INVESTIGATION OF THE ACCURACY OFGROVER'S METHOD WHEN SOLVING FOR THE MUTUAL INDUCTANCE OF TWO SINGLE-LAYER COAXIAL COILS

A Thesis presented to the Faculty of the Graduate School
University of Missouri-Columbia

In Partial Fulfillment
of the Requirements for the Degree

Master of Science

---

by

STACY ROHE

Dr. Thomas G. Engel, Thesis Supervisor

DECEMBER 2005

The undersigned, appointed by the Dean of the Graduate School, have examined the thesis entitled

INVESTIGATION OF GROVER'S ACCURACY WHEN SOLVING FOR THE MUTUAL INDUCTANCE OF TWO SINGLE-LAYER COAXIAL COILS

presented by Stacy Rohe

a candidate for the degree of Master of Science in Electrical Engineering

and hereby certify that in their opinion it is worthy of acceptance.

Thomas Engel

Dr. Thomas G. Engel (chairman)

Dr. John M. Gahl

John Farmer

Dr. John W. Farmer

INVESTIGATION OF THE ACCURACY OF GROVER'S METHOD WHEN SOLVING FOR THE MUTUAL INDUCTANCE OF TWO SINGLE-LAYER COAXIAL COILS

Stacy Rohe

Dr. Thomas G. Engel, Thesis Supervisor

ABSTRACT

In 1933 Grover introduced a convenient method of calculating the mutual inductance between two single-layer coaxial coils that is still widely accepted as the standard today. He produced tables of numeric values from Clem's series formula and used them in a single equation to calculate the mutual inductance. In this investigation, the accuracy of Grover's method is presented and discussed. Using the same geometries, examples from Grover's literature are compared to answers acquired using a field code software and the direct calculations of elliptical integrals. Also by fixing various dimensional parameters while varying others, practical interest geometries such as equal radii, equal lengths, different radii, different lengths, and different separation distance between the coils are achieved. The mutual inductances are found and compared for Grover's method, the field code method, and direct elliptical integral calculation. The maximum percent error found when comparing Grover's method to the field code method was 1.0715% and 9.799% for the literature and varying parameter examples respectively. While the maximum percent error found when comparing Grover's method to the direct calculation of the elliptical integrals was 98.87% and 58.92% for the literature and varying parameter examples respectively. Thus, Grover's method compares more favorability to the field code method over the direct elliptical integral calculation method.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1 Introduction

In recent years Grover's mutual inductance tables have been used to calculate the force in such applications as helical coil launchers (HCL) and other pulse induction launchers [1-3]. These applications use two helical coils that have an inductance gradient up to two times that of simple rail guns. Since the propulsion force is proportional to the inductance gradient the larger inductance gradient results in a higher electromagnet force, and hence higher acceleration of the projectile. To find the inductance gradient the mutual inductance of two coaxial single-layer coils must be found. This is done using Grover's method. Given that the force calculation depends on the mutual inductance, it is important for the mutual inductance to be accurate. Thus it is crucial to know how realistic Grover's tables' answers are compared to actual values.

In 1933, Grover created a convenient way to calculate the mutual inductance of any two coaxial single-layer coils. Before Grover's method, the mutual inductance values of these coils were found using elliptical integrals. Nagaoka, Olshausen, and Terezawa, derived absolute formulas for the general case and Kirchhoff and Cohen for the concentric case [4] using elliptical integrals. These formulas involved elliptical integrals of the first and second kinds, which are defined in [5] as:

$$F(k,\theta)=\int_0^{\theta}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}, \qquad (1.1)$$

$$E(k,\theta)=\int_0^{\theta}\sqrt{1-k^2\sin^2\theta}\;\;d\theta\,,$$

(1.2)

where

$$k = \sin\theta,$$
$$and \quad 0 \le \theta \le \frac{\pi}{2}.$$

For elliptical integrals k is called the modulus and ϕ is the amplitude. Solving elliptical integrals is often time-consuming and demanding, leading Grover to create a moderately accurate simpler method.

Grover introduced his standard tables which in addition with a single formula produce the mutual inductance calculation to cover all possible pairs of coils [4]. To achieve higher accuracies, Grover used Clem's series solution formula but his tables were computed by a combination of different series-closed form solutions [6]. Grover created three tables, one table that includes values of B_n for α and ρ^2 from zero to unity, the second table for the case when α and ρ^2 are greater than or equal to 0.9 and the third table for the case when the radii are equal, $\alpha = 1$. The relative error for these tables is claimed to be 10^{-5} to 10^{-4}, with the higher errors occurring when the two coils are loosely coupled [7]. When the coils are loosely or poorly coupled, it creates a situation where the terms in the tables nearly cancel and the accuracy of the mutual inductance is difficult to obtain. Today Grover's tables are widely accepted as the standard for calculating the mutual inductance of coaxial single-layer coils.

There have been only a small number of studies involving the accuracy of Grover's mutual inductance in the past, the most recent being 1979. Fawzi and Burke first presented a new algorithm for the calculation of the self, mutual inductance of coaxial coils based on Bartky's transformation [7]. Grover's mutual inductance and Olshausin's formula for mutual inductance are compared to the new algorithm for 7 and 16 significant digits over a wide range of coils. Fawzi and Burke claim, compared to their new algorithm, that Grover's tables results are usually correct to their claimed accuracy of four or five significant digits, but in some cases the tables results are only precise to only two or three digits. A follow-up paper was then written by Fawzi, Gohar, and AbdelAal on a new approach to solving for the force between coaxial single-layer coils [8]. They utilized the basic force expression and the direct application of Bartky's transformations for the mutual inductance. From the results in [8] the authors declare that when the four values of B_n in Grover's tables are close together the results will only be correct to one significant digit when compared to their algorithm. Such cases include loosely coupled coils, and very short coils. These papers did not compare Grover to complete elliptical equations or real-world results. No previous papers were found comparing Grover's tables to field code, or actual results.

Grover's tables for mutual inductance are mainly used because individuals want a quick solution or do not have access to field code software. This investigation reports the accuracy of Grover's mutual inductance when compared to field code software and elliptical integrals of the first two kinds. As earlier stated no previous studies have been completed that compare Grover's tables results to actual results. For practicality reasons this experiment uses field code software to simulate real world measurements.

To obtain the field code mutual inductance, MagNet [9], an electromagnetic field simulation software, was used as the accepted standard for actual measurements for practicality reasons. A script interface was developed that numerically calculates and graphically charts the coil parameters of the mutually coupled coils. Two different studies were done, one comparing Grover to the field code using examples from Grover's books

[4,6], the other comparing Grover to the field code results for varying separation distances of different α values.

Examples from Grover's literature [4, 6] are incorporated into MagNet and compared to his results. The results show that for the case when the coils are concentric, separation distance zero, the error is insignificantly, approximately 0.13026% and under. In addition when there is a separation distance the error only increases slightly. The largest error being 1.0715%, occurring when the coils radii are small, 2 and 3 cm, the lengths are 10 cm and 6 cm and the separation distance is large, 18 cm, leaving an air space of 10 cm between the coils, making the coils loosely coupled.

Grover's mutual inductance was also compared to MagNet's field code software for varying ranges of separation distance and ratio of the radii of the coils. The values of the lengths, turn densities, and the radius of the larger coils were fixed while the separation distance, s, is varied for different values of α. Alpha is altered by changing the small radius. Two different coils sets were studied, one with equal length coils, and one with non-equal length coils. A MatLab [10] program was developed to solve for the mutual inductance using Grover's tables. For both cases a small amount of error is achieved, under 10 percent, over the wide ranges of s and α. As alpha becomes larger the error rises in both cases. Generally the error is less when the lengths of the coils are not equal over the case when they are equal.

This investigation also compared Grover's mutual inductance, to the mutual inductance achieved when using elliptical integrals of the first and second kind. The elliptical integral equation was taken from [11], and is Maxwell's formula for the mutual inductance of two coaxial circles. This study was completed in the interest of observing where Grover may have adjusted his tables from the straight elliptical integral answers. A MatLab program was developed to solve elliptical integrals for the coil sets. To ensure the accuracy of the MatLab program written, the mutual inductances obtained were contrasted to examples in [11], and agreed with these published examples with an accuracy of 6 to7 significant digits. The design variables used in the program are the small and large radii, small and large lengths, turn densities, and the separation distance. The comparison was done identically as previously done for the field code.

The percent errors when comparing Grover's examples [4,6] to the elliptical method using MatLab are significantly larger than when Grover's results were compared to MagNet. The largest being 98.87% when the coils are small, 4.435 cm and 6.44 cm and the separation distance is large, 31.165 cm. The errors range between a maximum of 98.87% to a minimum of 2.17%. In Grover's literature, the inaccuracies are sizable when the coils are concentric, or when the coils are loosely coupled for the selected example geometries.

As in the case of the MagNet field code, Grover's results were compared to elliptical integrals for varying separation distances and radii ratios for the cases when the lengths of the coils are equal and unequal. In the instance that the lengths are equal the percent error is less than 2% when the radius ratio is small, .6 or less. As the ratio increases above 0.6 the error increases to a maximum of approximately 23% when alpha is 0.99. When unequal lengths occurred the overall error was substantially larger with a max error of roughly 60%. The max error for both cases at every alpha, except when alpha = 1, is at a separation distance of 0. This is not the case for when alpha = 1 because the coils cannot overlap if they are the same radius, so the s distance of 0 does not exist.

Chapter 2 Mutual Inductance Theory

2.1 Grover's Mutual Inductance Theory

Grover used the geometry seen in figure 2.1, to find the mutual inductance of two coaxial single-layer coils. Figure 2.1 shows a, $2m_1$ and n_1 are the radius, axial length, and winding density, respectively for the smaller coil and the counterpart quantities, A, $2m_2$, and n_2 for the larger coil. The radius A is assumed to be larger than the radius a. The distance s is the separation between the two coil centers. Depending on s, the coils could be partially inside, completely inside, or completely outside.

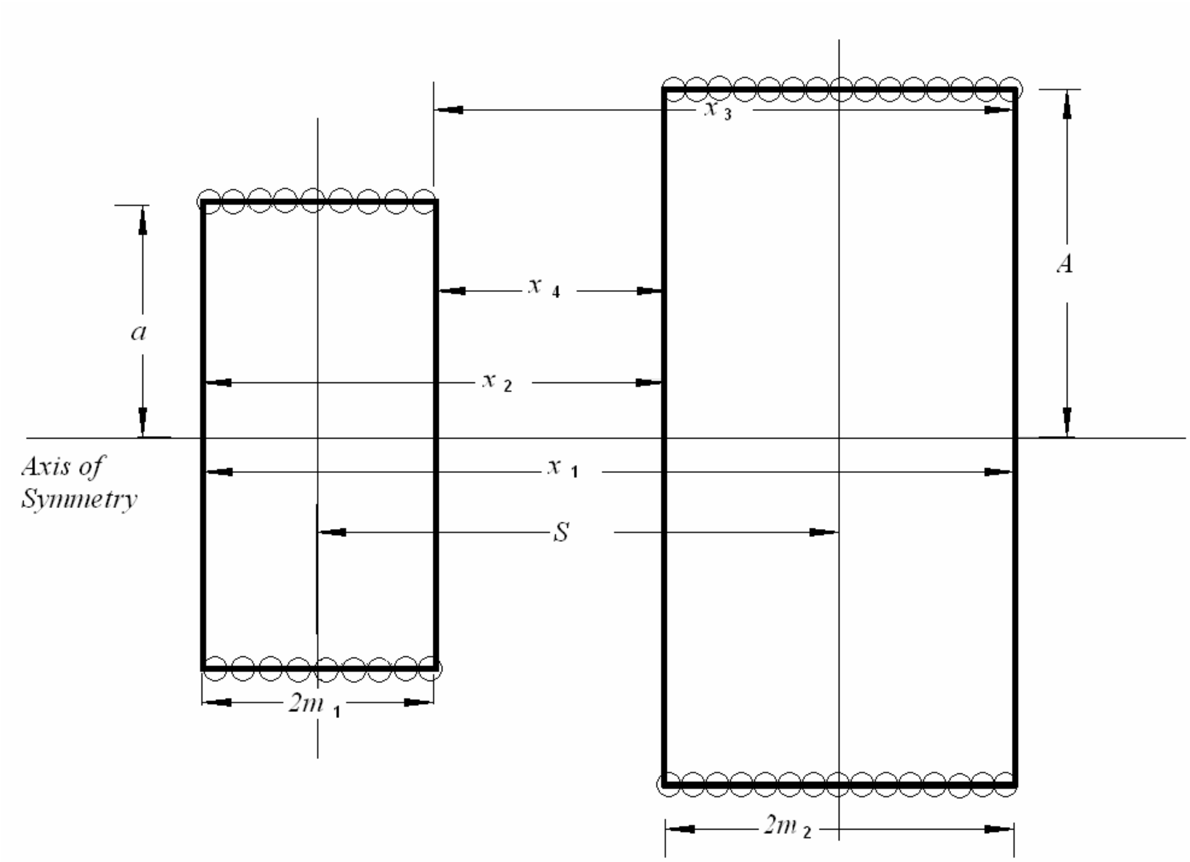

Figure 2. 1 Geometry for the mutual inductance of two coaxial single-layer coils.

Grover derived the mutual inductance from the sum of four integrals, which are functions of the four distances measured between the coils. These distances as seen in Fig. 1 are

$$\begin{aligned} x_1 &= s+(m_1+m_2), \\ x_2 &= s+(m_1-m_2), \\ x_3 &= s-(m_1-m_2), \\ x_4 &= s-(m_1+m_2). \end{aligned} \tag{2.1}$$

For use in the general formula the four corresponding diagonals can then be calculated by

$$\begin{aligned} r_1 &= \sqrt{A^2+x_1^2}, \qquad & r_3 &= \sqrt{A^2+x_3^2}, \\ r_2 &= \sqrt{A^2+x_2^2}, \qquad & r_4 &= \sqrt{A^2+x_4^2}. \end{aligned} \tag{2.2}$$

The mutual inductance of coaxial single-layer coils is then given by the general formula

$$M = .002\pi^2 a^2 n_1 n_2 [r_1 B_1 - r_2 B_2 - r_3 B_3 + r_4 B_4]. \tag{2.3}$$

Where constants B_n in (2.3) are functions of

$$\rho_n^2 = \frac{A^2}{r_n^2} \qquad and \qquad \alpha = \frac{a}{A}, \tag{2.4}$$

α being the ratio of the radii and ρ_n^2 being the squares of the sine of the angle subtended by the radius of the larger coil at the various axial distances x_n. B_n is found using α and ρ_n^2 by interpolating the data given in Grover's tables.

In computing the tables Grover used a series formula given by Clem to find the values of B_n. In the cases when both the values of α and ρ^2 were as great as 0.7 or larger a recourse to the absolute elliptical integral formulas had to be made [4]. Table 2.1 consists of all values of B_n for α and ρ^2 from zero to unity by increments of 0.05. Table 2.2 covers the case when α and ρ_n^2 are large and interpolation of Table 2.1 is difficult, the range of values for α and ρ^2 is from 0.9 to 1 with a smaller increment of 0.01. Table 2.3 consists of B_n values when the radii are equal, α is equal to 1, and ρ^2 extends from zero to unity by 0.01.

Table 2. 1 Grover's Table: values of B_n as a function of α and ρ^2.

α	**ρ^2 = 1**	**0.95**	**0.9**	**0.85**	**0.8**	**0.75**	**0.7**	**0.65**	**0.6**	**0.55**	**ρ^2 = .5**	**α**
1	0.84833	0.87727	0.89552	0.9102	0.92264	0.93345	0.94298	0.95144	0.959	0.96576	0.9718	**1**
0.95	0.86783	0.88982	0.90561	0.91859	0.92971	0.93944	0.94805	0.95573	0.96261	0.96877	0.97428	**0.95**
0.9	0.88418	0.90175	0.91531	0.92666	0.93655	0.94524	0.95298	0.9599	0.96612	0.97169	0.97668	**0.9**
0.85	0.8987	0.91296	0.92456	0.93444	0.94314	0.95085	0.95774	0.96393	0.96951	0.97452	0.97901	**0.85**
0.8	0.91176	0.92344	0.93329	0.94185	0.94944	0.95622	0.96231	0.96781	0.97276	0.97723	0.98124	**0.8**
0.75	0.92356	0.93318	0.9415	0.94885	0.95542	0.96132	0.96668	0.97151	0.97588	0.97983	0.98338	**0.75**
0.7	0.93426	0.94217	0.94917	0.95543	0.96107	0.96618	0.97082	0.97503	0.97884	0.9823	0.98541	**0.7**
0.65	0.94394	0.95045	0.95629	0.96157	0.96637	0.97074	0.97472	0.97835	0.98164	0.98464	0.98732	**0.65**
0.6	0.9527	0.95803	0.96286	0.96727	0.9713	0.97499	0.97837	0.98146	0.98427	0.98683	0.98913	**0.6**
0.55	0.9606	0.96492	0.96888	0.97252	0.97586	0.97894	0.98176	0.98435	0.98672	0.98887	0.99082	**0.55**
0.5	0.96769	0.97115	0.97434	0.9773	0.98003	0.98256	0.98488	0.98702	0.98897	0.99076	0.99237	**0.5**
0.45	0.974	0.97673	0.97927	0.98163	0.98382	0.98584	0.98772	0.98945	0.99103	0.99248	0.99379	**0.45**
0.4	0.97958	0.98169	0.98366	0.9855	0.98721	0.9888	0.99028	0.99164	0.99289	0.99404	0.99508	**0.4**
0.35	0.98444	0.98603	0.98751	0.9889	0.9902	0.99142	0.99254	0.99358	0.99454	0.99542	0.99622	**0.35**
0.3	0.98862	0.98976	0.99084	0.99186	0.9928	0.99369	0.99451	0.99527	0.99598	0.99662	0.99721	**0.3**
0.25	0.99212	0.99291	0.99365	0.99435	0.995	0.99561	0.99618	0.99671	0.9972	0.99765	0.99806	**0.25**
0.2	0.99498	0.99547	0.99594	0.99638	0.9968	0.99719	0.99755	0.99789	0.99821	0.99849	0.99875	**0.2**
0.15	0.99718	0.99746	0.99772	0.99797	0.9982	0.99842	0.99862	0.99881	0.99899	0.99915	0.9993	**0.15**
0.1	0.99875	0.99887	0.99899	0.9991	0.9992	0.9993	0.99939	0.99947	0.99955	0.99962	0.99969	**0.1**
0.05	0.99969	0.99972	0.99975	0.99977	0.9998	0.99982	0.99999	0.99987	0.99989	0.99991	0.99992	**0.05**
0	1	1	1	1	1	1	1	1	1	1	1	**0**

Table 2.1 Grover's Table: values of B_n as a function of α and ρ^2 (concluded).

