Lecture #10

ANNOUNCEMENT

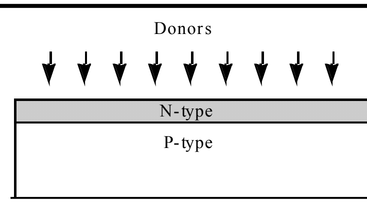
– Quiz #2 next Thursday (2/27) covering
  - carrier action (drift, diffusion, R-G)
  - continuity & minority-carrier diffusion equations
  - metal-semiconductor contacts

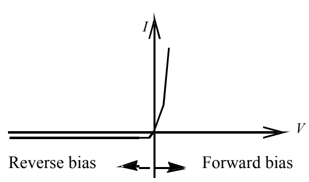
OUTLINE

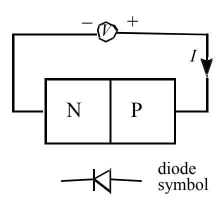
- pn junction electrostatics

Reading: Chapter 5

pn Junctions

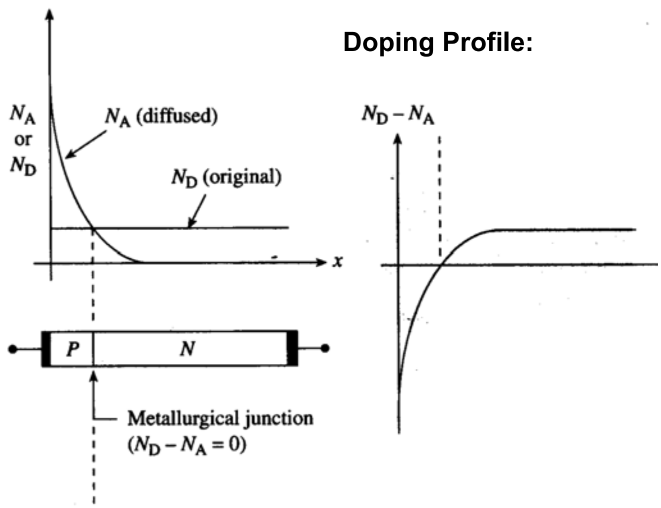




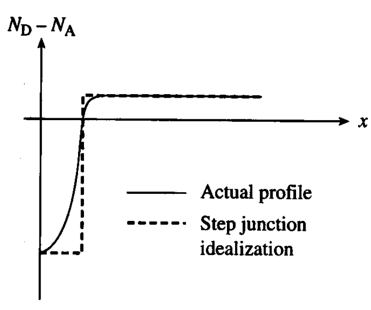


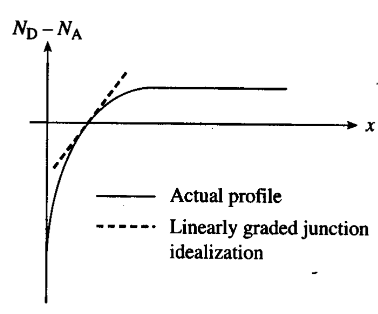
Terminology

Doping Profile:



Idealized Junctions





Energy Band Diagram

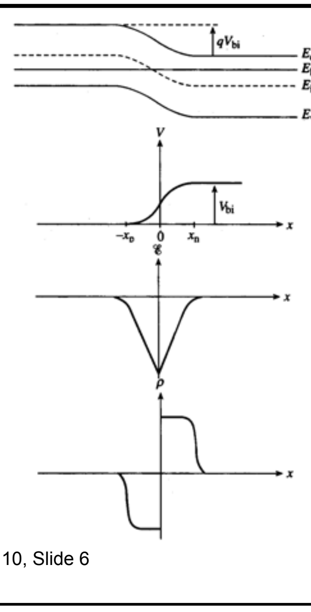
Qualitative Electrostatics

Band diagram

Electrostatic potential

Electric field

Charge density

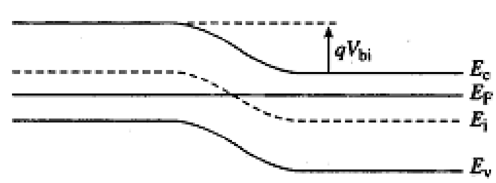


“Game Plan” for Obtaining $\rho(x)$, $\mathcal{E}(x)$, $V(x)$

- Find the built-in potential V_{bi}
- Use the depletion approximation $\rightarrow \rho(x)$
  (depletion-layer widths x_p, x_n unknown)
- Integrate $\rho(x)$ to find $\mathcal{E}(x)$
  - boundary conditions $\mathcal{E}(-x_p)=0$, $\mathcal{E}(x_n)=0$
- Integrate $\mathcal{E}(x)$ to obtain $V(x)$
  - boundary conditions $V(-x_p)=0$, $V(x_n)=V_{bi}$
- For $\mathcal{E}(x)$ to be continuous at $x=0$, $N_A x_p = N_D x_n$
  $\rightarrow$ solve for x_p, x_n

