$

Threshold Condition ($V_G = V_T$)

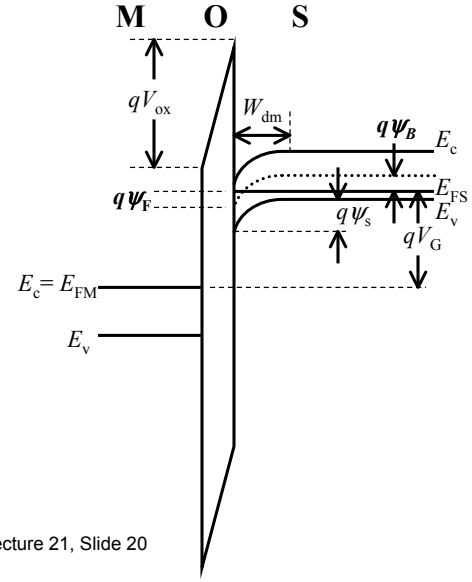
- When V_G is increased to the point where ψ_s reaches $2\psi_B$, the surface is said to be strongly inverted.
  (The surface is n-type to the same degree as the bulk is p-type.)
  This is the threshold condition.

$$V_G = V_T \Rightarrow \psi_s = 2\psi_B$$
$$E_i(bulk) - E_i(surface) = 2\left[E_i(bulk) - E_F\right]$$
$$E_i(surface) - E_F = -\left[E_i(bulk) - E_F\right]$$
$$\Rightarrow n_{surface} = N_A$$

MOS Band Diagram at Threshold (p-type Si)

$$\psi_s = 2\psi_B = 2\frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right)$$

$$W_d = W_{dm} = \sqrt{\frac{2\varepsilon_{Si}(2\psi_B)}{qN_A}}$$



Threshold Voltage

- For p-type Si:

$$V_G = V_{FB} + \psi_s + V_{ox} = V_{FB} + \psi_s + \frac{\sqrt{2qN_A\varepsilon_{si}\psi_s}}{C_{ox}}$$

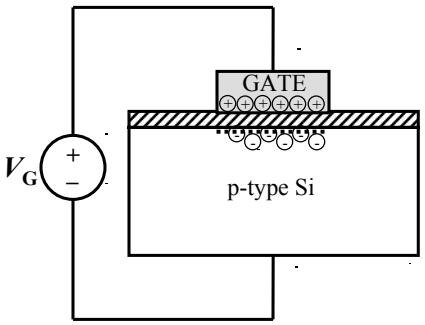
$$V_T = V_{FB} + 2\psi_B + \frac{\sqrt{2qN_A\varepsilon_{Si}(2\psi_B)}}{C_{ox}}$$

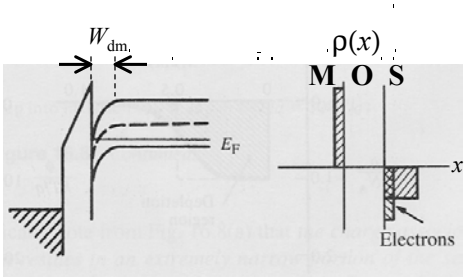
- For n-type Si:

$$V_T = V_{FB} + 2\psi_B - \frac{\sqrt{2qN_D\varepsilon_{Si}|2\psi_B|}}{C_{ox}}$$

Strong Inversion (p-type Si)

As V_G is increased above V_T, the negative charge in the Si is increased by adding mobile electrons (rather than by depleting the Si more deeply), so **the depletion width remains ~constant at $W_d = W_{dm}$**





$$\psi_s \cong 2\psi_B$$

Significant density of mobile electrons at surface (surface is n-type)

$$W_d \cong W_{dm} = \sqrt{\frac{2\varepsilon_{si}(2\psi_B)}{qN_A}}$$

Inversion Layer Charge Density (p-type Si)

$$V_G = V_{FB} + \psi_S + V_{ox}$$

$$= V_{FB} + 2\psi_B - \frac{(Q_{dep} + Q_{inv})}{C_{ox}}$$

$$= V_{FB} + 2\psi_B + \frac{\sqrt{2qN_A\varepsilon_s(2\psi_B)}}{C_{ox}} - \frac{Q_{inv}}{C_{ox}}$$

$$= V_T - \frac{Q_{inv}}{C_{ox}}$$

$$\therefore \quad \boxed{Q_{inv} = -C_{ox}(V_G - V_T)}$$