α	ρ^2 = .5	0.45	0.4	0.35	0.3	0.25	0.2	0.15	0.1	0.05	ρ^2 = 0	α
1	0.9718	0.97718	0.98194	0.98612	0.98974	0.99282	0.99535	0.99735	0.9988	0.99969	1	**1**
0.95	0.97428	0.97919	0.98354	0.98736	0.99066	0.99346	0.99577	0.99759	0.99891	0.99972	1	**0.95**
0.9	0.97668	0.98114	0.98509	0.98855	0.99155	0.99409	0.99618	0.99783	0.99902	0.99975	1	**0.9**
0.85	0.97901	0.98302	0.98658	0.9897	0.9924	0.99469	0.99657	0.99805	0.99912	0.99978	1	**0.85**
0.8	0.98124	0.98483	0.98801	0.9908	0.99322	0.99526	0.9997	0.99827	0.99922	0.9998	1	**0.8**
0.75	0.98338	0.98656	0.98938	0.99185	0.99399	0.99581	0.9973	0.99847	0.99931	0.99983	1	**0.75**
0.7	0.98541	0.9882	0.99068	0.99285	0.99473	0.99633	0.99764	0.99866	0.9994	0.99985	1	**0.7**
0.65	0.98732	0.98975	0.9919	0.9938	0.99543	0.99682	0.99795	0.99884	0.99948	0.99987	1	**0.65**
0.6	0.98913	0.99121	0.99306	0.99467	0.99608	0.99727	0.99825	0.99901	0.99956	0.99989	1	**0.6**
0.55	0.99082	0.99257	0.99413	0.9955	0.99669	0.9977	0.99852	0.99916	0.99963	0.9999	1	**0.55**
0.5	0.99237	0.99383	0.99512	0.99626	0.99725	0.99809	0.99877	0.99931	0.99969	0.99992	1	**0.5**
0.45	0.99379	0.99498	0.99603	0.99696	0.99776	0.99844	0.999	0.99944	0.99975	0.99994	1	**0.45**
0.4	0.99508	0.99601	0.99685	0.99759	0.99823	0.99877	0.99921	0.99955	0.9998	0.99995	1	**0.4**
0.35	0.99622	0.99694	0.99758	0.99815	0.99864	0.99905	0.99939	0.99966	0.99985	0.99996	1	**0.35**
0.3	0.99721	0.99774	0.99822	0.99864	0.999	0.9993	0.99955	0.99975	0.99989	0.99997	1	**0.3**
0.25	0.99806	0.99843	0.99876	0.99905	0.9993	0.99952	0.99969	0.99982	0.99992	0.99998	1	**0.25**
0.2	0.99875	0.99899	0.9992	0.99939	0.99955	0.99969	0.9998	0.99989	0.99995	0.99999	1	**0.2**
0.15	0.9993	0.99943	0.99955	0.99966	0.99975	0.99982	0.99989	0.99994	0.99997	0.99999	1	**0.15**
0.1	0.99969	0.99975	0.9998	0.99985	0.99989	0.99992	0.99995	0.99997	0.99999	1	1	**0.1**
0.05	0.99992	0.99994	0.99995	0.99996	0.99997	0.99998	0.99999	0.99999	1	1	1	**0.05**
0	1	1	1	1	1	1	1	1	1	1	1	**0**

Table 2. 2 Grover's auxiliary table: B_n for large α and ρ^2.

α	ρ^2 = 1	0.99	0.98	0.97	0.96	ρ^2 = .95	α
1	0.84883	0.85698	0.86298	0.8682	0.87292	0.87727	**1**
0.99	0.85294	0.86035	0.86606	0.87107	0.87562	0.87982	**0.99**
0.98	0.85686	0.86366	0.8691	0.87391	0.87829	0.88236	**0.98**
0.97	0.86063	0.86693	0.8721	0.87672	0.88094	0.88487	**0.97**
0.96	0.86428	0.87014	0.87506	0.87949	0.88356	0.88736	**0.96**
0.95	0.86783	0.87329	0.87798	0.88223	0.88615	0.88982	**0.95**
0.94	0.87127	0.87639	0.88086	0.88494	0.88872	0.89226	**0.94**
0.93	0.87462	0.87944	0.8837	0.88761	0.89125	0.89468	**0.93**
0.92	0.87788	0.88242	0.88649	0.89024	0.89375	0.89706	**0.92**
0.91	0.88107	0.88536	0.88924	0.89285	0.89622	0.89942	**0.91**
0.9	0.88418	0.88824	0.89195	0.89541	0.89866	0.90175	**0.9**

α	ρ^2 = .95	0.94	0.93	0.92	0.91	ρ^2 = .9	α
1	0.87727	0.88133	0.88515	0.88877	0.89222	0.89552	**1**
0.99	0.87982	0.88376	0.88747	0.891	0.89436	0.89757	**0.99**
0.98	0.88236	0.88617	0.88978	0.8932	0.89647	0.8996	**0.98**
0.97	0.88487	0.88857	0.89207	0.89539	0.89858	0.90162	**0.97**
0.96	0.88736	0.89094	0.89433	0.89757	0.90066	0.90362	**0.96**
0.95	0.88982	0.89329	0.89658	0.89972	0.90273	0.90561	**0.95**
0.94	0.89226	0.89562	0.89881	0.90186	0.90478	0.90759	**0.94**
0.93	0.89468	0.89792	0.90102	0.90397	0.90681	0.90954	**0.93**
0.92	0.89706	0.9002	0.9032	0.90607	0.90883	0.91148	**0.92**
0.91	0.89942	0.90246	0.90536	0.90815	0.91082	0.9134	**0.91**
0.9	0.90175	0.90469	0.9075	0.9102	0.9128	0.91531	**0.9**

This research is essential, a method can be chosen depending on the error a person can manage. No previous studies obtain the amount of error in Grover's tables. The investigation shows when it would be appropriate to use Grover's tables for a quick answer compared to when an individual would want to use a field code, or elliptical integrals to acquire a more exact answer. The MagNet interface can also be used to discover the situation when the parameters create the maximum mutual inductance gradient. This is very valuable in the design of a coil gun system since the force is directly proportional to the mutual inductance gradient.

Table 2. 3 B_n for coils of equal radii ($\alpha = 1$).

ρ^2	B_n	ρ^2	B_n	ρ^2	B_n	ρ^2	B_n
0	1	**0.25**	0.992815	**0.5**	0.971802	**0.75**	0.933448
0.01	0.999987	**0.26**	0.992244	**0.51**	0.970649	**0.76**	0.931397
0.02	0.99995	**0.27**	0.99165	**0.52**	0.969469	**0.77**	0.929294
0.03	0.999889	**0.28**	0.991035	**0.53**	0.968262	**0.78**	0.927135
0.04	0.999804	**0.29**	0.990399	**0.54**	0.967027	**0.79**	0.924918
0.05	0.999695	**0.3**	0.989742	**0.55**	0.965763	**0.8**	0.922639
0.06	0.999562	**0.31**	0.989062	**0.56**	0.964471	**0.81**	0.920297
0.07	0.999407	**0.32**	0.98836	**57**	0.963149	**0.82**	0.917886
0.08	0.999228	**0.33**	0.987637	**0.58**	0.961798	**0.83**	0.915403
0.09	0.999026	**0.34**	0.986891	**0.59**	0.960416	**0.84**	0.912843
0.1	0.998802	**0.35**	0.986123	**0.6**	0.959002	**0.85**	0.910202
0.11	0.998556	**0.36**	0.985332	**0.61**	0.957558	**0.86**	0.907472
0.12	0.998287	**0.37**	0.98452	**0.62**	0.95608	**0.87**	0.904648
0.13	0.997996	**0.38**	0.983684	**0.63**	0.95457	**0.88**	0.901721
0.14	0.997684	**0.39**	0.982826	**0.64**	0.953024	**0.89**	0.898683
0.15	0.997349	**0.4**	0.981944	**0.65**	0.951443	**0.9**	0.895522
0.16	0.996992	**0.41**	0.981039	**0.66**	0.949826	**0.91**	0.892225
0.17	0.996614	**0.42**	0.98011	**0.67**	0.948172	**0.92**	0.888774
0.18	0.996214	**0.43**	0.979158	**0.68**	0.94648	**0.93**	0.885151
0.19	0.995793	**0.44**	0.978182	**0.69**	0.944748	**0.94**	0.881327
0.2	0.995351	**0.45**	0.977181	**0.7**	0.942975	**0.95**	0.877266
0.21	0.994886	**0.46**	0.976156	**0.71**	0.941161	**0.9**	0.872917
0.22	0.994401	**0.47**	0.975106	**0.72**	0.939302	**0.97**	0.868201
0.23	0.993894	**0.48**	0.974031	**0.73**	0.937398	**0.98**	0.862983
0.24	0.993366	**0.49**	0.97293	**0.74**	0.935448	**0.99**	0.85698
0.25	0.992815	**0.5**	0.971802	**0.75**	0.933448	**1**	0.848826

2.2 Elliptical Integral Mutual Inductance Theory

Before Grover's method was introduced the conventional method of elliptical integrals was used to calculate the mutual inductance of two coaxial single-layer coils [11]. The theory of elliptical integrals for the mutual inductance of two coaxial single-layer coils begins by calculating the magnetic flux through a wire loop due to the magnetic field produced my another wire loop[12].

$$\Phi_{B2} = \int_{S_2} \mathbf{B}_1 \bullet d\mathbf{a}_2 = M_{1\to 2} I_1 \tag{2.5}$$

where Φ_{B2} is proportional to the current flowing in loop 1 and the mutual inductance. Similarly

$$\Phi_{B1} = \int_{S_1} \mathbf{B}_2 \bullet d\mathbf{a}_1 = M_{2\to 1} I_2 \tag{2.6}$$

where Φ_{B1} is proportional to the current flowing in loop 2 and the mutual inductance. Now replacing the magnetic field **B** with the vector potential by

$$B_j = \nabla \times A_j \tag{2.7}$$

where

$$\mathbf{A}_j = \frac{\mu_o}{4\pi} I_j \oint_{\partial S_j} \frac{d\mathbf{l}_j}{\left|\mathbf{r} - \mathbf{r}_j\right|} . \tag{2.8}$$

The magnetic flux equation then becomes

$$\Phi_{Bk} = \oint_{\partial S_k} \mathbf{A}_j \bullet d\mathbf{l}_k = \frac{\mu_o}{4\pi} I_j \oint_{\partial S_j} \oint_{\partial S_k} \frac{d\mathbf{l}_j}{\left|\mathbf{r} - \mathbf{r}_j\right|} = M_{j \to k} I_j \tag{2.9}$$

Hence the mutual inductance is symmetric $M_{1\to 2} = M_{12} = M_{2\to 1}$ so

$$M_{12} = \frac{\mu_o}{4\pi} \oint_{\partial S} \oint_{\partial S'} \frac{d\mathbf{l}_1 \bullet d\mathbf{l}_2}{\left|\mathbf{r} - \mathbf{r}'\right|} = \frac{\mu_o}{4\pi I_1 I_2} \int_V dr \int_{V'} dr' \frac{\mathbf{J}_1(\mathbf{r}) \bullet \mathbf{J}_2(\mathbf{r}')}{\left|\mathbf{r} - \mathbf{r}'\right|}. \tag{2.10}$$

The mutual inductance between two coaxial coils, one with radius *a*, and another with radius *A* with the distance between centers *s* can then be written as:

$$M_{12} = \frac{\mu_o}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{Aa\cos(\varphi - \varphi')\,d\varphi\,d\varphi'}{(A^2 + a^2 + s^2 - 2Aa\cos(\varphi - \varphi'))^{1/2}}. \tag{2.11}$$

This integral can be exactly solved in the elliptical integral form

$$M_{12} = \mu_o \sqrt{Aa} \left[\left(\frac{2}{k} - k \right) K + \frac{2}{k} E \right] \tag{2.12}$$

where

$$k = \frac{2\sqrt{Aa}}{\sqrt{(A+a)^2 + s^2}}, \tag{2.13}$$

$$K(m) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{(1 - m\sin^2\theta)}}, \tag{2.14}$$

and

$$E(m) = \int_0^{\pi/2} \sqrt{(1 - m\sin^2\theta)}\; d\theta. \tag{2.15}$$

Chapter 3 Design of Experiment

For two coaxial single-layer coils Grover indicates there are certain points of interest when solving for the mutual inductance. These situations occur when the coils are: concentric, loosely coupled, have equal radii, and concentric with equal lengths. This investigation details these situations, by fixing the lengths, turn densities, the radius of the large coil, the spacing of both coils, the current, and the wire thickness, then varying the separation distance for different values of α, where α is changed by changing the values of the small coils radius. Results were achieved using MagNet as the field code software and MatLab to solve for the elliptical integrals and the Grover methods. Two mutual inductance examples were created to cover all the points of interest, table 3.1. Two examples were performed to demonstrate the difference between not equal and equal length coils.

Table 3. 1 Parameters for experiment.

Example	**$2m_1$ [cm]**	**$2m_2$ [cm]**	**n_1 [Turns/cm]**	**n_2 [Turns/cm]**	**A [cm]**	**α =a/A**	**S [cm]**
1	1	1	10	10	10	.1-1	-4 to 4
2	10	16	10	20	20	.1-1	-40 to 40

Additionally example problems solving for mutual inductances have been drawn from [4, 6] and will be compared to the numeric results obtained by elliptical integral method and field code method.

3.1 MagNet Design

A visual basic script was programmed in MagNet (see Appendix A) in which various input parameters can be entered, Figure 3.1. The known data parameters are entered or chosen into the top boxes, in centimeters. To create a visual of the two coils and obtain outputs the "Make Coils" button is clicked.

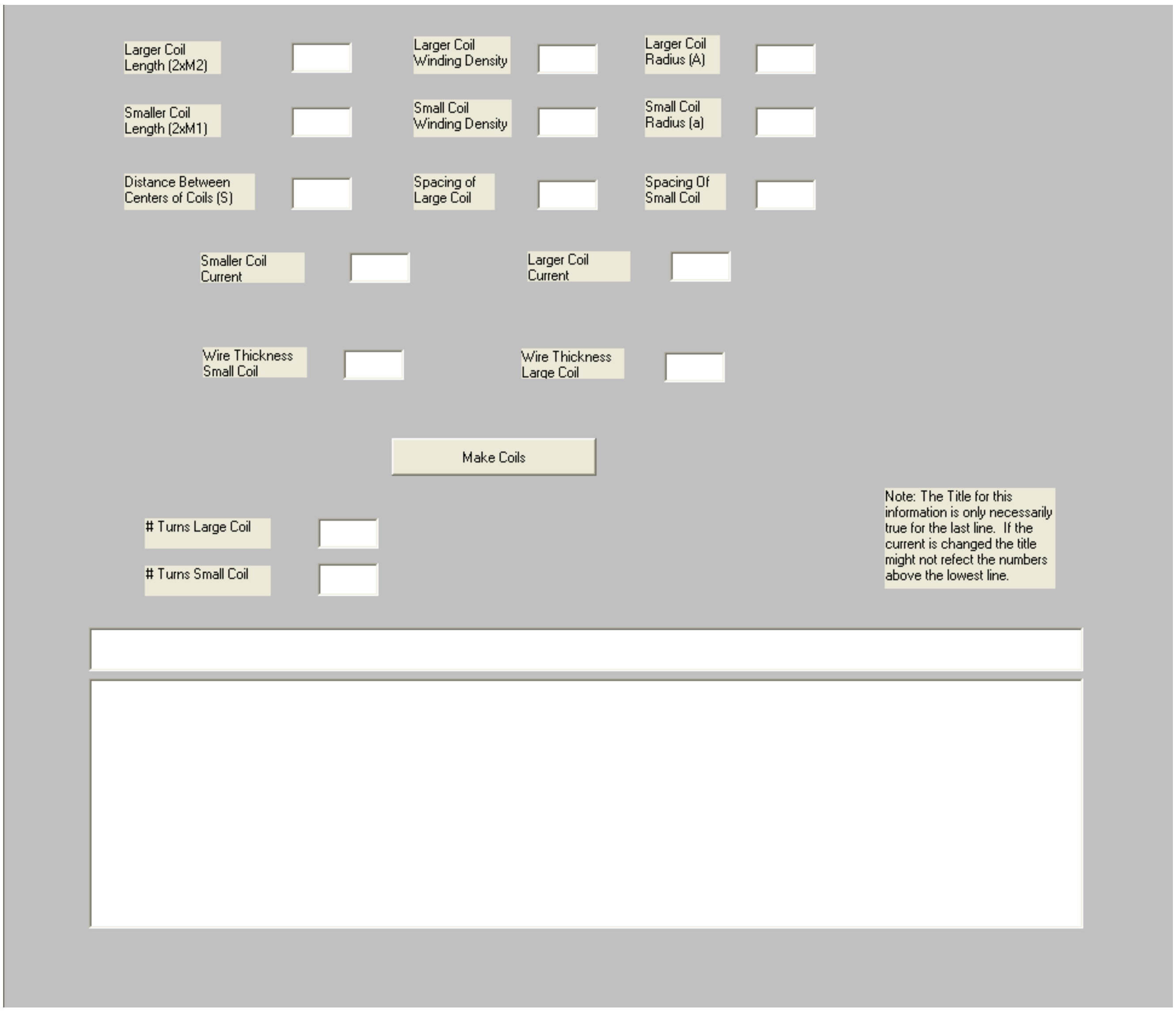

Figure 3. 1 MagNet interface – creates coils from input parameters.

MagNet utilizes the input values to create two coils contained in an air box, Figure 3.2. The air box is created to be much larger than the coils so that the flux function contour lines are not affected by the air box, as shown in figure 3.3.

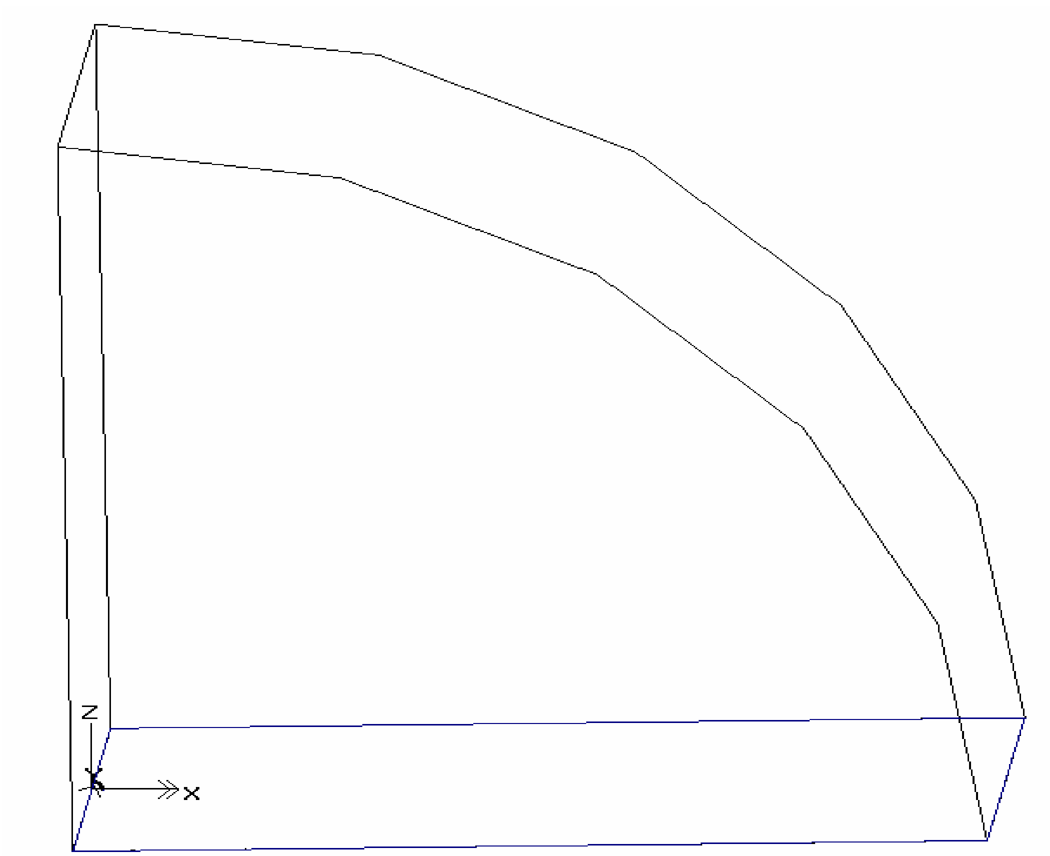

Figure 3. 2 Two coils created inside an air box.

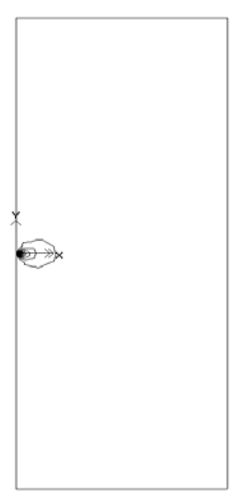

Figure 3. 3 Flux contour lines around coils in an air box.

When the "Make Coils" button is clicked the coils are created and solved using the input parameters and the static 2D method in MagNet. During its analysis MagNet uses a finite element method accomplished through the division of the model into a mesh of elements, seen in figure 3.4. These mesh elements are shaped like triangles are defined by three nodes.

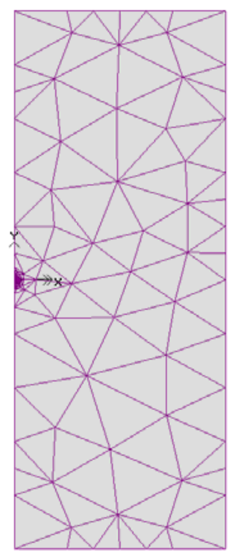

Figure 3. 4 Division of the model into a mesh of elements.