Built-In Potential V_{bi}

$$qV_{bi} = \Phi_{S\,p\text{-side}} - \Phi_{S\,n\text{-side}} = (E_i - E_F)_{p\text{-side}} + (E_F - E_i)_{n\text{-side}}$$



For **non-degenerately doped** material:

$$(E_i - E_F)_{p-side} = kT\ln\left(\frac{p}{n_i}\right) = kT\ln\left(\frac{N_A}{n_i}\right)$$

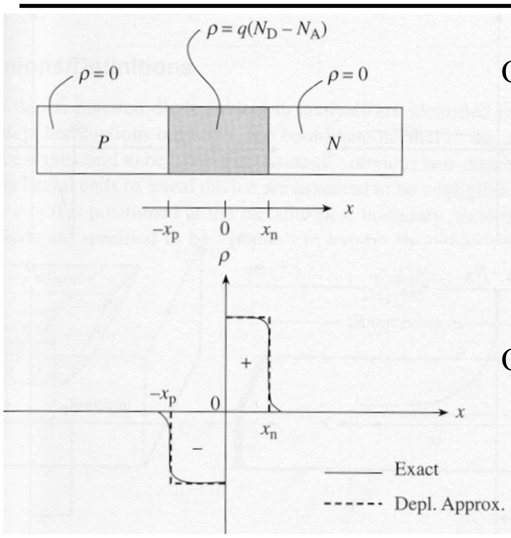
$$(E_F - E_i)_{n-side} = kT\ln\left(\frac{n}{n_i}\right) = kT\ln\left(\frac{N_D}{n_i}\right)$$

V_{bi} for "One-Sided" pn Junctions

$$qV_{bi} = (E_i - E_F)_{p-side} + (E_F - E_i)_{n-side}$$

p⁺n junction **n⁺p junction**

The Depletion Approximation



On the ***p-side***, $\rho = -qN_A$

$$\frac{d\mathcal{E}}{dx} = -\frac{qN_A}{\varepsilon_s}$$

$$\mathcal{E}(x) = -\frac{qN_A}{\varepsilon_s}x + C_1 = \frac{-qN_A}{\varepsilon_s}(x + x_p)$$

On the ***n-side***, $\rho = qN_D$

$$\mathcal{E}(x) = \frac{-qN_D}{\varepsilon_s}(x_n - x)$$

Electric Field in the Depletion Layer

The electric field is continuous at *x* = 0

$\Rightarrow \quad N_A x_p = N_D x_n$

Electrostatic Potential in the Depletion Layer

On the ***p-side***:

$$V(x) = \frac{qN_A}{2\varepsilon_s}(x + x_p)^2 + D_1$$

(arbitrarily choose the voltage at $x = x_p$ to be 0)

On the ***n-side***:

$$V(x) = -\frac{qN_D}{2\varepsilon_s}(x_n - x)^2 + D_2 = V_{bi} - \frac{qN_D}{2\varepsilon_s}(x_n - x)^2$$

• At $x = 0$, expressions for p-side and n-side must be equal:

• We also know that $N_A x_p = N_D x_n$

Depletion Layer Width

- Eliminating x_p, we have:

$$x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{q}\left(\frac{N_A}{N_D(N_A + N_D)}\right)}$$

- Eliminating x_n, we have:

$$x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{q}\left(\frac{N_D}{N_A(N_A + N_D)}\right)}$$

- Summing, we have:

$$x_n + x_p = W = \sqrt{\frac{2\varepsilon_s V_{bi}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

One-Sided Junctions

If $N_A >> N_D$ as in a p$^+$n junction:

$$\boxed{W = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}}} \approx x_n$$

$$x_p = x_n \, N_D / N_A \cong 0$$

What about a n$^+$p junction?

$$W = \sqrt{2\varepsilon_s V_{bi} / qN} \quad \text{where} \quad \frac{1}{N} = \frac{1}{N_D} + \frac{1}{N_A} \approx \frac{1}{\textit{lighter dopant density}}$$

Peak Electric Field

$$\left| \int \mathcal{E} \, dx \right| = \frac{1}{2} |\mathcal{E}(0)| W = V_{bi} - V_A$$

- For a one-sided junction, $W \cong \sqrt{\frac{2\varepsilon_s}{qN}(V_{bi} - V_A)}$

so $\mathcal{E}(0) = \frac{2(V_{bi} - V_A)}{W} \cong \sqrt{\frac{2qN(V_{bi} - V_A)}{\varepsilon_s}}$

Example

A p^+n junction has $N_A=10^{20}$ cm^{-3} and $N_D=10^{17}$cm^{-3}. What is a) its built in potential, b)W, c)x_n, and d) x_p ?

