Lecture #11

OUTLINE

- pn Junctions
  - – reverse breakdown
  - – ideal diode analysis
    - » current flow (qualitative)
    - » minority carrier distributions

Reading: Chapter 6

A Note of Caution

- Typically, pn junctions in IC devices are formed by counter-doping. The equations derived in class (and in the textbook) can be readily applied to such diodes if

$N_A \equiv$ net acceptor doping on p-side $(N_A\text{-}N_D)_{\text{p-side}}$

$N_D \equiv$ net donor doping on n-side $(N_D\text{-}N_A)_{\text{n-side}}$

pn Junction Electrostatics, $V_A \neq 0$

- Built-in potential V_{bi} (non-degenerate doping):

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right) + \frac{kT}{q}\ln\left(\frac{N_D}{n_i}\right) = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^{\,2}}\right)$$

- Depletion width W :

$$W = x_p + x_n = \sqrt{\frac{2\varepsilon_s}{q}(V_{bi} - V_A)\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

$$x_p = \frac{N_D}{N_A + N_D}W \qquad x_n = \frac{N_A}{N_A + N_D}W$$

- Electric field distribution $\mathcal{E}(x)$

- Potential distribution $V(x)$

Note that $V(0) = \dfrac{N_D}{N_A + N_D}(V_{bi} - V_A)$

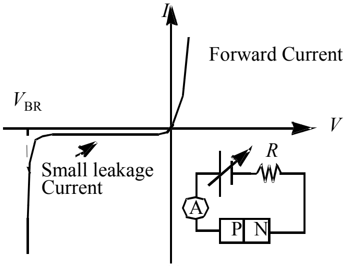
Peak Electric Field

$$\left|\int \mathcal{E}\, dx\right| = \frac{1}{2}\left|\mathcal{E}(0)\right|W = V_{bi} - V_A$$

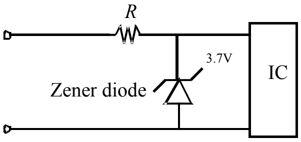
- For a **one-sided junction**: $W \cong \sqrt{\frac{2\varepsilon_s}{qN}(V_{bi} - V_A)}$

therefore $\mathcal{E}(0) = \frac{2(V_{bi} - V_A)}{W} \cong \sqrt{\frac{2qN(V_{bi} - V_A)}{\varepsilon_s}}$

Junction Breakdown



(a)



(b)

A ***Zener diode*** is designed to operate in the breakdown mode.

- If $V_{\text{reverse}} = -V_A$ is so large such that the peak electric field exceeds a critical value $\mathcal{E}_{\text{crit}}$, then the junction will break down (large reverse current will flow)

$$\mathcal{E}_{crit} = \sqrt{\frac{2qN(V_{bi} + V_{BR})}{\varepsilon_s}}$$

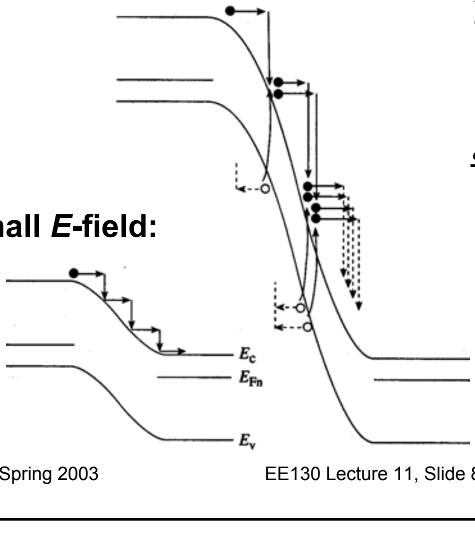
- Thus, the reverse bias at which breakdown occurs is

$$V_{BR} = \frac{\varepsilon_s \mathcal{E}_{crit}{}^2}{2qN} - V_{bi}$$

Avalanche Breakdown Mechanism

High *E*-field:

Small *E*-field:



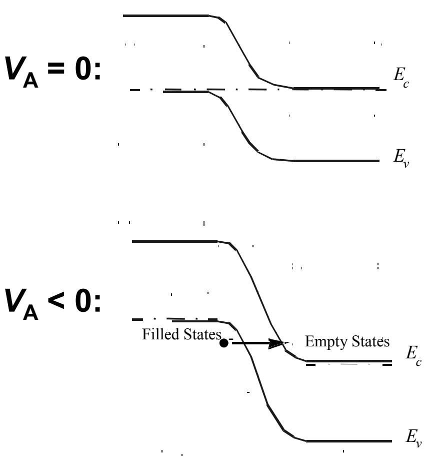
$$V_{BR} \approx \frac{\varepsilon_s \mathcal{E}_{crit}{}^2}{2qN} \text{ if } V_{\text{BR}} >> V_{\text{bi}}$$

$\mathcal{E}_{\text{crit}}$ increases slightly with *N*:

For 10^{14} cm^{-3} < *N* < 10^{18} cm^{-3},

10^5 V/cm < $\mathcal{E}_{\text{crit}}$ < 10^6 V/cm

Tunneling (Zener) Breakdown Mechanism



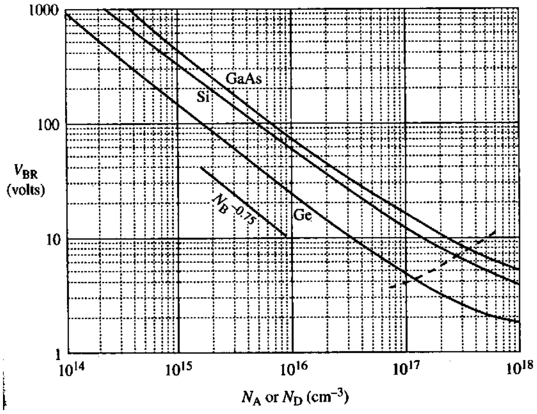
Dominant breakdown mechanism when both sides of a junction are very heavily doped.

$$V_{BR} = \frac{\varepsilon_s \mathcal{E}_{crit}^{\ 2}}{2qN} - V_{bi}$$

$$\mathcal{E}_{crit} \approx 10^6 \text{ V/cm}$$

Typically, $V_{BR} < 5$ V for Zener breakdown

Empirical Observations of V_{BR}



- V_{BR} decreases with increasing N
- V_{BR} decreases with decreasing E_G

Breakdown Temperature Dependence

- For the **avalanche mechanism**: V_{BR} increases with increasing T
  - Mean free path decreases
- For the **tunneling mechanism**: V_{BR} decreases with increasing T
  - Flux of valence-band electrons available for tunneling increases

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Current Flow in a pn Junction Diode

- When a forward bias (V_A>0) is applied, the potential barrier to diffusion across the junction is reduced
  - Minority carriers are “injected” into the quasi-neutral regions => $\Delta n_p > 0$, $\Delta p_n > 0$
- Minority carriers diffuse in the quasi-neutral regions, recombining with majority carriers

- Current density $J = J_n(x) + J_p(x)$

$$J_n(x) = q\mu_n n\mathcal{E} + qD_n \frac{dn}{dx} = q\mu_n n\mathcal{E} + qD_n \frac{d(\Delta n)}{dx}$$

$$J_p(x) = q\mu_p p\mathcal{E} - qD_p \frac{dp}{dx} = q\mu_p p\mathcal{E} - qD_p \frac{d(\Delta p)}{dx}$$

- J is constant throughout the diode, but $J_n(x)$ and $J_p(x)$ vary with position

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Ideal Diode Analysis: Assumptions

- Non-degenerately doped step junction
- Steady-state conditions
- Low-level injection conditions prevail in the quasi-neutral regions
- Recombination-generation is negligible in the depletion region

$$\Rightarrow \quad \frac{dJ_n}{dx} = 0, \quad \frac{dJ_p}{dx} = 0$$

i.e. J_n & J_p are constant inside the depletion region

Ideal Diode Analysis: Approach

- Solve the minority-carrier diffusion equations in quasi-neutral regions to obtain $\Delta n_p(x,V_A)$,$\Delta p_n(x,V_A)$
  - apply boundary conditions
    - p-side: $\Delta n_p(-x_p)$, $\Delta n_p(-\infty)$
    - n-side: $\Delta p_n(x_n)$, $\Delta p_n(\infty)$
- Determine minority-carrier current densities in quasi-neutral regions

$$J_n(x,V_A) = qD_n \frac{d(\Delta n_p)}{dx} \qquad J_p(x,V_A) = -qD_p \frac{d(\Delta p_n)}{dx}$$