The accuracy of MagNet's solutions depends on the nature of the field and the size of the mesh elements. In the regions where the direction or magnitude of the field is changing rapidly, high accuracy requires small mesh elements. Figure 3.5 is a focused picture of figure 3.4, showing the mesh around the 2 coils. The coils are shown as the two vertical lines towards the middle of the picture. The figure illustrates how the mesh is more refined to improve the accuracy around the ends of the coils where the fields are changing.

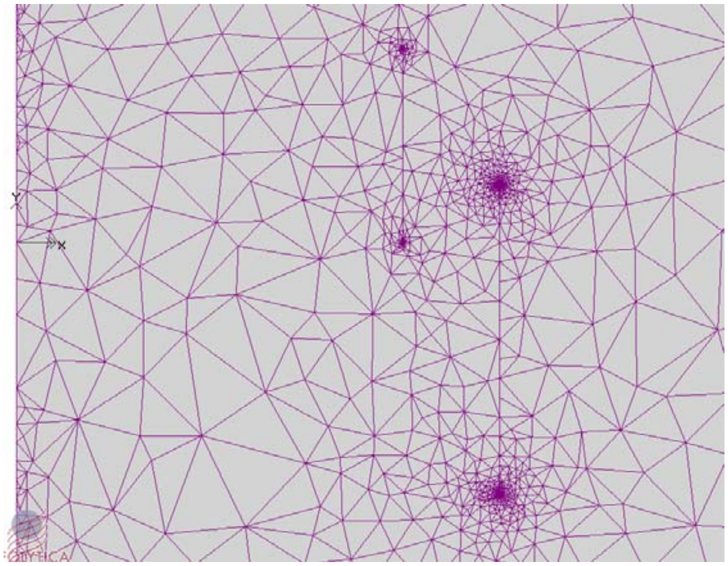

Figure 3. 5 Mesh analysis of two 2D coils.

MagNet has built in features that can be adjusted to increase the accuracy of its' solutions. To refine the mesh and achieve higher accuracies the h-adaptation was set to refine 25 percent of the elements with a tolerance of 0.01%. The polynomial order was set to 2 with a convergence tolerance of 0.01 percent. The type of coil utilized were stranded because the current and turns are separated as seen in the coil parameters in figure 3.6. Detailed information on MagNet can be found in [9].

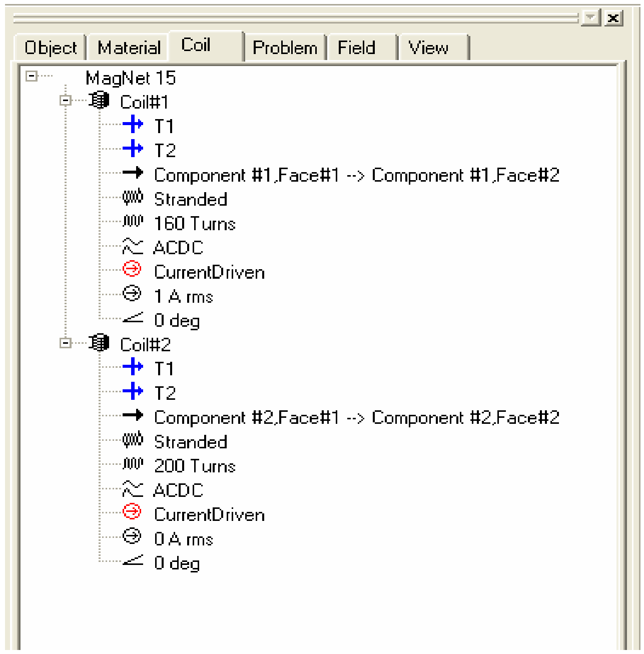

Figure 3. 6 Coil parameters.

During analysis of the coils some parameters were appointed values that they would hold throughout the entire experiment. To solve for the mutual inductance in a straightforward manner the large coil was driven with 1 amp of current, while the small coil's current was set to 0 amps. The wire thickness of both coils was set to .01 cm, while the spacing of the coils was established to be 0. The coils are modeled as one piece visually and MagNet incorporates the turns into its calculations, figure 3.7 and figure 3.8. Figures 3.7 and 3.8 are two different geometries that were created using MagNet. Figure 3.7 is a 2D look at the coils, much like Grover's geometry shown in figure 2.1. Figure 3.8 shows a 2D rotational view with the coils overlapping and the direction of the currents displayed.

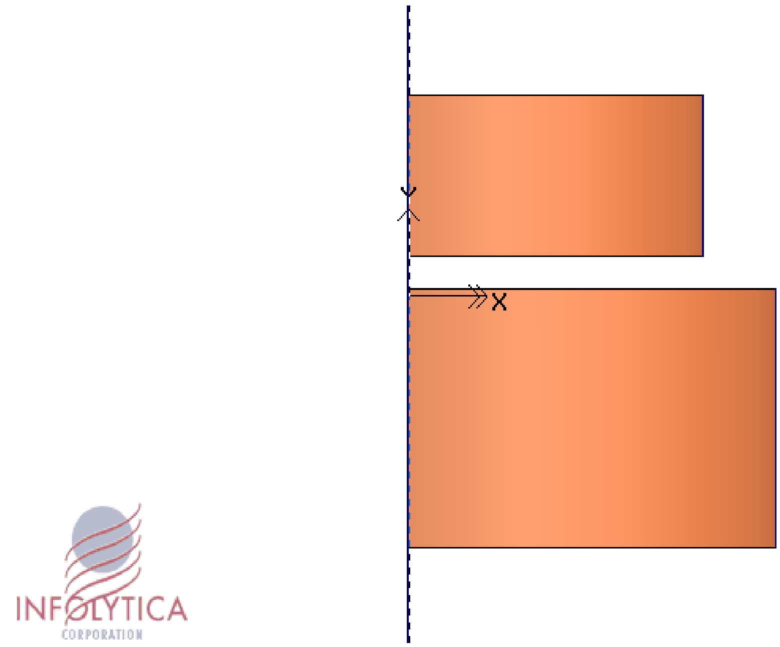

Figure 3. 7 Geometry of 2D coils created in MagNet.

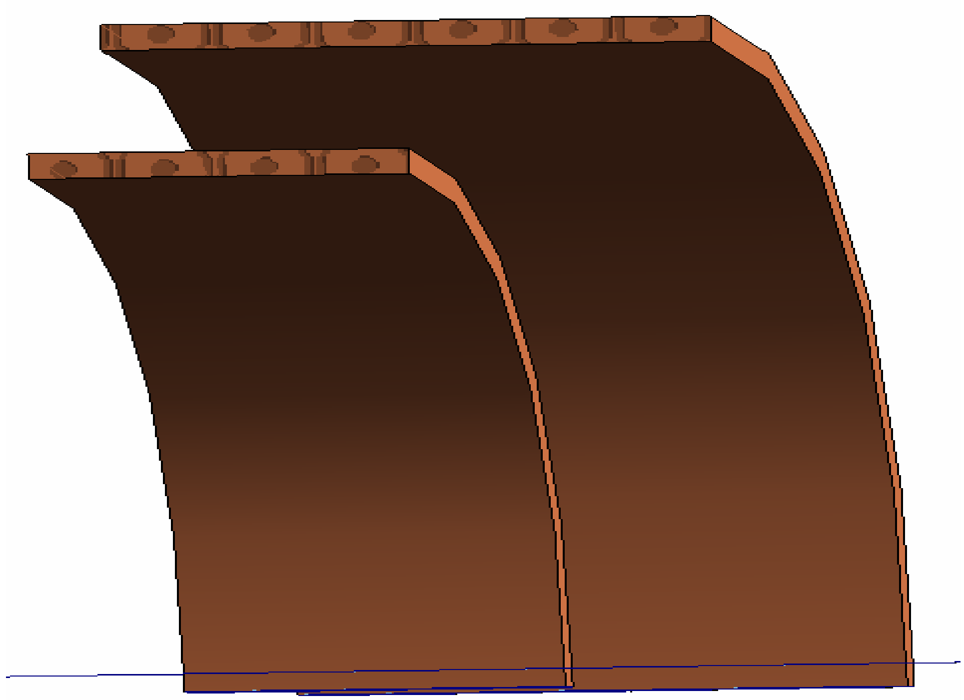

Figure 3. 8 2D rotational coils illustrating current direction.

When MagNet solves the coils it outputs a post-processing bar that displays energy, force, flux linkage, power loss, and current for each coil, bottom of figure 3.9. The interface solves for the mutual inductance, self inductances depending on parameters, and the force on each coil and place them into a text box.

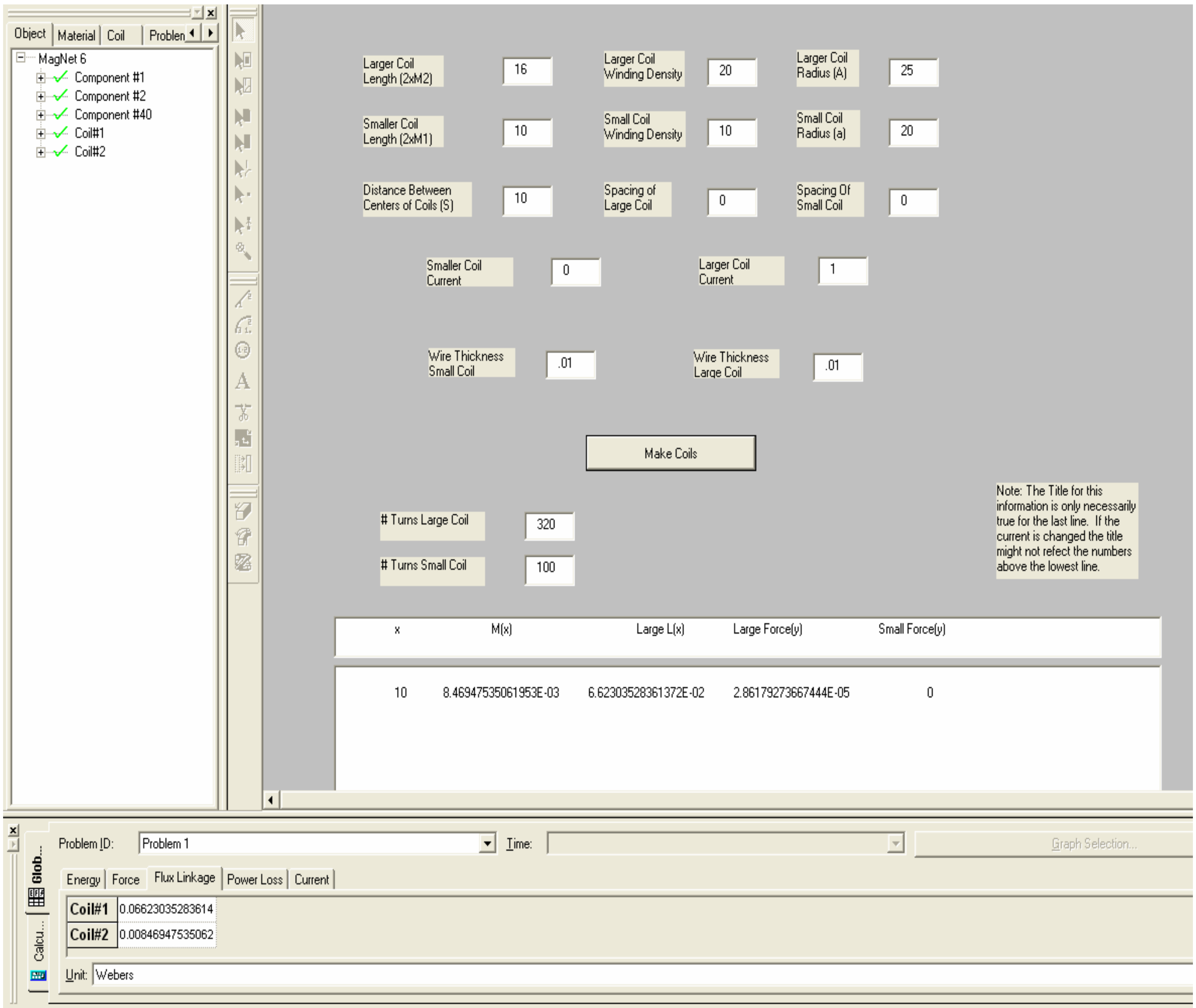

Figure 3. 9 MagNet interface – inputs are above the "Make Coils" button, outputs turns, separation distance, mutual inductance, self inductance, and the forces on each coil. Also MagNet's post processing bar shown at bottom outputs various data.

The separation distance between the two coils is outputted from the input data. Using the flux linkage parameter MagNet previously obtained, the mutual inductance is then solved in the interface using

$$M_1 = \frac{\lambda_{21}}{I_1} \quad = \quad M_2 = \frac{\lambda_{12}}{I_2} \tag{3.1}$$

where λ_{21} is the flux linkage of the small coil, λ_{12} is the flux linkage of the large coil, I_1 is large coil current, and I_2 is the small coil current. Flux linkage is the sum of all the fluxes for all the turns of the coil. The self inductance is

$$L_{large} = \frac{\lambda_{11}}{I_1} \qquad L_{small} = \frac{\lambda_{22}}{I_2} \tag{3.2}$$

where λ_{11} is the flux linkage of the large coil, λ_{22} is the flux linkage of the small coil, I_1 is large coil current, and I_2 is the small coil current. For this experiment to solve for the mutual inductance one of the current values is set to 0. As stated earlier I_1 is set to 1 and I_2 is 0 so the self inductance of the small coil does not exist. The force for each coil is calculated by MagNet and placed into the interface output textbox. Figure 3.9 shows input values, fixed input data, and the output data for the given parameters. The mutual inductance values for varying s for the many values of α were recorded and can be found in Appendix C.

3.2 MatLab Design

MatLab was used to solve for Grover's mutual inductance and the mutual inductance using elliptical integrals. The values obtained using MatLab were then evaluated against know values from published literature [4, 6, 11]. The comparison to published work was made to establish the validly of the data obtained from MatLab. Because of the complexity of incorporating both mutual inductance methods many subprograms were written for simplicity, Appendix B. For each set of programs the large radius, lengths, and turn densities were set, while the separation distance varied depending on the example (table 3.1) for each value of α from .1 to 1. The numerical values of the mutual inductances' for each separation distance at every value of α can be found in Appendix C.

3.2.1 Solving for Grover's Mutual Inductance

Finding the mutual inductance using Grover's method is completed first. Once the mutual inductance answer was obtained it was then compared to Grover's results and compared well. This was completed to confirm the answers provided by the MatLab program. Since solving for Grover's mutual inductance involves using multiple tables several subprograms were used. The main program, grovermutual, receives the radii, lengths, and turn densities as user inputs. It then solves for x_n and r_n values, and then uses these values to find α and ρ^2. From the values of α and ρ^2 a subprogram is then chosen. In MatLab three subprograms, induct4, largegrover, and equalradii were created to each contain a table. Each of these subprograms obtains the correct values from their respected table and uses theses values to solve for B_n's by interpolation. The values for each B_n are then passed back into the main program and used to solve Grover's mutual inductance equation (2.3). To solve for the mutual inductance values for each separation distance, a program was created (changingS) to call the main program for each value of s. This enables MatLab to output the mutual inductance for every s value when only run once. The small radius is then changed to modify α, and the program is run again. This is done for all values of α from 0.1 to 0.9 by increments of 0.1 and from 0.95 to 1 by increments of 0.01 for each example. Grover's mutual inductance answers for each example at the varying s and α values can be found in Appendix C.

3.2.2 Solving for Mutual Inductance using Elliptical Integrals

Hand calculations of the complete elliptical integrals of the first and second kinds can be tedious and time consuming. MatLab make use of equations (1.1) and (1.2) by solving for $\phi = \frac{\pi}{2}$.

In MatLab the complete elliptical integral of the first kind is

$$F(m) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{(1 - m\sin^2\theta)}}, \tag{3.3}$$

and the complete elliptical integral of the second kind is

$$E(m) = \int_0^{\pi/2} \sqrt{(1 - m\sin^2\theta)} \quad d\theta, \tag{3.4}$$

where

$$k^2 = m, \tag{3.5}$$

$$m = 1 - \left(\frac{y_2}{y_1}\right)^2, \tag{3.6}$$

and

$$\begin{aligned} y_1 &= \sqrt{(A+a)^2 + d^2}, \\ y_2 &= \sqrt{(A-a)^2 - d^2}. \end{aligned} \tag{3.7}$$

The values of *A* and *a* are the radius of the large and small coils respectively, and *d* is the separation distance. These equations can be utilized by a MatLab function called ellipke that returns F and E, which are the complete elliptical integrals of the first and second kinds with *m* as the input [10]. The values of F and E are then used to solve for the mutual inductance of two central turns given by Maxwell as,

$$M_o = 4\pi\sqrt{aA}\left[\left(\frac{2}{k} - k\right)F - \frac{2}{k}E\right] \tag{3.8}$$

found in [11].

The mutual inductance of the two coils with N_1 and N_2 turns will then be

$$M = N_1 N_2 M_o. \tag{3.9}$$

where

$$\begin{aligned} N_1 &= 2m_1 n_1 \\ N_2 &= 2m_2 n_2. \end{aligned} \tag{3.10}$$

The values obtained using equation (3.8) were compared directly to examples of Maxwell's mutual inductance equation found in [11]. When comparing elliptical integrals solved using MatLab to those found in [11] the accuracy was found to be 6-7 significant digits. This is a better accuracy than Grover has claimed for his tables.

As when Grover was solved using MatLab, the user inputs the radii, lengths and turn densities, and the program (ellmutual, Appendix B) applies these inputs to the above equations to acquire the mutual inductance. Once again, an addition program (mutualchange, Appendix B) is created to output the main program's mutual inductance from the elliptical integrals for each different separation distance. The user then changes the smaller radius and runs the program again for the values of s found in table 3.1. The mutual inductance values when using elliptical integral method for each example can be found in Appendix C.

Chapter 4 Experimental Results

Two different types of studies were done. The first study established the accuracies of the experimental code written. The second study demonstrated the separation distance locations where Grover's tables are most accurate for a range of ratios of the radii between 0.1 and 1.

By comparing Grover's literature examples [4, 6] to the answers obtained using MagNet one can provide evidence of the accuracy of the experimental interface created in MagNet. This can also be done to confirm the experimental program written in MatLab to solve for the mutual inductance using elliptical integrals. The examples chosen are over a wide range of coil lengths, radii and separation distances.

The approximate errors of Grover's method compared to field code software and elliptical integral methods are examined for the set values of s and alpha. Surface plots were created to get an overall picture of the error over all alpha and s distances. Plots of various ratios of the radii are given below for each comparison for the situations of equal and not equal lengths of the coils.

Both studies examine the points of interest; concentric, loosely coupled, equal radii and concentric with equal lengths for two coaxial single-layer coils. The results are demonstrated using percent error calculations. Percent error was calculated using

$$\%error = \left| \frac{experimental - actual}{actual} \right| \; x \;\; 100 \qquad (4.1)$$

where the experimental value is Grover's mutual inductance and the actual is either the mutual inductance when using MagNet or the mutual inductance when using elliptical integrals. The numeric values for the percent error when comparing Grover to MagNet and Grover to Elliptical while varying s can be found in Appendix D.

4.1 Grover vs. MagNet – Examples from Grover's Literature

In [4] and [6] Grover illustrated examples for several different geometries. Table 4.1 shows the calculated percent error (4.1) for varying geometries when Grover's book examples are compared to MagNet.