***Solution**:*

a)

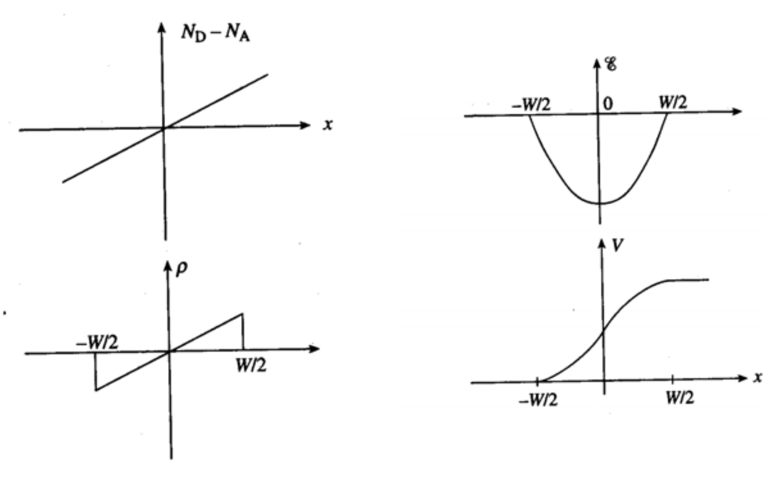
$$V_{bi} = \frac{E_G}{2q} + \frac{kT}{q}\ln\frac{N_D}{n_i} \approx 1\,\text{V}$$

b) $$W \approx \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}} = \left(\frac{2\times 12\times 8.85\times 10^{-14}\times 1}{1.6\times 10^{-19}\times 10^{17}}\right)^{1/2} = 0.12\,\mu\text{m}$$

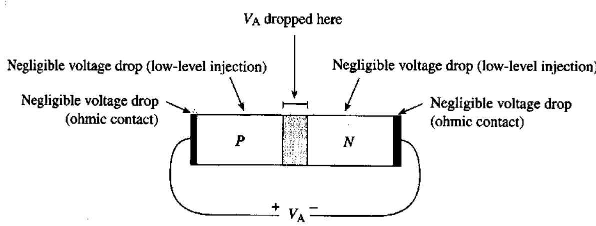
c) $x_n \approx W = 0.12\,\mu\text{m}$

d) $x_p = x_n N_D / N_A = 1.2\times 10^{-4}\mu\text{m} = 1.2\,\text{Å} \approx 0$

Linearly Graded Junction

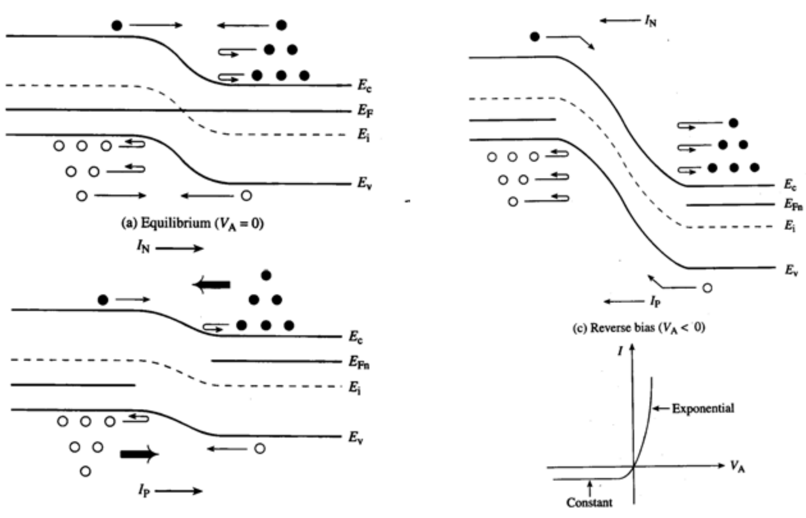


Biased PN Junctions



REQUIREMENT: $V_A < V_{bi}$, otherwise, we cannot assume low-level injection

Current Flow - Qualitative



Effect of Bias on Electrostatics