- Evaluate J_n at $x=-x_p$ and J_p at $x=x_n$

$$J(V_A) = J_n(V_A)|_{x=-xp} + J_p(V_A)|_{x=xn}$$

Carrier Concentrations at $-x_p$, x_n

Consider the **equilibrium** (V_A = 0) carrier concentrations:

p-side	**n-side**
$p_{p0}(-x_p) = N_\mathrm{A}$	$n_{n0}(x_n) = N_\mathrm{D}$
$n_{p0}(-x_p) = \frac{n_i^2}{N_\mathrm{A}}$	$p_{n0}(x_n) = \frac{n_i^2}{N_\mathrm{D}}$

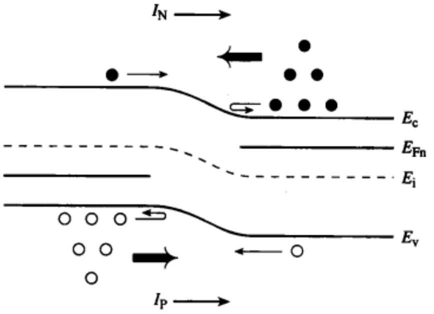
If low-level injection conditions prevail in the quasi-neutral regions when $V_A \neq 0$, then

$$p_p(-x_p) = N_\mathrm{A} \qquad n_n(x_n) = N_\mathrm{D}$$

"Law of the Junction"

The voltage V_A applied to a pn junction falls mostly across the depletion region (assuming that low-level injection conditions prevail in the quasi-neutral regions).

We can draw 2 quasi-Fermi levels in the depletion region:



$$p = n_{\mathrm{i}} e^{(E_{\mathrm{i}} - F_{\mathrm{P}})/kT}$$

$$n = n_{\mathrm{i}} e^{(F_{\mathrm{N}} - E_{\mathrm{i}})/kT}$$

$$pn = n_{\mathrm{i}}^2 e^{(E_i - F_P)/kT} e^{(F_N - E_i)/kT}$$
$$= n_{\mathrm{i}}^2 e^{(F_N - F_P)/kT}$$

$$\boxed{pn = n_{\mathrm{i}}^2 e^{qV_{\mathrm{A}}/kT}}$$

Excess Carrier Concentrations at $-x_p$, x_n

p-side

$$p_p(-x_p) = N_{\mathrm{A}}$$

$$n_p(-x_p) = \frac{n_i^2 e^{qV_{\mathrm{A}}/kT}}{N_{\mathrm{A}}}$$
$$= n_{p0} e^{qV_{\mathrm{A}}/kT}$$

$$\Delta n_p(-x_p) = \frac{n_i^2}{N_{\mathrm{A}}}\left(e^{qV_{\mathrm{A}}/kT} - 1\right)$$

n-side

$$n_n(x_n) = N_{\mathrm{D}}$$

$$p_n(x_n) = \frac{n_i^2 e^{qV_{\mathrm{A}}/kT}}{N_{\mathrm{D}}}$$
$$= p_{n0} e^{qV_{\mathrm{A}}/kT}$$

$$\Delta p_n(x_n) = \frac{n_i^2}{N_{\mathrm{D}}}\left(e^{qV_{\mathrm{A}}/kT} - 1\right)$$

Example: Carrier Injection

A pn junction has N_A=10^{18} cm^{-3} and N_D=10^{16} cm^{-3}. The applied voltage is 0.6 V.

Question: What are the minority carrier concentrations at the depletion-region edges?

Answer:

$$n_p(-x_p) = n_{po}e^{qV_A/kT} = 100 \times e^{0.6/0.026} = 10^{12}\ \text{cm}^{-3}$$

$$p_n(x_n) = p_{no}e^{qV_A/kT} = 10^4 \times e^{0.6/0.026} = 10^{14}\ \text{cm}^{-3}$$

Question: What are the excess minority carrier concentrations?