Table 4. 1 Comparison between Grover's examples and MagNet.

problem	a [cm]	A [cm]	2m1 [cm]	2m2 [cm]	n1 [Turns/cm]	n2 [Turns/cm]	S Distance [cm]	MagNet [H]	Grover [H]	% Error
1m	3	4	50	4	10	50	0	7.0154E-04	7.0170E-04	0.0224
2m	5	10	4	16	20	10	0	5.0624E-04	5.0690E-04	0.1303
3m	3	4	4	50	50	10	0	7.0160E-04	7.0170E-04	0.0138
4m	4.9	5	1	1	10	10	0	1.8073E-05	1.8087E-05	0.0791
5m	2	5	30	24	10	40	0	1.3696E-03	1.3701E-03	0.0349
6m	5	5	1	1	10	10	2	6.8826E-06	6.9003E-06	0.2568
7m	20	20	4	6	10	10	10	5.4840E-04	5.4884E-04	0.0798
8m	20	25	10	16	10	20	10	8.4695E-03	8.4580E-03	0.1355
9m	2	3	10	6	10	10	18	8.5938E-07	8.5017E-07	1.0715
10m	4.435	6.44	27.38	20.55	0.7296	2.737	31.165	1.0819E-06	1.0862E-06	0.3963

It can be observed from table 4.1 that the error is only a slight amount for all values tested.

Overall the error is less when the coils are concentric, having no separation distance. The case with the largest amount of error, 1.0715% occurs when there is a large separation distance and the coils are fairly small, problem 9m. It can also be noted that when the coils have the same radius, same lengths and the coils are situated directly next to each other, such as in problem 6m, a higher percent error occurs compared to when the radius are equal but the lengths are not and the separation distance yields coils with air box between them, such as in problem 7m. The percent error information leads one to believe that using Grover in situation such as those presented can be a realistic approximation.

4.2 Grover vs. Elliptical Integrals – Examples from Grover's Literature

As above Grover's example problems from [4] and [6] are examined. The percent error, (4.1) is found when comparing the example problems to the complete elliptical integrals using MatLab, table 4.2.

Table 4. 2 Comparison between Grover's examples and elliptical integrals.

problem	a [cm]	A [cm]	2m1 [cm]	2m2 [cm]	n1 [Turns/cm]	n2 [Turns/cm]	S Distance [cm]	Elliptical [H]	Grover [H]	% Error	$k^2\sin^2\theta$
1e	3	4	50	4	10	50	0	5.9566E-03	7.0170E-04	88.22	0.9596002
2e	5	10	4	16	20	10	0	7.0223E-04	5.0690E-04	27.82	0.7901235
3e	3	4	4	50	50	10	0	5.9566E-03	7.0170E-04	88.22	0.9596002
4e	4.9	5	1	1	10	10	0	2.48E-05	1.8087E-05	26.97	0.9997959
5e	2	5	30	24	10	40	0	4.8517E-03	1.3701E-03	71.76	0.666389
6e	5	5	1	1	10	10	2	6.7537E-06	6.9003E-06	2.17	0.9245562
7e	20	20	4	6	10	10	10	5.34E-04	5.4884E-04	2.77	0.8858131
8e	20	25	10	16	10	20	10	7.9612E-03	8.4580E-03	6.24	0.8858131
9e	2	3	10	6	10	10	18	6.8960E-07	8.5017E-07	23.28	0.004729
10e	4.435	6.44	27.38	20.55	0.7296	2.737	31.165	5.4619E-07	1.0862E-06	98.87	0.0109953

When comparing Grover's method to the Elliptical method the errors are significantly larger than the case of Grover versus MagNet. The errors range between a maximum of 98.87% to a minimum of 2.17%. One explanation comes from [4] where Grover states "for values of α and ρ_n^2 both as great as 0.7, and larger, recourse had to be made to the absolute elliptical integral formulas." This indicates that Grover's tables have been adjusted from directly applying elliptical integrals and will account for some of the errors. Looking at table 4.2, the larger inaccuracies exist when the coils are concentric, and when the coils are loosely coupled. In the above table it can be seen that in problem 7e, a 10 cm separation between the centers, with 5 cm of air between the coils does not product a large amount of error, 2.77%. On the other hand in problem 9e the separation distance between the centers is 18 cm, and the air between them is 10 cm, producing a sizable percent error of 23.28%. The larger separation distance along with the small size coils contribute to the larger error in 9e.

There could be a concern with (3.3) diverging. This could suggest that as $k^2\sin^2\theta$ goes to 1 the mutual inductance answers when solving using elliptical integrals could be erroneous. Through an investigation of comparing $k^2\sin^2\theta$ to the percent error this is not the case. Looking at the values of $k^2\sin^2\theta$ where $m=k^2$ there is not a trend that would signify that as $k^2\sin^2\theta$ becomes closer to 1 that the error is higher, or increasing. This can be observed in figure 4.1.

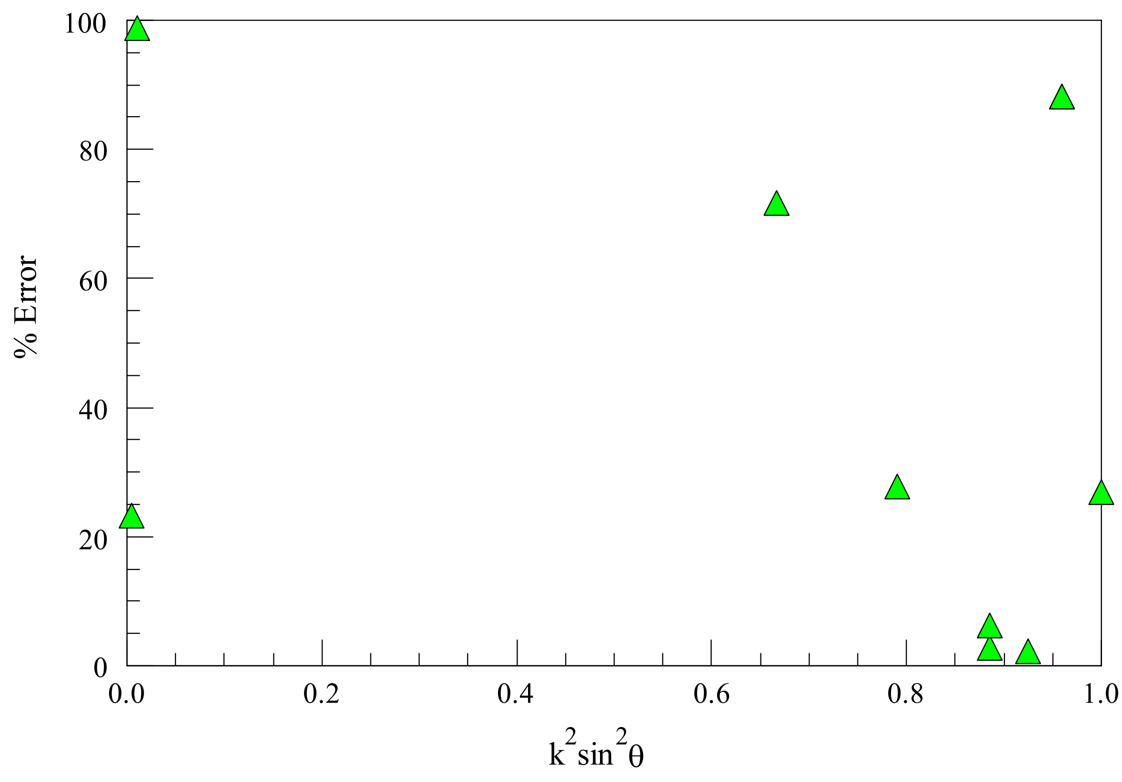

Figure 4. 1 $k^2\sin^2\theta$ versus % error

4.3 Grover vs. MagNet – Varying Separation Distances for Several α Values

The analysis was set up as stated above in table 3.1 and evaluated for equal and not equal lengths. For this investigation the acceptable error amount for the use of Grover's tables is 1 percent or under. Over the wide ranges detailed, the data shows that when comparing Grover's mutual inductance to MagNet's mutual inductance little error is achieved, below 10 percent for when the coil lengths are not equal and equal. In both cases the error seems to rise as alpha becomes larger. This can be observed in figures 4.1, 4.2 and 4.7.

4.3.1 Grover vs. MagNet – Equal Lengths

For the case when the lengths are equal the error is negligible when alpha is under 0.6. When alpha is between 0.6 and 0.8 the error is more significant, consistently over 1%, and when alpha is larger than 0.8 the error is a great deal larger, above 5% as can be seen in figure 4.1. It can be seen from figure 4.1 that when comparing Grover to MagNet, for the case of equal length coils the error is trivial when alpha is under 0.6. However, when alpha is larger than 0.9 the error increase considerably.

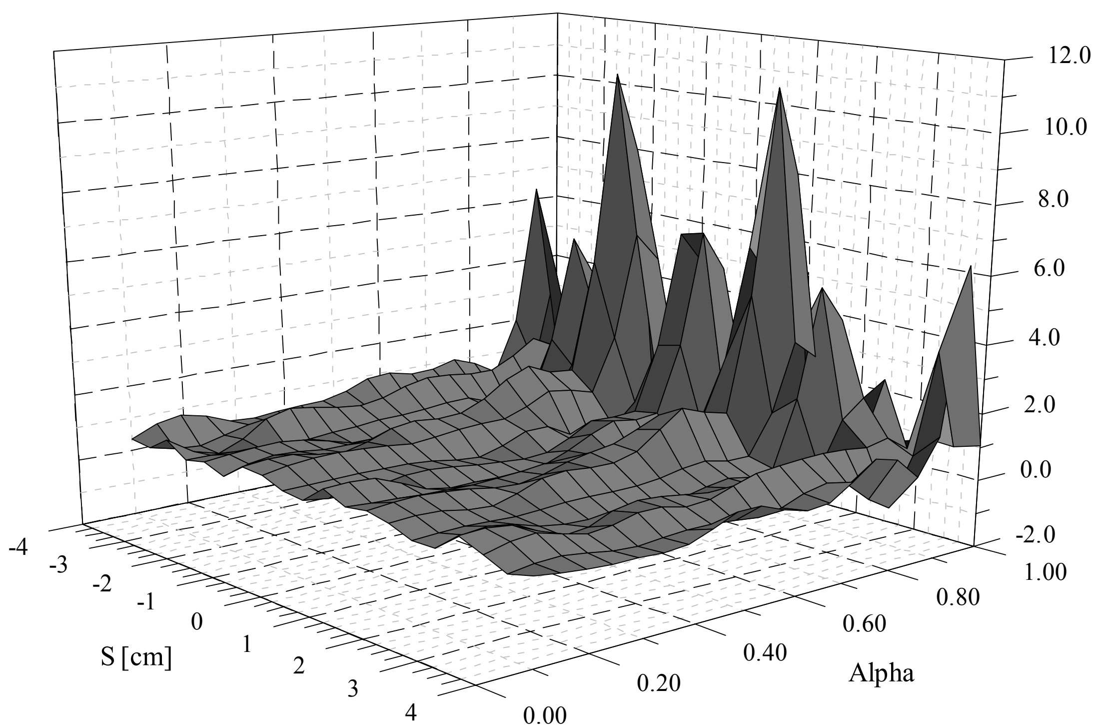

Figure 4. 2 Percent error between Grover's mutual inductance and MagNet's measured mutual inductance for two coils of equal lengths.

Figure 4.2 give you a better idea about the error when alpha is large. The plot illustrates figure 4.1 from alpha of .95 to 1. From the graph the occurrence of loosely coupled coils can be better comprehended. It reveals that when the separation distance increase the error also increases, and also confirms that the error also gets larger as alpha increases.

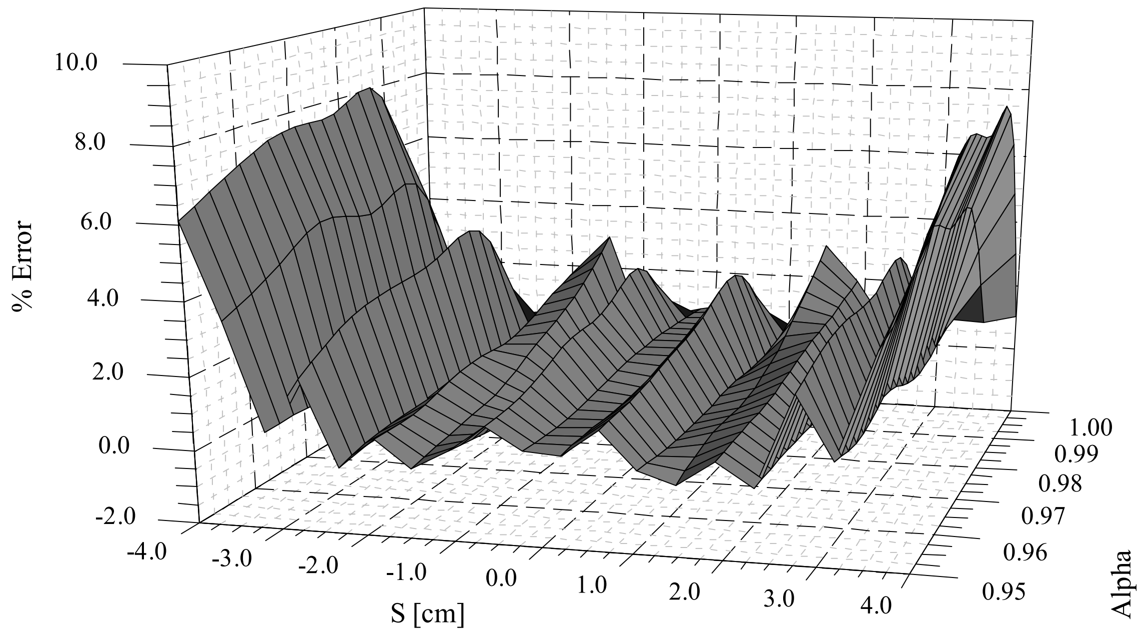

Figure 4. 3 Error occurring when alpha is large for the case when the lengths of the coils are equal when comparing Grover to MagNet.

The mutual inductance results were graphed onto a 2D plot for all values of alpha. The typical graphs (figures 4.3, 4.4, 4.5 and 4.6) for different values of alpha are illustrated to give you an idea about their nature as the separation distances change. As mention earlier the error will increase as alpha increases so for each of the actual graphs the errors are not the same, only the nature of the waveform.

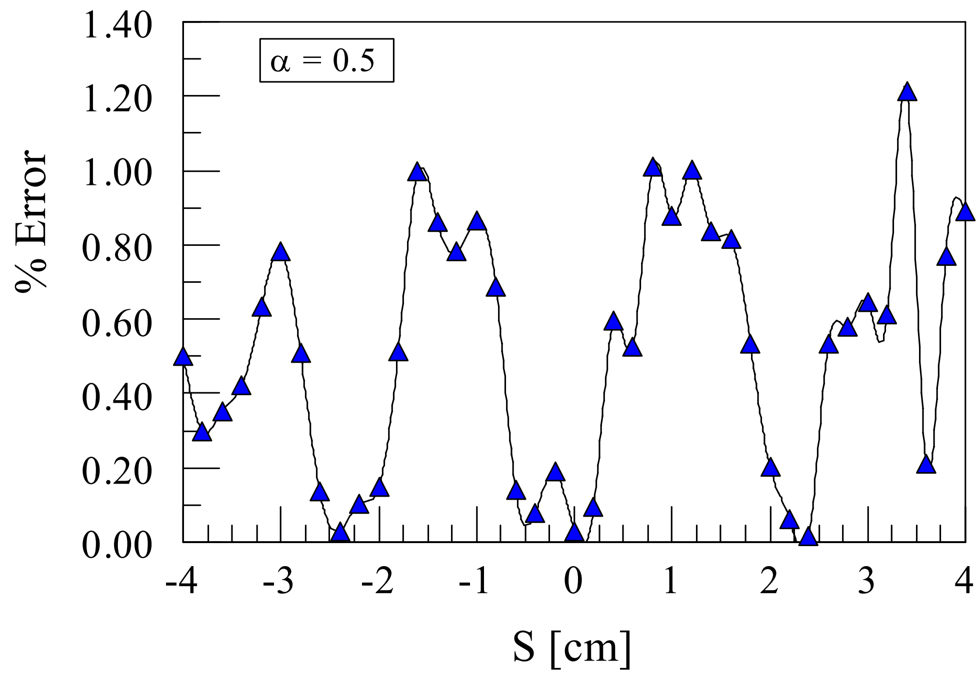

Figure 4. 4 Grover vs. MagNet -Typical waveform for alpha = 0.1 – 0.6 and equal lengths.

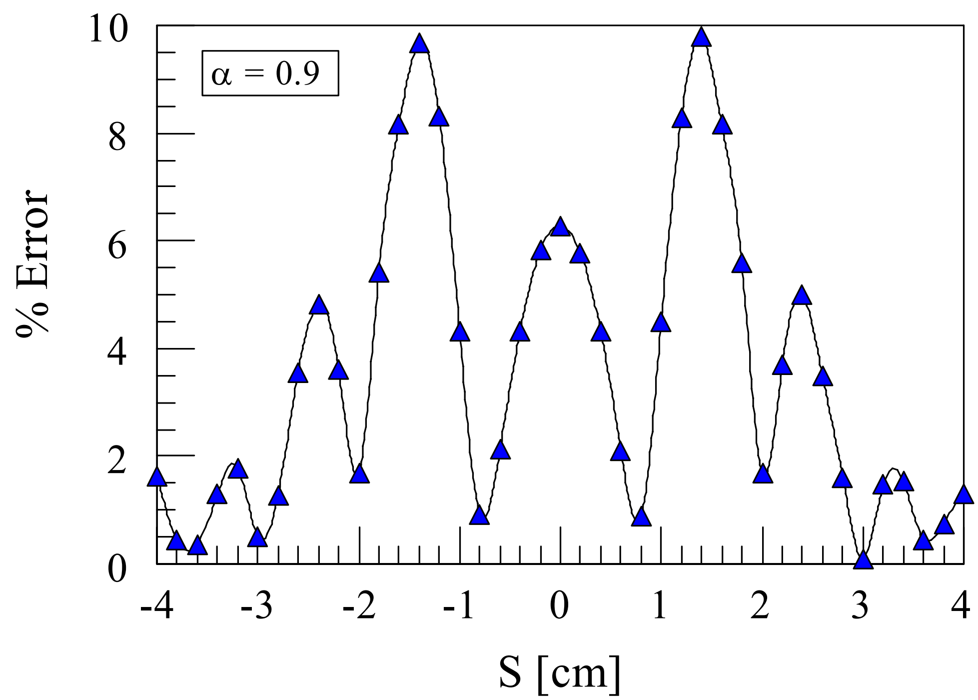

Figure 4. 5 Grover vs. MagNet - Typical waveform for alpha = 0.7 - 0.9 and equal lengths.

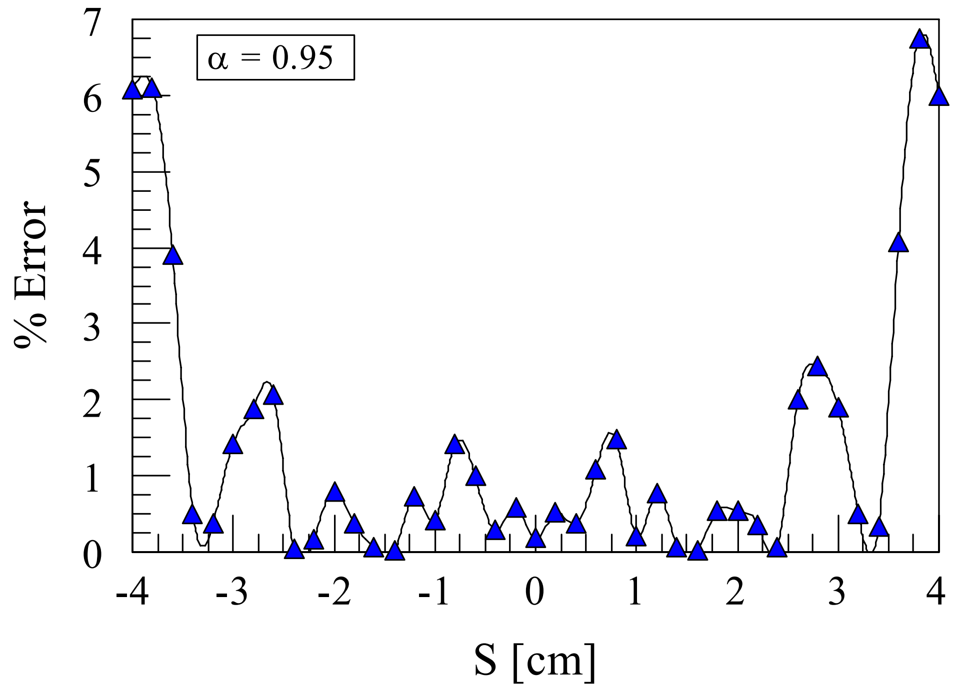

Figure 4. 6 Grover vs. MagNet - Typical waveform for alpha = 0.95 – 0.99 and equal lengths.

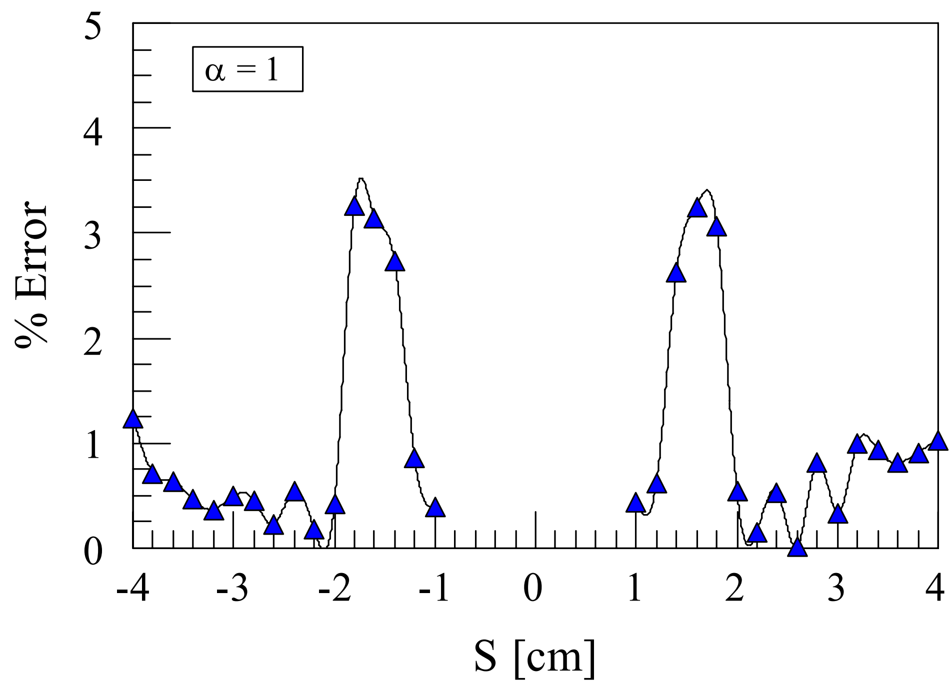

Figure 4. 7 Grover vs. MagNet - Waveform for alpha = 1 and equal lengths.

The maximum error for when the lengths are equal is 9.799% which occurs at s = 1.4 cm when alpha = 0.9, figure 4.4. The minimum error of 0.00095% takes place when s = -1.4 cm and alpha = 0.95, figure 4.5. A noticeable trend arises when alpha is between 0.95 and 0.98. There is virtual no error, 1% or less when the s distances are between -2.4 cm to -1 cm, -0.4 cm to 0.4 cm and 1 cm to 2.4 cm.

4.3.2 Grover vs. MagNet – Unequal Lengths

In the case when the lengths of the coils are not equal there is very little percent error between Grover's mutual inductance and the field code mutual inductance from MagNet, figure 4.7.

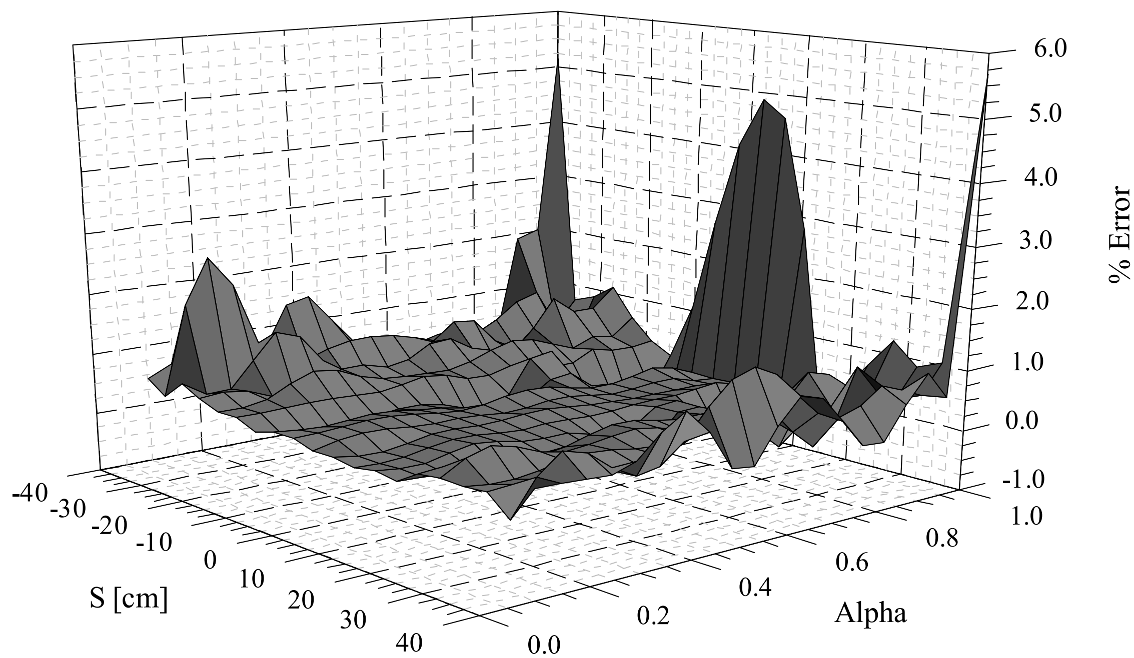

Figure 4. 8 Percent Error between Grover's mutual inductance and MagNet's measured mutual inductance for two coils of unequal lengths.

In figure 4.7 from the s distance perspective the middle of the graph is low, while the elevated portions begin to exist as the separation distance increases. When the separation distance is between -10 cm and 10 cm there is practically no error, approximately .2% or less. The majority of the sizeable error is when s is less than -30 cm or greater than 30 cm. The typical plots for every alpha are seen in figures 4.8, 4.9, 4.10 and 4.11.

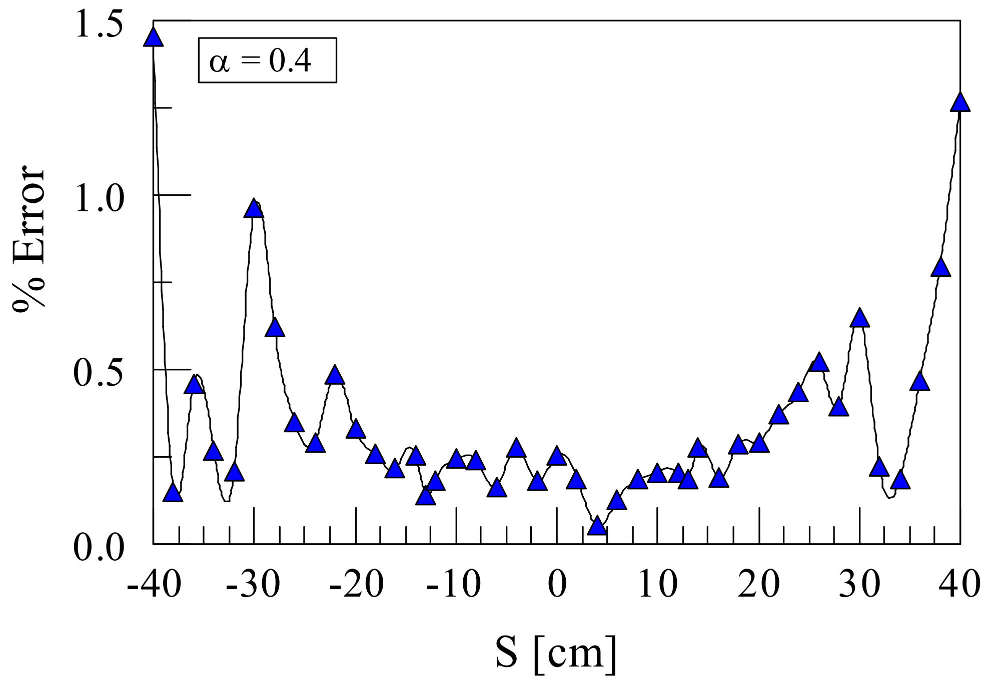

Figure 4. 9 Grover vs. MagNet - Typical plot for alpha = 0.1 – 0.6 and not equal lengths.

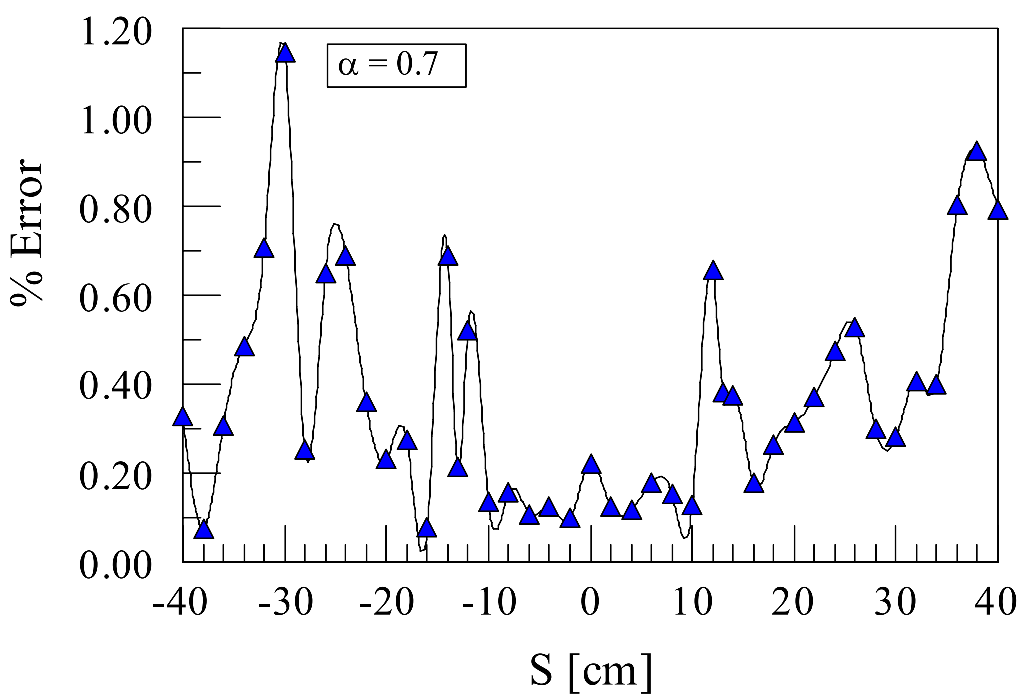

Figure 4. 10 Grover vs. MagNet - Typical plot for alpha = 0.7 – 0.9 and not equal lengths.

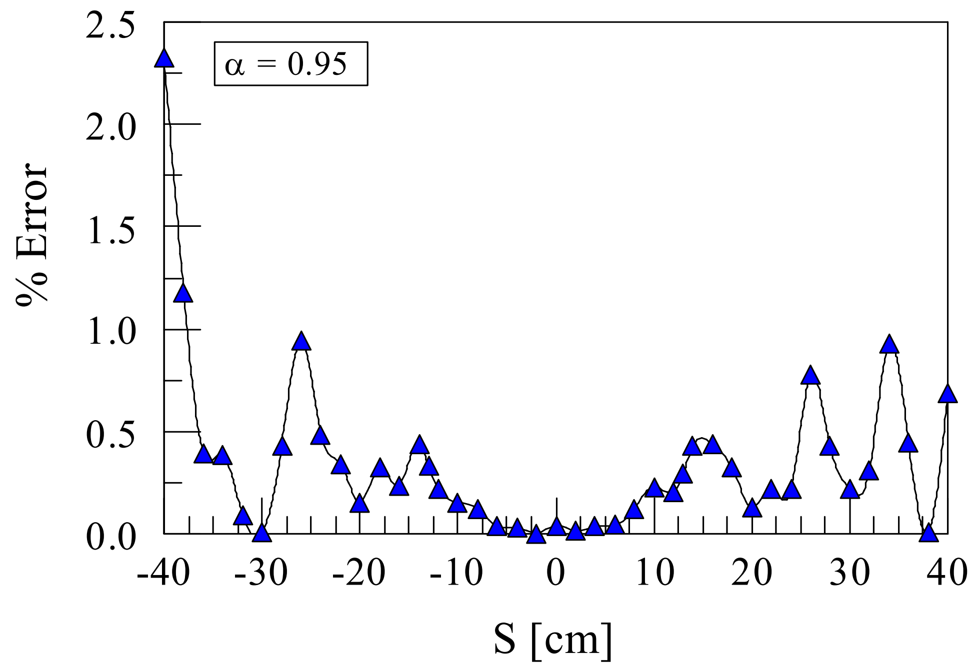

Figure 4. 11 Grover vs. MagNet - Typical plot for alpha = 0.95 – 0.99 and not equal lengths.

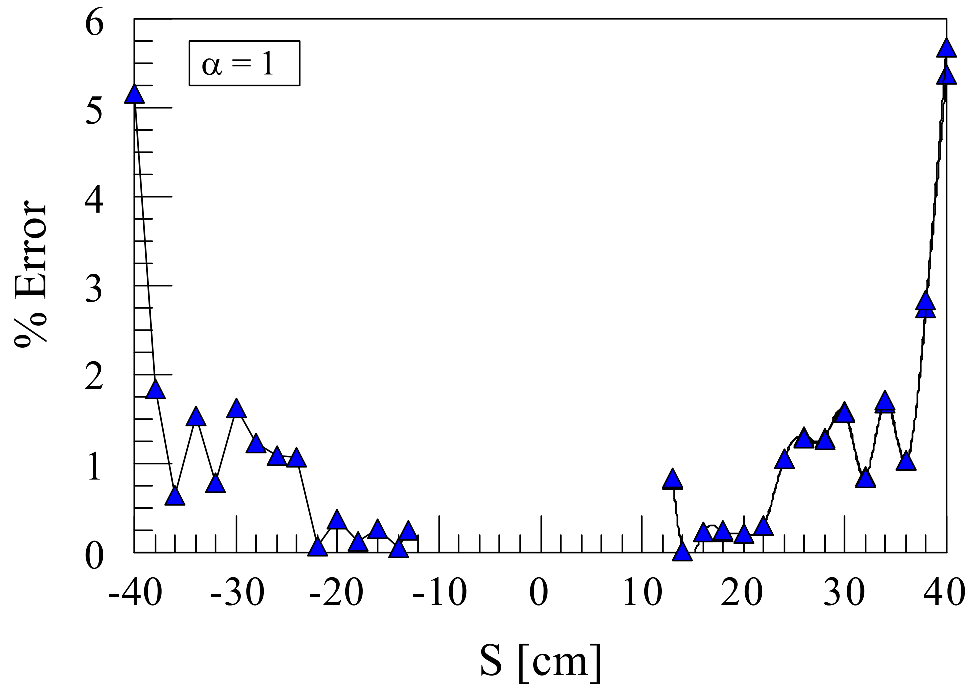

Figure 4. 12 Grover vs. MagNet - Plot for alpha = 1 and not equal lengths.

The maximum error of 5.684% takes place when alpha = 1 at s = 40 cm, figure 4.11. While the minimum error is 0.00171% occurring at s = -2 cm when alpha = 0.95, figure 4.10. When the lengths are not equal, then overall the errors are significantly less than when the lengths of the two coils are equal.

4.4 Grover vs. Elliptical Integrals – Varying Separation Distances for Several α Values

Analysis of Grover's mutual inductance compared to the mutual inductance achieved when solving elliptical integrals was set up as stated above in table 3.1 and evaluated for equal and unequal lengths. The acceptable error amount is again 1% or under. When the lengths are equal the percent error is regularly in the acceptable range for various values of s and α, as shown in figures 4.12, 4.13. In the instance when the lengths are not equal the acceptable error is seldom reached as in figures 4.19.

4.4.1 Grover vs. Elliptical Integrals – Equal Lengths

When alpha is under 0.6, the error for the whole s distance range is in an acceptable range of 1% or under. While alpha is between 0.6 and 0.8 the error is slightly higher with the maximum point approximately 4%. As for when alpha is larger than 0.8 the maximum error increase significantly as alpha approaches 0.99, with the maximum error being roughly 23%. Figure 4.12 illustrates that when alpha is smaller than 0.6 the error is trivial, while it increases considerably when alpha is large, 0.9 or higher.

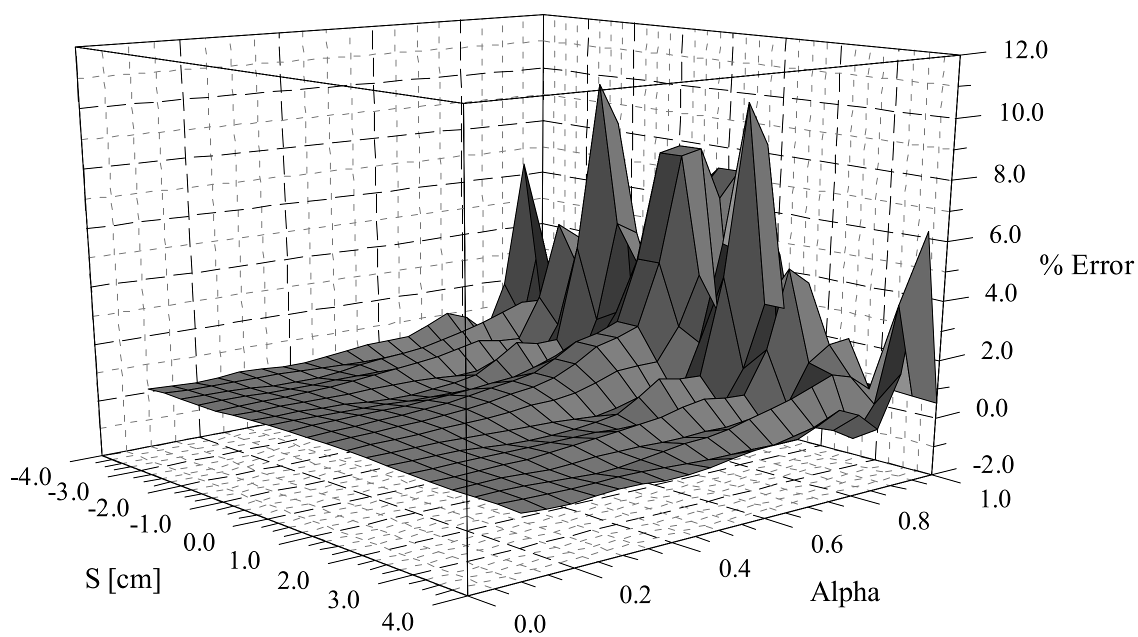

Figure 4. 13 Percent error between Grover's mutual inductance and the mutual inductance from elliptical integral for two coils of equal lengths.

Now examining the range when the error is significant, alpha is .95 or larger, the largest error points are when s is equal to zero. Figure 4.13 reveals the large amount of error around the zero separation distance point. There is also a rise in error between Grover's mutual inductance and the mutual inductance from elliptical integrals when the separation distance becomes large, and the coils are loosely coupled.