Answer:

$$\Delta n_p(-x_p) = n_p(-x_p) - n_{po} = 10^{12} - 100 = 10^{12}\ \text{cm}^{-3}$$

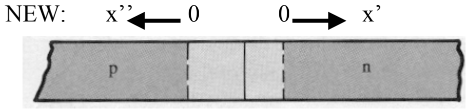
$$\Delta p_n(x_n) = p_n(x_n) - p_{no} = 10^{14} - 10^4 = 10^{14}\ \text{cm}^{-3}$$

Excess Carrier Distribution

- From the minority carrier diffusion equation: $\frac{d^2\Delta p_n}{dx^2} = \frac{\Delta p_n}{D_p\tau_p} = \frac{\Delta p_n}{L_p{}^2}$
- We have the following boundary conditions:

$$\Delta p_n(x_n) = p_{no}(e^{qV_A/kT} - 1) \qquad \Delta p_n(\infty) \rightarrow 0$$

- For simplicity, we will develop a new coordinate system:



- Then, the solution is of the form:

$$\Delta p_n(x') = A_1 e^{x'/L_p} + A_2 e^{-x'/L_p}$$

$$\Delta p_n(x') = A_1 e^{x'/L_p} + A_2 e^{-x'/L_p}$$

From the $x = \infty$ boundary condition, $A_1 = 0$.
From the $x = x_n$ boundary condition, $A_2 = p_{no}(e^{qV_A/kT} - 1)$

Therefore, $\Delta p_n(x') = p_{no}(e^{qV_A/kT} - 1)e^{-x'/L_p},\ x' > 0$

Similarly, we can derive

$$\Delta n_p(x'') = n_{po}(e^{qV_A/kT} - 1)e^{-x''/L_n},\ x'' > 0$$

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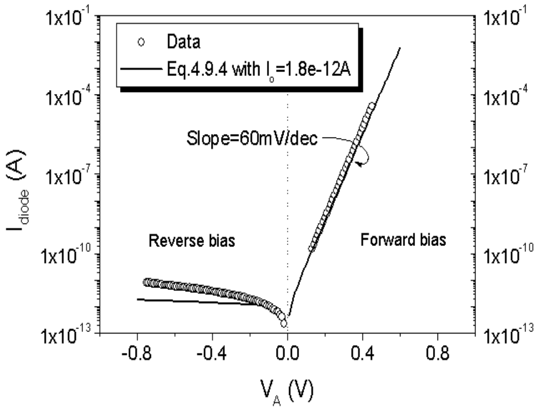
pn Diode *I-V* Characteristic

p-side: $J_n = -qD_n \dfrac{d\Delta n_p(x'')}{dx''} = q\dfrac{D_n}{L_n} n_{p0}(e^{qV_\mathrm{A}/kT} - 1)e^{-x''/L_n}$

n-side: $J_p = -qD_p \dfrac{d\Delta p_n(x')}{dx'} = q\dfrac{D_p}{L_p} p_{n0}(e^{qV_\mathrm{A}/kT} - 1)e^{-x'/L_p}$

$$J = J_n\big|_{x=-x_p} + J_p\big|_{x=x_n} = J_n\big|_{x''=0} + J_p\big|_{x'=0}$$

$$J = qn_\mathrm{i}^2\left[\frac{D_n}{L_n N_\mathrm{A}} + \frac{D_p}{L_p N_\mathrm{D}}\right](e^{qV_\mathrm{A}/kT} - 1)$$



$$I = I_0(e^{qV_A/kT} - 1)$$

$$I_0 = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$$

Diode Saturation Current I_0

$$I_0 = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$$

- I_0 can vary by orders of magnitude, depending on the semiconductor material
- In an asymmetrically doped pn junction, the term associated with the more heavily doped side is negligible:
  - If the p side is much more heavily doped, $I_0 \cong Aqn_i^2\left(\frac{D_p}{L_pN_D}\right)$
  - If the n side is much more heavily doped, $I_0 \cong Aqn_i^2\left(\frac{D_n}{L_nN_A}\right)$