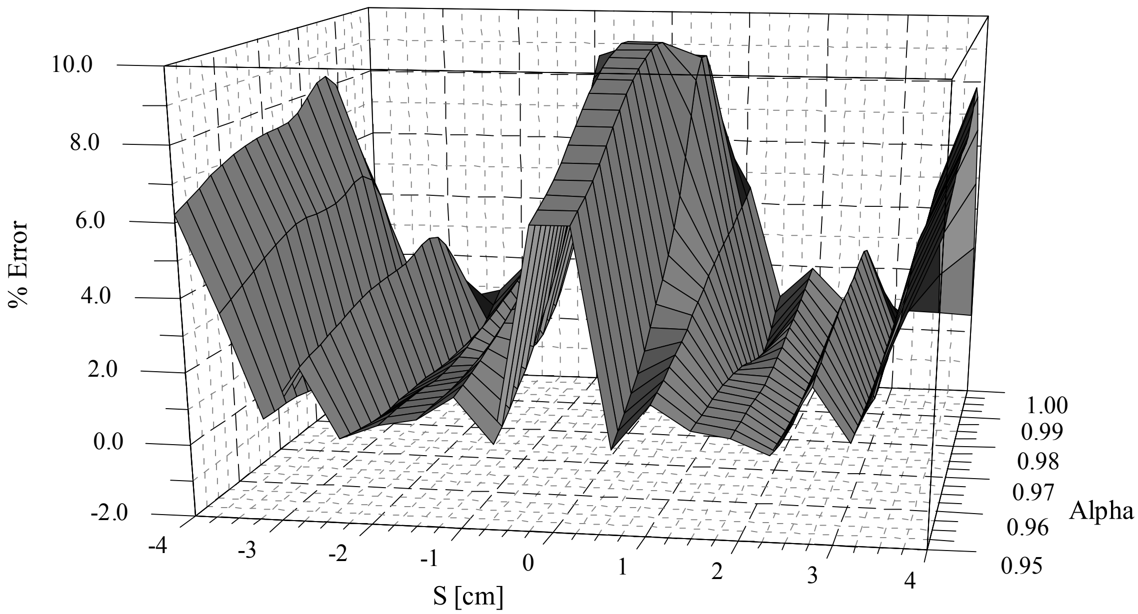

Figure 4. 14 Error occurring when comparing Grover to Elliptical for large values of alpha when the two coils are equal.

A large contrast occurs between comparing Grover to MagNet and Grover to elliptical integrals. When MagNet was compared, there was little error when the coils separation distance was surrounding zero, figure 4.2, while from figure 4.13 it can be seen the highest error when comparing elliptical integrals is surrounding zero. This contrast can also be seen in the 2D cases.

The results were graphed onto a 2D plot for all values of alpha. The individual graphs for each value of α are loosely similar to an exponential sinusoid waveform. The typical graphs (figures 4.14, 4.15, 4.16 and 4.17) for different values of alpha are illustrated to give you an idea about their nature as the separation distances change. As mention earlier the error will increase as alpha increases so for the actual graphs the errors are not the same, only the nature of the waveform.

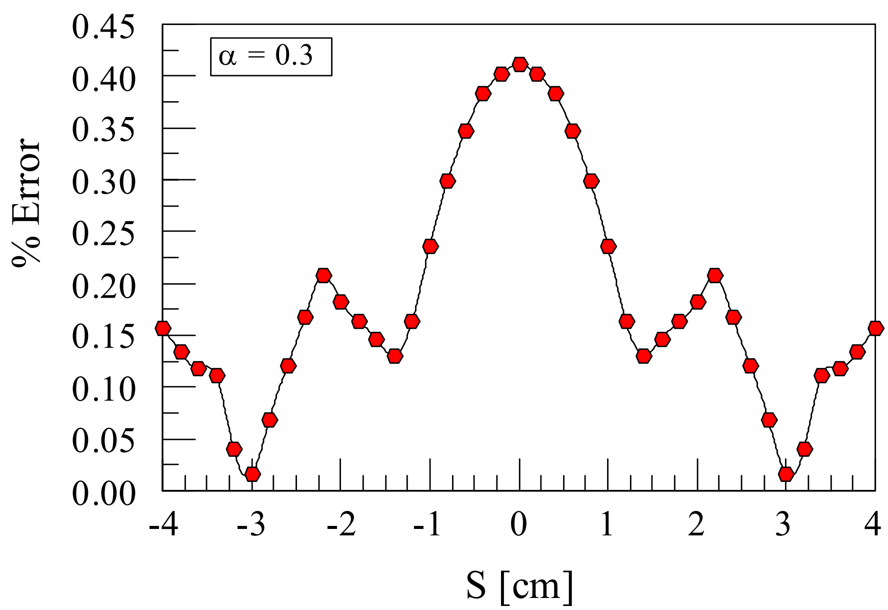

Figure 4. 15 Grover vs. Elliptical - Typical plot for alpha = 0.1 – 0.3 and equal lengths.

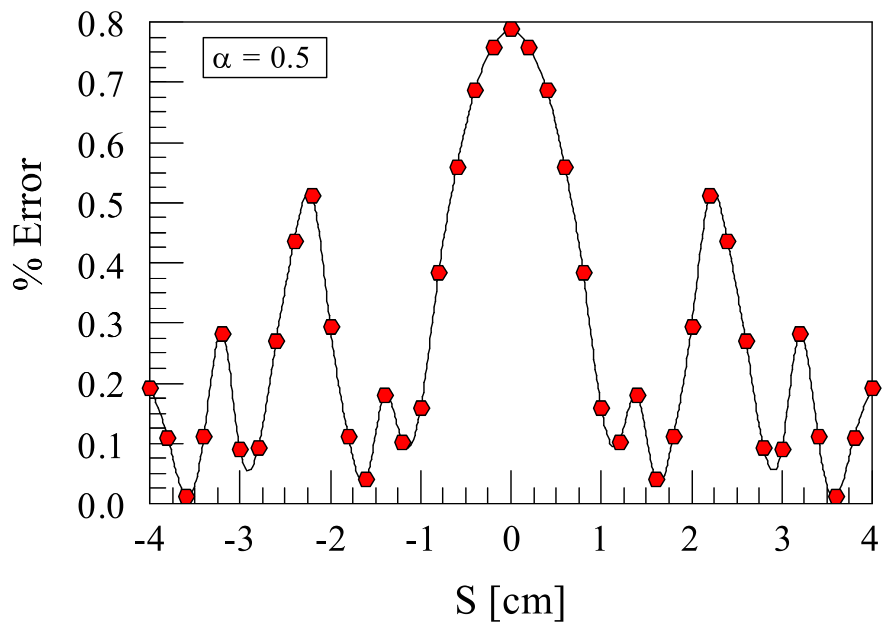

Figure 4. 16 Grover vs. Elliptical - Typical plot for alpha = 0.4 - 0.6 and not equal lengths.

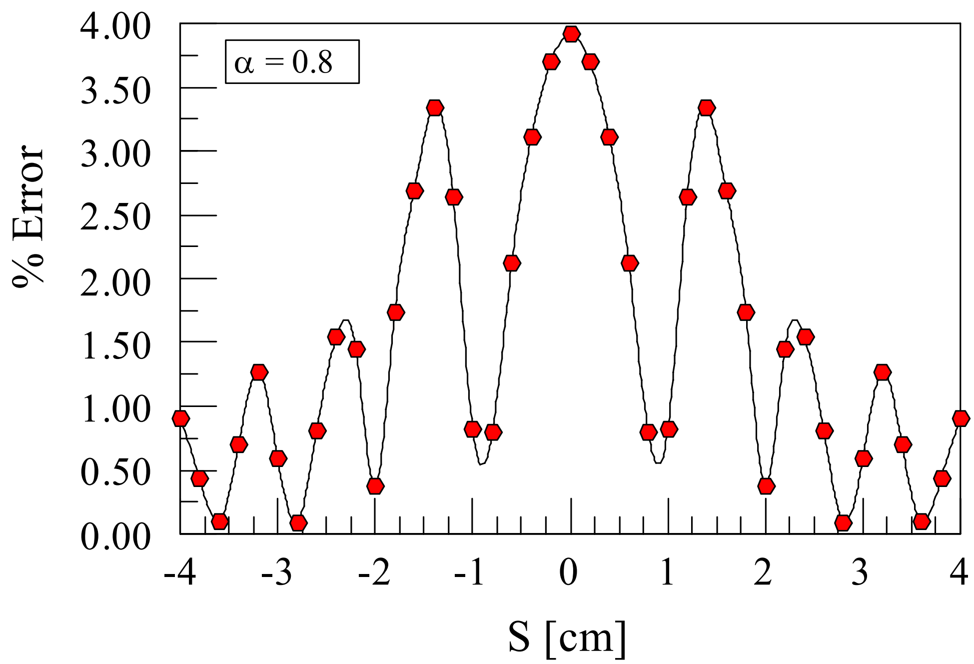

Figure 4. 17 Grover vs. Elliptical - Typical plot for alpha = 0.7 – 0.9 and equal lengths.

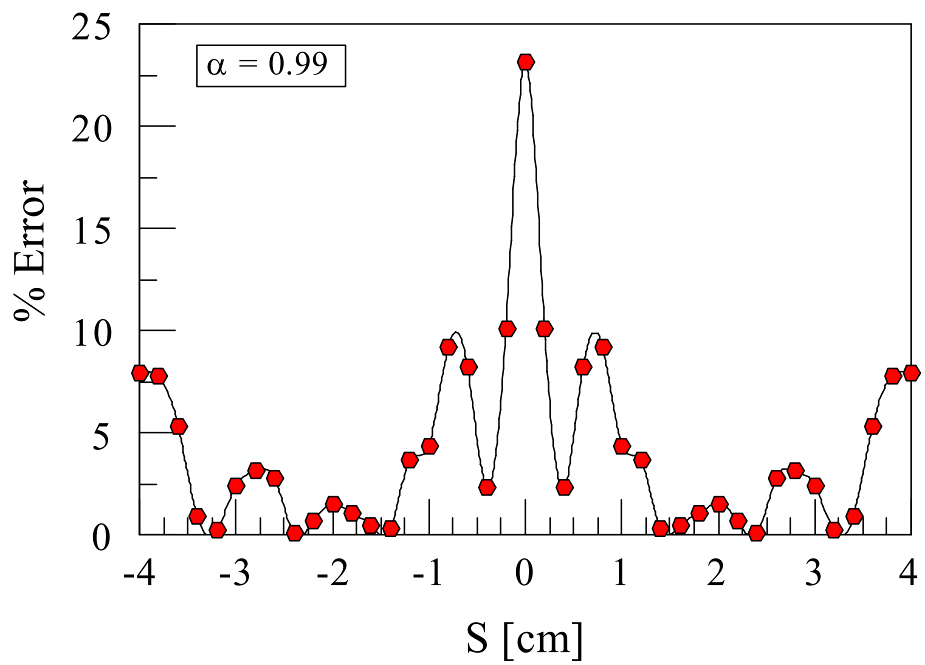

Figure 4. 18 Grover vs. Elliptical - Typical plot for alpha = 0.95-0.99 and equal lengths.

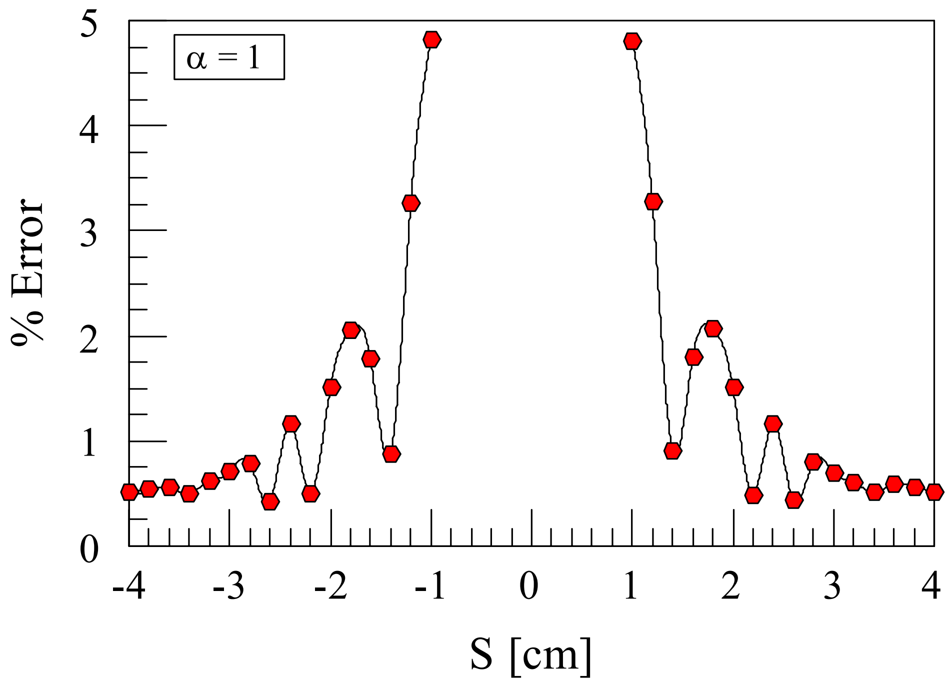

Figure 4. 19 Grover vs. Elliptical - Plot for alpha = 1 and equal lengths.

The maximum error for when the lengths are equal is 23.11% which occurs at s = 0 cm when alpha = 0.99, figure 4.17. The minimum error of 3.64e-06% takes place when s = ±3 cm and alpha = 0.3, figure 4.14.

4.4.2 Grover vs. Elliptical Integrals – Unequal Lengths

When the lengths of the coils are unequal the error is substantially higher. The shape of the error waveforms for varying s and α are similar to exponential sinusoidal waveforms. The error peaks when the separation distance is 0 cm in every case. Figure 4.19 demonstrates the sinusoidal features, with the smallest error points ranging around ±10 cm.

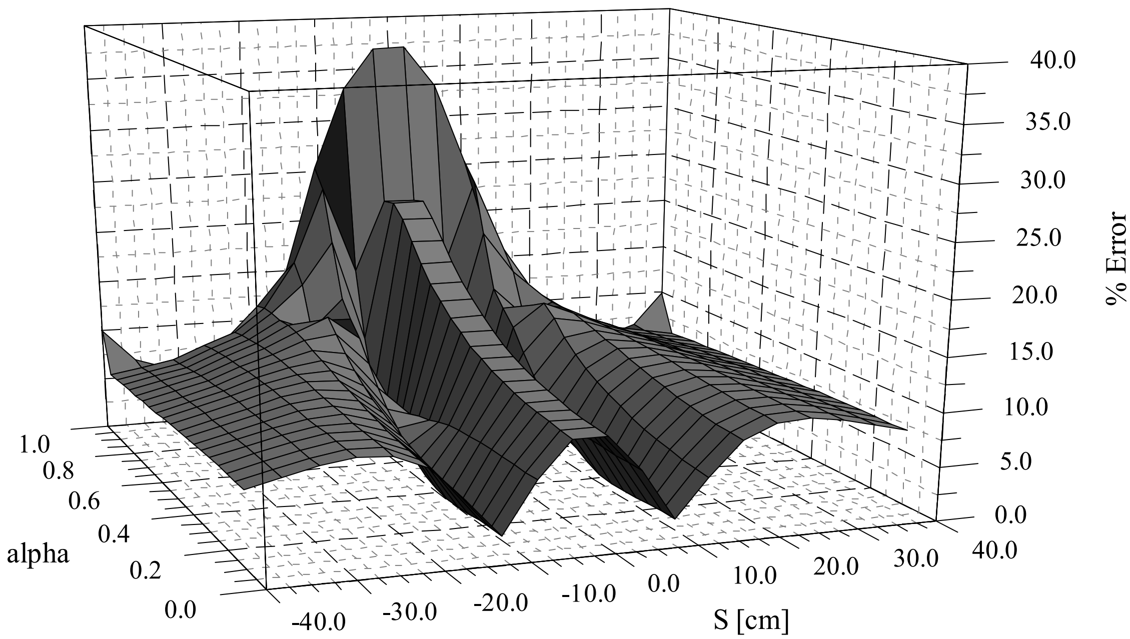

Figure 4. 20 Percent error between Grover's mutual inductance and the mutual inductance from elliptical integral for two coils of non-equal lengths.

The results were graphed onto a 2D plot for all values of alpha. The typical graphs (figures 4.20, 4.21, and 4.22) for different values of alpha are illustrated to give you an idea about their nature as the separation distance change. As the ratio of radii increases the minimum points of the main waveform moves towards s = 0 on the x-axis. As these shift, it creates less points between the maximum and minimum of the wave, creating a steep waveform. This transformation can be seen when observing figures 4.20 and 4.21.

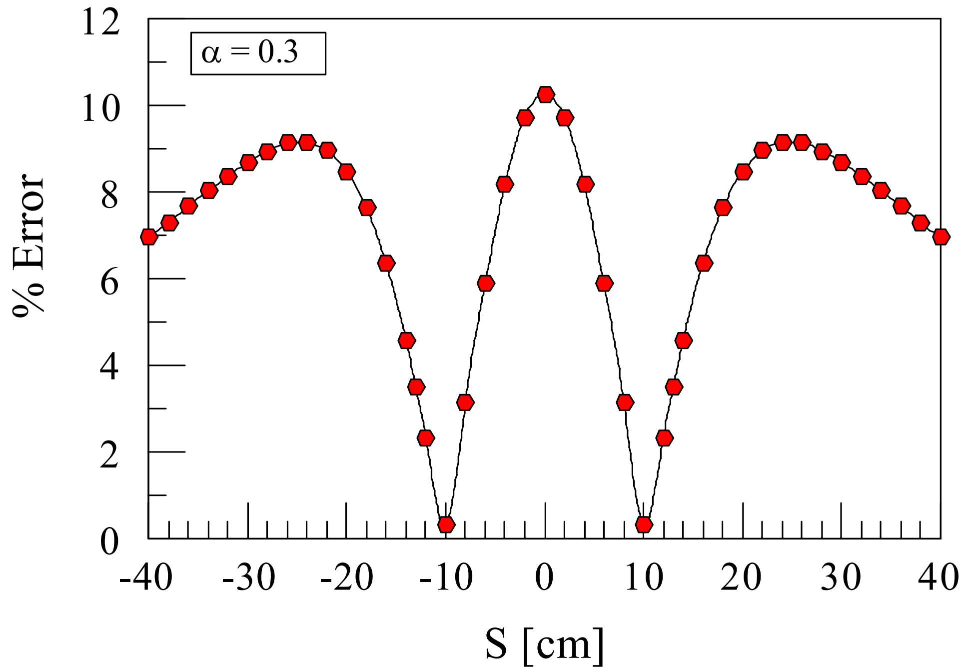

Figure 4. 21 Grover vs. Elliptical - Typical plot for alpha = 0.1 – 0.9 and not equal lengths.

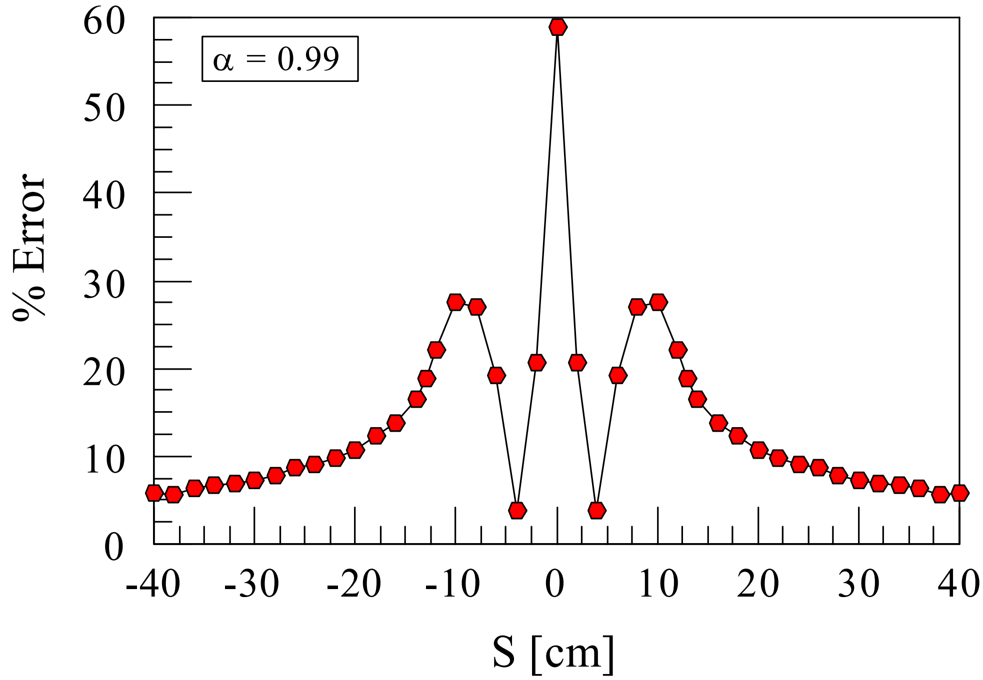

Figure 4. 22 Grover vs. Elliptical - Typical plot for alpha = 0.95 – 0.99 and not equal lengths.

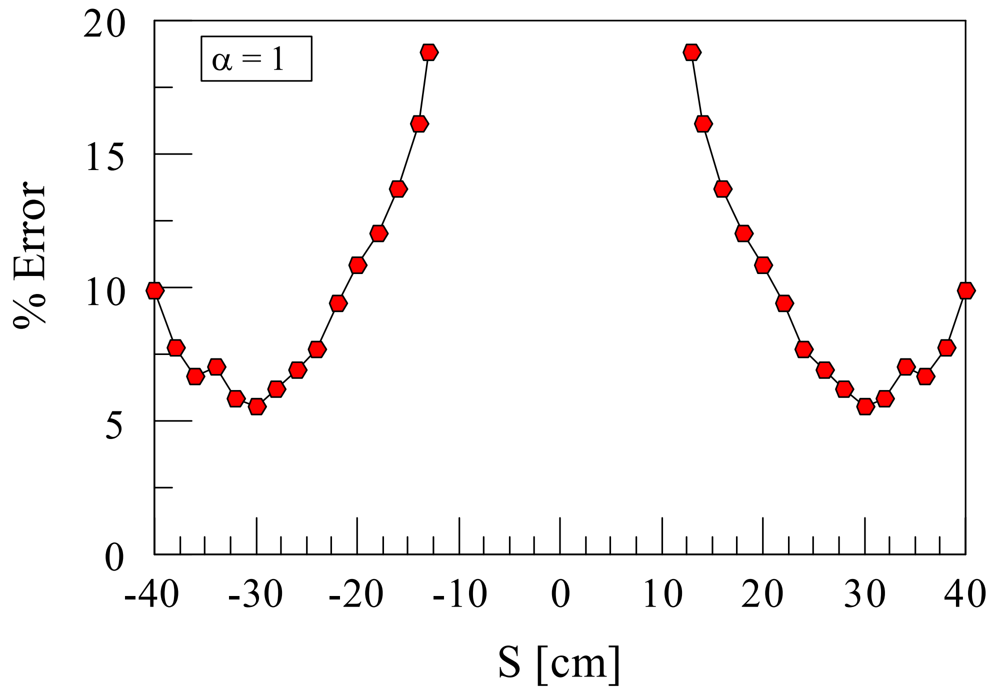

Figure 4. 23 Grover vs. Elliptical - Plot for alpha = 1 and not equal lengths.

The maximum error for when the lengths are not equal is 58.92% which occurs at s = 0 cm when alpha = 0.99, figure 4.21. The minimum error between Grover's mutual inductance and the mutual inductance from elliptical integrals for two coils of equal lengths is 0.319% and takes place when s = ±10 cm and alpha = 0.3, figure 4.20.

Chapter 5 Results Summary

Comparing the examples from Grover's literature with the results given using the MagNet interface the mutual inductances are in good agreement. The largest error is 1.0715% and the lowest is 0.0138%. Table 5.1 re-illustrates that MagNet is consistent with Grover's previous calculations. The errors may be from rounding predicaments. Grover's tables are rounded to 5 significant digits, then the interpolated answer B_n was rounded and lastly in the case of Grover, the mutual inductance, M, was rounded to 7 significant digits. Slight rounding error within MagNet could also exist when solving for data within the program. The mutual inductance found by MagNet was also rounded to 7 significant digits.

Table 5. 1 Error between Grover and MagNet when using Grover's literature examples

MagNet [H]	**Grover [H]**	**% Error**
7.0154E-04	7.0170E-04	0.0224
5.0624E-04	5.0690E-04	0.1303
7.0160E-04	7.0170E-04	0.0138
1.8073E-05	1.8087E-05	0.0791
1.3696E-03	1.3701E-03	0.0349
6.8826E-06	6.9003E-06	0.2568
5.4840E-04	5.4884E-04	0.0798
8.4695E-03	8.4580E-03	0.1355
8.5938E-07	8.5017E-07	1.0715
1.0819E-06	1.0862E-06	0.3963

Grover's literature results compare very different to the mutual inductance of elliptical integrals with the same input parameters. The error ranges from 98.87% to 2.17% as re-illustrated in table 5.2. Grover modified the elliptical integrals to solve for his initial equations and tables. This adjustment can account for the errors when comparing his examples to elliptical integrals.

Table 5. 2 Mutual inductance and percent error between Grover's literature and elliptical integrals.

Elliptical [H]	**Grover [H]**	**% Error**
5.9566E-03	7.0170E-04	88.22
7.0223E-04	5.0690E-04	27.82
5.9566E-03	7.0170E-04	88.22
2.48E-05	1.8087E-05	26.97
4.8517E-03	1.3701E-03	71.76
6.7537E-06	6.9003E-06	2.17
5.34E-04	5.4884E-04	2.77
7.9612E-03	8.4580E-03	6.24
6.8960E-07	8.5017E-07	23.28
5.4619E-07	1.0862E-06	98.87

When Grover was compared to MagNet for varying s distances, varying alphas and equal lengths the error was approximately 1% or below for all s distances when alpha was less than 0.6. As alpha increased the peaks of the waveforms also increased. The peaks also seem to shift towards the outer separation distances as alpha increases. The minimum error of 0.00095% takes place when s = -1.4 cm and alpha = 0.95. The maximum error for when the lengths are equal is 9.799% which occurs at s = 1.4 cm when alpha = 0.9. A noticeable trend arises when alpha is between 0.95 and 0.98. There is virtual no error, 1% or less when the s distances are between -2.4 cm to -1 cm, -0.4 cm to 0.4 cm and 1 cm to 2.4 cm. When alpha = 1 the error is sufficient until s reaches under 2 cm or over -2 cm, it then climbs up to about 3.5% then decreases until they are directly next to each other.

In the situation where Grover was compared to MagNet for varying s distances, varying alphas and different lengths the errors were far less than in the case of equal length coils. The majority of the points are under 1% for all values of alpha, except alpha = 1, with one or two points on each plot as an exception. The data points in the plots that are over 1% are located out where the separation distances become large, below -30 or above 30. When alpha = 1 the error decrease as the s distances proceed towards 0. For alpha = 1 the data points do satisfy the error criteria between -20 and 20 when they exist. The minimum error is 0.00171% found at s = -2 cm when alpha = 0.95. The maximum error of 5.684% takes place when alpha = 1 at s = 40 cm.

Table 5. 3 Results summary of Grover vs MagNet

Grover vs MagNet	**Alpha**	**Acceptable error**
Equal	0.5 and under	all s
	0.6-0.9	when loosely coupled
	0.95-0.99	around s = 0, moderately coupled
	1	loosely coupled, becoming adjacent
Unequal	all	when coils are not loosely coupled

Evaluating Grover's mutual inductance and the mutual inductance given by elliptical integrals to get a percent error by and large produces higher errors than when Grover was compared to MagNet. The maximum error of each waveform exists when the separation distance is zero. For the occurrence of equal lengths the error is acceptable while alpha is under 0.6. The error increases slightly between 0.6 and 0.8 and then begins to ramp up when alpha is 0.95 to a maximum of 23% when alpha is 0.99. The minimum error of 3.64e-06% takes place when s = ±3 cm and alpha = 0.3. The maximum error for when the lengths are equal is 23.11% which occurs at s = 0 cm when alpha = 0.99. When alpha is 1 the error is acceptable at the outer separation distances until about ±2.5 cm, where it starts increases like a exponential sinusoidal function until the coils become directly next to each other.

When the coils have unequal lengths and Grover is compared to Elliptical the waveform peaks are substantially larger in error than all the previously mentioned cases. The error peaks again when the separation distance is zero. The smaller errors range around ±10 cm, and gradually shifts closer to 0 as alpha increases. The minimum error between Grover's mutual inductance and the mutual inductance from elliptical integrals for two coils of equal lengths 0.319% takes place when s = ±10 cm and alpha = 0.3. The maximum error for when the lengths are unequal is 58.92% which occurs at s = 0 cm when alpha = 0.99. When alpha is 1 the percent error never reaches any lower than 5%, hence is never in the acceptable range.

Table 5. 4 Results summary of Grover vs Elliptical

Grover vs Elliptical	**Alpha**	**Acceptable error**
Equal	0.5 and under	all s
	0.6-0.9	moderately coupled through loosely coupled
	0.95-0.99	moderately coupled
	1	loosely coupled, min points when moderately coupled
Unequal	various	minimum points

Chapter 6 Conclusion

A MagNet computer model that predicts measured mutual inductances was used to evaluate the accuracy of Grover's mutual inductance equation. In addition Grover's mutual inductance was weighed against the mutual inductance found using the complete elliptical integrals of the first and second kinds to observe the possible instances where Grover modified his tables away from the complete elliptical integral equation. Two different types of studies were completed. The first compared known Grover data to the solutions given by MagNet and elliptical integrals for the given geometries of two coils. The second calculated the mutual inductance for the three methods while varying the separation distances between the centers of the two coils for varying ratios of the two radii. These mutual inductances were then used to find the percent error between Grover-MagNet, and Grover-Elliptical.

The results show that Grover's claim of accuracy to 4 or 5 significant digits is almost never correct. When comparing Grover's literature to calculated answers, MagNet achieves a maximum of 3 significant digits, while Elliptical's best case is only 1 digit. Higher digit accuracies are reached when using the varying s distances method.

For Grover versus MagNet, when the lengths are equal and the coils are concentric the accuracy is 3 digits, but when any type of separation distance occurs the accuracy drops to 1 or 2 digits. When the lengths are not equal the accuracy typically ranges from 1 to 3 significant digits. However for the unequal case when alpha is between 0.95 and 0.99 and the s distance is ±2 the inductances are found to 4 significant digits.

For Grover versus Elliptical when the lengths are unequal, Grover's claim of 4 or 5 digits is never correct. It only comes close with 3 significant digits at the minimum points on the error waveform at alpha = 0.4. Otherwise the digits range from 0 to 2. When the lengths are equal and alpha is less than 0.6, the mutual inductance is accurate to 2 or 3 digits. This is also the case for higher values of alpha at the minimum points of each waveform. Otherwise the digits range from 0 to 2. Grover's claim is accurate for a number of data points when comparing the mutual inductances of Grover and elliptical integrals for coils of equal lengths. These occurred when alpha = 0.3 and s = ±3 cm, alpha = 0.6 and s = ±3.6 cm, and also when alpha = 0.7 and s = ±3.6 cm.

Although Grover's mutual inductance equation is not as accurate as its claim, it still generally has little significant error. As a whole the errors of Grover compared with MagNet are far less than Grover compared with elliptical integrals. Observing from the literature comparison data MagNet's highest error is only 1.0715% while Elliptical's error is a huge 98.87%. Using the varying distances method the largest error obtained for Grover-MagNet is 9.799% while the largest error obtained for Grover-Elliptical is 58.92%.

Even though the maximum errors for Grover versus Elliptical are quite large, there are various locations where the error is at an acceptable level for this investigation, this error level being 1% or less. This happens specifically when examining two coils for varying alpha and s distances when the coil lengths are equal. When alpha is less than 0.6, the ratio of the radii is one half or less, the error produced from the mutual inductances is always below 1%. As alpha becomes larger, 0.6 to 0.9 and 1, the intensity of the waveform increases, lessening the acceptable data points when the separation distances are small, making the satisfactory range when the coils are loosely coupled. Then alpha increases to between 0.95 and 0.99 the errors are large around zero separation and when the separation is large. The errors are within the limits for these values of alpha when s is -2.4 to -1.4, and 1.4 to 2.4 for equal length coils. For the most part when the coils are unequal lengths and Grover is compared to Elliptical the errors obtained are undesirable. However when the coils are slightly overlapped (around s =10 cm) a minimum point is created and most of the time this point is approximately 1% or less.

Grover's mutual inductance more closely compares with the mutual inductance achieved using the field code software, MagNet, with a maximum error of only 9.799%. When the coil lengths are equal and alpha is less than 0.6 the error is almost exclusively in the acceptable error range. When alpha is between 0.6-0.9 the acceptable areas are when the coils are loosely coupled and around zero separation. Many of the acceptable data points are eliminated because the error around the zero separation point increases as alpha increases towards 0.9. At alpha = 0.95 the waveform complete changes from low error when the separation distance is large, higher errors when s = 0 and large errors in between to low errors when the separation is small and larger errors as the coils move apart from each other.

For coils of unequal lengths, when comparing Grover to MagNet, the larger errors, above 1%, always occur when the coils are loosely separated with the majority of the error existing when s is less than -30 cm or greater than 30 cm. Between -10 cm and 10 cm there is practically no error for every alpha except alpha =1. This trivial error is approximately 0.2% or less. The error is typically under 1% for every alpha between -36 to 36.

Grover states that his tables are less accurate when the coils are loosely coupled [6], this is supported when Grover is compared to MagNet for when the lengths are both equal and not equal, although when the lengths are equal it is only true for alpha between 0.95 and 0.99. This loss in accuracy at the greater separation distances occurs because the four B_n values are close to each other and nearly cancel each other out. Errors are not higher for loosely coupled coils when Grover is compared to elliptical integrals. The considerable error points are located around s = 0 while the points of less error are when the separation distances are large. A possible reason for this is because Grover altered the elliptical integrals by using series expansions to achieve improved accuracies. Other errors could include rounding errors of the computer software programs and Grover's rounding in his tables.

Summing up the Grover-Elliptical investigation, the Grover and Elliptical methods only compare well when the lengths are equal and the ratio of the radii is half or less when comparing varying separation distances. When comparing elliptical integrals to Grover's literature the acceptable error is never achieved, although when the coil lengths are equal the errors are around 2%. The modifications Grover made to the elliptical integral equation in his tables can account for a number of the errors obtained. The error waveforms are similar to exponential sinusoidal waveforms with the maximum errors located when the separation distance is zero. The error then decreases as the separation distances increase in either the positive or negative direction. The percent error is less in the case when the lengths of the coils are equal. For both cases the error is less when alpha is smaller.

In summary of the Grover-MagNet investigation the evaluation using Grover's literature provided inaccuracies of roughly 1% or less. This illustrates that Grover and MagNet compare well using known data. For the investigation of varying s distances both equal and unequal lengths achieved many situations where the error is an acceptable level. Coils of equal lengths accomplish the goal when alpha is less than 0.6, while coils of different lengths reach the objective for all values of alpha, except alpha =1, with some data anomalies occurring. When alpha = 1 for differing lengths the loosely coupled points have errors above 1%. The higher errors occur when the coils are not equal lengths and loosely coupled for all alpha and also when alpha is between 0.95 and 0.99 for equal coil lengths that are loosely coupled. Otherwise when the coils lengths are equal and alpha is between 0.6 and 0.9 the higher error are generally located between -2 to -1 and 1 to 2. The results reveal that the accuracy of Grover's tables decreases as the separation increases and also as alpha increases.

Grover compares best when compared to MagNet, a field code software, for unequal length coils. When the coils are overlapped or inside one another the percent error is around 0.2% or less. The error does increase as alpha increase when the coils are loosely coupled, less than -30 cm or greater than 30 cm, but the peak error was only 5.684% which in some experiments can be an acceptable amount of error.

References

[1] T. G. Engel, W. C. Nunnally, and J. M. Neri, "Research Progress in the Development of a Medium-Bore High-Efficiency Helical Coil Electromagnetic Launcher," 12th Symposium on Electromagnetic Launch Technology, pp. 49-52, 2005.

[2] T. G. Engel, W. C. Nunnally, and J. M. Neri, "Development of a Medium-Bore High-Efficiency Helical Coil Electromagnetic Launcher," IEEE Transactions on Plasma Science, vol. 32. no. 5, pp. 1893-1895, 2004.

[3] T. G. Engel, D. Surls, and W. C. Nunnally, "Prediction and Verification of Electromagnetic Forces in Helical Coil Launchers," IEEE Transactions on Magnetics, vol. 39, no. 1, pp. 112-115, 2003.

[4] F. W. Grover, "Tables for Calculation of the Mutual Inductance of Any Two Coaxial Single Layer Coils", Proc., I.R.E., Vol. 21, No. 7, 1933, pp. 1039-1049.

[5] M. L. Boas, "*Mathematical Methods in the Physical Sciences*," John Wiley & Sons, New York 1983. pp. 474-479.

[6] F. Grover, "*Inductance Calculations; Working Formulas and Table,*" NY: D. Van Nostrand Publishing, 1946.

[7] T. H. Fawzi and P.E. Burke, "The Accurate Computation of Self and Mutual Inductances of Circular Coils," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-97, No. 2, pp. 464-468, 1978.

[8] T. H. Fawzi and P.E. Burke, "The Accurate Computation of Forces Between Circular Coils," IEEE Transactions on Magnetics, vol. 15, no. 6, pp. 1491-1493, 1979.

[9] MagNet, version. 6.20. Infolytica Corporation, P.O. Box 1144 Station Place du Parc, Montreal, Quebec H2W 2P4 Canada.

[10] MatLab, version 7 release 14, The MathWorks, Inc., 3 Apple Hill Drive Natick, MA 01760-2098.

[11] E. B. Rosa and F. W. Grover, "Formulas and Tables for the Calculation of Mutual and Self Inductances", Bulletin of Bureau of Standards, vol. 8, no. 1, 1911.

[12] J. D. Jackson, "Classical Electrodynamics" John Wiley & Sons, Inc., New York 1999 pp 180-218

APPENDIX A MAGNET VARIABLES AND CODE

A.1 MAGNET model variables

Variable	Description	Units
A1	Radius of large coil	centimeters
A2	Radius of small coil	centimeters
Energy	Stored magnetic energy	Joules
force1_x	Force on large coil from the x direction	Newton
force1_y	Force on large coil from the y direction	Newton
force1_z	Force on large coil from the z direction	Newton
force2_x	Force on small coil from the x direction	Newton
force2_y	Force on small coil from the y direction	Newton
force2_z	Force on small coil from the z direction	Newton
LCS	Large coil spacing	centimeters
LCT	Large coil current	Amperes
LFlux	Large coil flux linkage	Webers
LMInductance	Large coil mutual inductance	Henry
LSInductance	Large coil self inductance	Henry
LTurns	Large coil turns	Turns
M1	Length of the small coil	centimeters
M2	Length of the large coil	centimeters
SCS	Small coil spacing	centimeters
SCT	Small coil current	Amperes
SFlux	Small coil flux linkage	Webers
SMInductance	Small coil mutual inductance	Henry
Sp	Spacing between the middle of the 2 coils	centimeters
SSInductance	Small coil self inductance	Henry
STurns	Small coil turns	Turns
WD1	Small coil winding density	Turns/centimeter
WD2	Large coil winding density	Turns/centimeter
WT1	Small coil wire thickness	centimeters
WT2	Large coil wire thickness	centimeters

A.2 MagnetInterface.frm file

```
'Function gets coils parameters, creates geometry for the 2 coils
'solves for mutual inductance, self inductance, force of each coil
'outputs solutions

Sub GoButton_Click

'initializes variables
Call newDocument()
DIM A2, A1, WD1, WD2
DIM M2, M1, Sp, p
DIM WT1, WT2, n
DIM Counter, m, even
DIM LWidth, SWidth, Line4
DIM Line1, Line2, Line3
DIM NumberofComponents1
DIM i, NumOfComp1, d, NumOfComp2
DIM NumberofComponents2, Energy
DIM SInductance, LInductance

'checks inputs for numerical data
      If(IsNumeric(S.text) And IsNumeric(LargeR.text) And
IsNumeric(LargeDia.text) _
        And IsNumeric(SmallR.text) And IsNumeric(SmallDia.text) And
IsNumeric(SmallTD.text) _
        And IsNumeric(LargeTD.text) And IsNumeric(SmallWT.text) And
IsNumeric(LargeWT.text) _
        And IsNumeric(LSpacing.text) And IsNumeric(SSpacing.text) And
IsNumeric(LCurrent.text) _
        And ISNumeric(SCurrent.text)) Then

Sp = CDbl(S.text)           'Spacing between the middle of the 2 coils
A2 = CDbl(LargeR.text)      'Radius of the large coil
M2 = CDbl(LargeDia.text)    'Diameter of the large coil
A1 = CDbl(SmallR.text)      'Radius of the small coil
M1 = CDbl(SmallDia.text)    'Diameter of the small coil
WD1 = CDbl(SmallTD.text)    'Small coil winding density
WD2 = CDbl(LargeTD.text)    'Large coil winding density
WT1 = CDbl(SmallWT.text)    'Small coil wire thickness
WT2 = CDbl(LargeWT.text)    'Large coil wire thickness
LCS = CDbl(LSpacing.text)   'Large coil spacing
SCS = CDbl(SSpacing.text)   'Small coil spacing
LCT = CDbl(LCurrent.text)   'Large coil current
SCT = CDbl(SCurrent.text)   'Small coil current

LWidth = (M2/2)-Sp/2
SWidth = Sp/2-M1/2

    Else
      MsgBox("A non-number has been detected, Please enter numbers in the
appropriate boxes")
    Exit Sub
    End If

Call getDocument().beginUndoGroup("Set Default Units", true)
Call getDocument().setDefaultLengthUnit("Centimeters")
Call getDocument().endUndoGroup()

'Checks to make sure coil exists
If (A2 or M2) <> 0 then

'Creates the large coil with no separation between turns
```

```
IF LCS = 0 THEN

'Large Coil, Top Coil Part
Call getDocument().getView().newLine((A2-WT2/2), (-Sp/2 + M2/2),
(A2+ WT2/2), (-Sp/2 + (M2/2)))
Call getDocument().getView().newLine((A2+ WT2/2), (-Sp/2 + M2/2),
(A2 + WT2/2), (-Sp/2 - M2/2))
Call getDocument().getView().newLine((A2 + WT2/2), (-Sp/2 - M2/2),
(A2 - WT2/2), (-Sp/2 - (M2/2)))
Call getDocument().getView().newLine((A2 - WT2/2), (-Sp/2 - M2/2),
(A2 - WT2/2), (-Sp/2+(M2/2)))

Call getDocument().getView().selectAt(A2, -Sp/2, infoToggleInSelection,
Array(infoSliceSurface))

REDIM ArrayOfValues(0)
ArrayOfValues(0)= "Component #1"
Call getDocument().getView().makeComponentInAnArc(0, 0, 0, -1, 90,
ArrayOfValues, "Name=Copper: 5.77e7 Siemens/meter", True)

'MAKING THE COMPONENT A COIL

Call getDocument().getView().selectObject("Component #1", infoSetSelection)
REDIM ArrayOfValues(0)
ArrayOfValues(0)= "Component #1"
Call getDocument().makeSimpleCoil(1, ArrayOfValues)

ELSE
' Creates the large coil when separation exists between turns
even = 0
Counter = 0
n=0
m=0
Call getDocument().getView().newLine((A2-WT2/2), (-Sp/2-M2/2), (A2+WT2/2),
(-Sp/2-M2/2))

'HORIZONTAL LINES - to create boxes for components
For Counter = 1 to 20
m = m+1
n = n+1
Line1 = ((-Sp/2)-(M2/2)+(n*WT2)+(m*LCS))
If (Line1 > LWidth) then
    Exit For
end if
Call getDocument().getView().newLine((A2-WT2/2), ((-Sp/2)-
(M2/2)+(n*WT2)+(m*LCS)), (A2+WT2/2), ((-Sp/2)-(M2/2)+(n*WT2)+(m*LCS)))
Next

n=1
m=0

For even = 1 to 20
Line2 = ((-Sp/2)-(M2/2)+(n*WT2)+(m*LCS))
If (Line2 > LWidth) then
    Exit For
end if
Call getDocument().getView().newLine((A2-WT2/2), ((-Sp/2)-
(M2/2)+(n*WT2)+(m*LCS)), (A2+WT2/2) ,((-Sp/2)-(M2/2)+(n*WT2)+(m*LCS)))
n = n+1
m = m+1
Next

'VERTICAL LINES - to create boxes for components
even = 0
```

```
Counter = 0
n=0
m=0
p=1
For Counter = 1 to 20
Line3 = ((-Sp/2)-(M2/2)+(p*WT2)+(m*LCS))
If (Line3 > LWidth) then
    Exit For
end if
Call getDocument().getView().newLine((A2-WT2/2), ((-Sp/2)-
(M2/2)+(n*WT2)+(m*LCS)), (A2-WT2/2), ((-Sp/2)-(M2/2)+(p*WT2)+(m*LCS)))
n = n+1
m = m+1
p = p+1
Next

n=0
m=0
p=1
For even = 1 to 20
Line4 = ((-Sp/2)-(M2/2)+(p*WT2)+(m*LCS))
If (Line4 > LWidth) then
    Exit For
end if
Call getDocument().getView().newLine((A2+WT2/2), ((-Sp/2)-
(M2/2)+(n*WT2)+(m*LCS)), (A2+WT2/2), ((-Sp/2)-(M2/2)+(p*WT2)+(m*LCS)))
n = n+1
m = m+1
p = p+1
Next


Counter = 0
n = 0
m = 0
NumOfComp1 = 0

For Counter = 1 to 20
Line2 = ((-Sp/2)-(M2/2)+(n*WT2)+(m*LCS))
If (Line2 > LWidth) then
    Exit For
end if
Call getDocument().getView().selectAt(A2, ((-Sp/2)-
(M2/2)+(WT2/1000)+(n*WT2)+(m*LCS)), infoAddToSelection,
Array(infoSliceSurface))
NumOfComp1 = NumOfComp1+1
n = n+1
m = m+1
Next

If NumOfComp1 Mod 2 = 0  then
NumberOfComponents1 = (NumOfComp1)
else
NumberOfComponents1 = (NumOfComp1-1)
end if


REDIM LArrayOfValues(NumberOfComponents1)
For i = 0 to NumberOfComponents1
    d = i+1
    LArrayOfValues(i)= "Component #" & d
Next
```

```
Call getDocument().getView().makeComponentInAnArc(0, 0, 0, -1, 90,
LArrayOfValues, "Name=Copper: 5.77e7 Siemens/meter", True)


END IF

END IF



'CREATING SMALL COIL
' checks to make sure smaller coils exists
IF (A1 or M1) <> 0 Then

' makes smaller coil if there is no separation between turns
IF SCS = 0 THEN
    'Small Coil, Top Coil Part
    Call getDocument().getView().newLine((A1 - WT1/2), (Sp/2 + M1/2),
(A1+WT1/2), (Sp/2+M1/2))
    Call getDocument().getView().newLine((A1+WT1/2), (Sp/2+M1/2),
(A1+WT1/2), (Sp/2-M1/2))
    Call getDocument().getView().newLine((A1+WT1/2), (Sp/2-M1/2),
(A1-WT1/2), (Sp/2-M1/2))
    Call getDocument().getView().newLine((A1-WT1/2), (Sp/2-M1/2),
(A1 - WT1/2), (Sp/2 + M1/2))

Call getDocument().getView().selectAt(A1, Sp/2, infoToggleInSelection,
Array(infoSliceSurface))


REDIM ArrayOfValues(0)
ArrayOfValues(0)= "Component #2"
Call getDocument().getView().makeComponentInAnArc(0, 0, 0, -1, 90,
ArrayOfValues, "Name=Copper: 5.77e7 Siemens/meter", True)


'MAKING THE COMPONENT A COIL

Call getDocument().getView().selectObject("Component #2", infoSetSelection)
REDIM ArrayOfValues(0)
ArrayOfValues(0)= "Component #2"
Call getDocument().makeSimpleCoil(1, ArrayOfValues)

' makes smaller coil if there is separation between turns
ELSE
'HORIZONTAL LINES - to create boxes for components
even = 0
Counter = 0
n=0
m=0
Call getDocument().getView().newLine((A1-WT1/2), (Sp/2+M1/2), (A1+WT1/2),
(Sp/2+M1/2))


For Counter = 1 to 20
m = m+1
n = n+1
Line1 = ((Sp/2)+(M1/2)-(n*WT1)-(m*SCS))
If (Line1 < SWidth) then
    Exit For
end if
Call getDocument().getView().newLine((A1-WT1/2), ((Sp/2)+(M1/2)-(n*WT1)-
(m*SCS)), (A1+WT1/2), ((Sp/2)+(M1/2)-(n*WT1)-(m*SCS)))
Next
```

```
n=1
m=0

For even = 1 to 20
Line2 = ((Sp/2)+(M1/2)-(n*WT1)-(m*SCS))
If (Line2 < SWidth) then
    Exit For
end if
Call getDocument().getView().newLine((A1-WT1/2), ((Sp/2)+(M1/2)-(n*WT1)-
(m*SCS)), (A1+WT1/2) ,((Sp/2)+(M1/2)-(n*WT1)-(m*SCS)))
n = n+1
m = m+1
Next

'VERTICAL LINES - to create boxes for components
even = 0
Counter = 0
n=0
m=0
p=1
For Counter = 1 to 20
Line3 = ((Sp/2)+(M1/2)-(p*WT1)-(m*SCS))
If (Line3 < SWidth) then
    Exit For
end if
Call getDocument().getView().newLine((A1-WT1/2), ((Sp/2)+(M1/2)-(n*WT1)-
(m*SCS)), (A1-WT1/2), ((Sp/2)+(M1/2)-(p*WT1)-(m*SCS)))
n = n+1
m = m+1
p = p+1
Next

n=0
m=0
p=1
For even = 1 to 20
Line4 = ((Sp/2)+(M1/2)-(p*WT1)-(m*SCS))
If (Line4 < SWidth) then
    Exit For
end if
Call getDocument().getView().newLine((A1+WT1/2), ((Sp/2)+(M1/2)-(n*WT1)-
(m*SCS)), (A1+WT1/2), ((Sp/2)+(M1/2)-(p*WT1)-(m*SCS)))
n = n+1
m = m+1
p = p+1
Next

Counter = 0
n = 0
m = 0
d = 0
NumOfComp2 = 0

For Counter = 1 to 20
Line1 = ((Sp/2)+(M1/2)-(n*WT1)-(m*SCS))
If (Line1 < SWidth) then
    Exit For
end if
Call getDocument().getView().selectAt(A1, ((Sp/2)+(M1/2)-(WT1/1000)-
(n*WT1)-(m*SCS)), infoAddToSelection, Array(infoSliceSurface))
NumOfComp2 = NumOfComp2+1
n = n+1
```

```
m = m+1
Next

If NumOfComp2 Mod 2 = 0  then
NumberOfComponents2 = (NumOfComp2 - 1)
else
NumberOfComponents2 = (NumOfComp2)
end if



REDIM SArrayOfValues(NumberOfComponents2)
For i = 0 to (NumberOfComponents2)
    d = i+1
    SArrayOfValues(i)= "Component #2" & d
Next

Call getDocument().getView().makeComponentInAnArc(0, 0, 0, -1, 90,
SArrayOfValues, "Name=Copper: 5.77e7 Siemens/meter", True)


END IF

END IF


'CREATING THE AIR BOX

    Call getDocument().getView().newLine(0, (150*M2), 0, (-150*M2))
    Call getDocument().getView().newLine( 0, (-150*M2), 75*A2,
(-150*M2))
    Call getDocument().getView().newLine(75*A2, (-150*M2), 75*A2, (150*M2))
    Call getDocument().getView().newLine(75*A2, (150*M2), 0, (150*M2))

    Call getDocument().getView().selectAt((8*A2)-1, ((8*M2)/2)-1,
infoToggleInSelection, Array(infoSliceSurface))


REDIM ArrayOfValues(0)
ArrayOfValues(0)= "Component #40"

    Call getDocument().getView().makeComponentInAnArc(0, 0, 0, -1, 90,
ArrayOfValues, "Name=AIR", True)


'creates large coil if separation between turns
If LCS <> 0 then

    If NumOfComp1 Mod 2 = 0  then
        NumberOfComponents1 = NumOfComp1
    else
        NumberOfComponents1 = (NumOfComp1-1)
    end if

    REDIM LCArrayOfValues(NumberOfComponents1)
    For i = 1 to NumberOfComponents1
        d = i
        Call getDocument().getView().selectObject(("Component #" )& d &
(",Face#1"), infoSetSelection)
        LCArrayOfValues(i) = (("Component #") & d &(",Face#1"))
    Next
    Call getDocument().makeSimpleCoil(1, LCArrayOfValues)
```

```
end if

'creates small coil if separation between turns
If SCS <> 0 then

    If NumOfComp2 Mod 2 = 0  then
        NumberOfComponents2 = NumOfComp2-2
    else
        NumberOfComponents2 = NumOfComp2-1
    end if

    REDIM SCArrayOfValues(NumberOfComponents2)
    For i = 0 to NumberOfComponents2
        d = i+1
        Call getDocument().getView().selectObject(("Component #2" )& d &
(",Face#1"), infoSetSelection)
        SCArrayOfValues(i) = (("Component #2") & d &(",Face#1"))
    Next

        Call getDocument().makeSimpleCoil(1, SCArrayOfValues)

End if


    TurnsL.text = (M2*WD2)
    TurnsS.text = (M1*WD1)

Call getDocument().setCoilNumberOfTurns("Coil#1", TurnsL)           'sets
turns for large coil
Call getDocument().setCoilCurrent("Coil#1", LCT, 0)                 'sets
current for large coil
Call getDocument().setCoilNumberOfTurns("Coil#2", TurnsS)           'sets
turns for small coil
Call getDocument().setCoilCurrent("Coil#2", SCT, 0)                 'sets
current for small coil
Call getDocument().beginUndoGroup("Set Solver Options", true)
Call getDocument().setPolynomialOrder("", 2)                        'set
polynomial order to 2
Call getDocument().endUndoGroup()
Call getDocument().beginUndoGroup("Set Adaption Options", true)     'sets
adaption tolerance .01%
Call getDocument().useHAdaption(True)                               'set
refinement 25%
Call getDocument().setAdaptionTolerance(0.0001)
Call getDocument().setHAdaptionRefinement(0.25)
Call getDocument().endUndoGroup()
    Title.text = clear

'Solves coils
Call getDocument().solveMagnetostatic2D()
'initialize variables
   Dim Total, SolutionType, Current
   Dim Magnitude, Phase
'  DIM SInductance, LInductance, Energy
   Dim SFlux, LFlux, LMag, LPhase
   Dim SMag, SPhase, LMInductance, SSInductance
   Dim SMInductance, LSInductance, force1_x, force1_y
   Dim force1_z, force2_x, force2_y, force2_z
   TotalPIDs = GetDocument.GetNumberofProblems()
   TotalC = GetDocument.GetNumberofCoils()


    Redim SolID(1)
On Error Resume Next
```

```
'gets current for each coil
    CALL getDocument.getParameter("Coil#1", "Current", LCT)
    CALL getDocument.getParameter("Coil#2", "Current", SCT)


 'check to see if problem has been solved
 for i = 1 to TotalPIDs
     if (getDocument.getSolution.isProblemSolved(i)) Then

' gets flux linkage through each coil
CALL getDocument().getSolution().getFluxLinkageThroughCoil(i,"Coil#1",
LFlux, LPhase)

CALL getDocument().getSolution().getFluxLinkageThroughCoil(i,"Coil#2",
SFlux, SPhase)

' gets force on each coil
CALL getDocument().getSolution().getForceOnBody(1, "Component #1",
force1_x, force1_y, force1_z)

CALL getDocument().getSolution().getForceOnBody(1, "Component #2",
force2_x, force2_y, force2_z)

'Solves for energy
Energy = GetDocument.getSolution.getStoredEnergy(i)

'if large current is zero solve for the mutual inductance and the self
inductance of the small coil.
         If LCT = 0 Then
               LMInductance = LFlux/(SCT)
               SSInductance = (SFlux)/ (SCT)


'outputs title of output table
Title.Text = Title.Text & Chr(9) & "x" & Chr(9) & Chr(9) & "M(x)" & Chr(9)
& Chr(9) & Chr(9) _
& "Small L(x)" & Chr(9) & Chr(9) & Chr(9) & "Large Force(y)" & Chr(9) &
Chr(9) & Chr(9) & "Small Force(y)"

'outputs answers in table
Output.Text = Output.Text & Chr(9) & Sp & Chr(9) & LMInductance & Chr(9) &
SSInductance _
& Chr(9) & Chr(9) & force1_y & Chr(9) & Chr(9) & Chr(9) & force2_y &
Chr(10)

'if small current is zero solve for the mutual inductance and the self
inductance of the large coil.
        Elseif SCT = 0 then
               LSInductance = (LFlux) / (LCT)
               SMInductance = (SFlux)/ (LCT)

'outputs title of output table
Title.Text = Title.Text & Chr(9) & "x" & Chr(9) & Chr(9) & "M(x)" & Chr(9)
& Chr(9) & Chr(9) _
& "Large L(x)" & Chr(9) & Chr(9) & "Large Force(y)" & Chr(9) & Chr(9) &
"Small Force(y)"

'outputs answers in table
Output.Text = Output.Text & Chr(9) & Sp & Chr(9) & SMInductance & Chr(9) &
LSInductance _
& Chr(9) & force1_y & Chr(9) & Chr(9) & force2_y & Chr(10)


Else
```

```
' neither currents are zero solve for self inductance of small and large
coils

    LSInductance = (LFlux)/(LCT)
    SSInductance = (SFlux)/(SCT)

Title.Text = Title.Text & Chr(9) & "x" & Chr(9) & Chr(9) & "Large L(x)"_
& Chr(9) & Chr(9) & Chr(9) & "Small L(x)" & Chr(9) & Chr(9) & "Large
Force(y)" _
& Chr(9) &Chr(9) & Chr(9) & "Small Force(y)"

Output.Text = Output.Text & Chr(9) & Sp & Chr(9) & LSInductance & Chr(9) &
SSInductance _
& Chr(9) & force1_y & Chr(9) & Chr(9) & force2_y & Chr(10)


 End if


End if
Next
End Sub
```

APPENDIX B MATLAB VARIABLES AND CODE

B.1 MATLAB GROVER model variables

Variable	Description	Units
Smalla	Small coil radius	centimeter
LargeA	Large coil radius	centimeter
twoM1	Length of small coil	centimeter
twoM2	Length of large coil	centimeter
s	separation distance between coils	centimeter
n1	small coil turn density	Turns/centimeter
n2	large coil turn density	Turns/centimeter
x1	distance of the length of the two coils and any spacing inbetween the two coils	centimeter
x2	distance of the length of the small coil and any spacing inbetween the two coils	centimeter
x3	distance of the length of the large coil and any spacing inbetween the two coils	centimeter
x4	distance of any spacing inbetween the two coils	centimeter
r1	diagonal with respect to large radius and x1	centimeter
r2	diagonal with respect to large radius and x2	centimeter
r3	diagonal with respect to large radius and x3	centimeter
r4	diagonal with respect to large radius and x4	centimeter
alpha	ratio of the small and large radii	-
rho1	squares of the sine of the angle subtended by the radius of the larger coil at the axial distance x1	-

rho2	squares of the sine of the angle subtended by the radius of the larger coil at the axial distance x2	-
rho3	squares of the sine of the angle subtended by the radius of the larger coil at the axial distance x3	-
rho4	squares of the sine of the angle subtended by the radius of the larger coil at the axial distance x4	-
B1	interpolated value from Grover's table for alpha and rho1	-
B2	interpolated value from Grover's table for alpha and rho2	-
B3	interpolated value from Grover's table for alpha and rho3	-
B4	interpolated value from Grover's table for alpha and rho4	-
M	Grover's mutual inductance	Henry

B.2 changingS.m file

```
% changingS.m program file
% User inputs values for small radius, large radius,
% small coil length, large coil length, small turn density,
% and large turn density.
% S is set to either .04 to -.04 with steps of -.002 or
% .4 to -.4 with steps of .02.
% Program calls grovermutual function with inputs and the various values
% of s, then returns a value of M

function K = spacing(Smalla, LargeA, twoM1, twoM2, n1, n2)
digits(5)

for s= .04:-.002:-.04
    array = vpa(grovermutual(Smalla, LargeA, twoM1, twoM2, s, n1, n2))
    s
end
```

B.3 grovermutual.m file

```
% grovermutual.m program file
% Receives user input of the small radius, large radius,
% small coil length, large coil length, separation distance,
% turn density of the small coil, turn density of the large coil.
% Solves for Grover's parameters, xn, rn, then depending on the
% values of alpha and rho it chooses which program to call to solve
% for Bn, receives values for every Bn and solves for the mutual
% inductance using Grover's method.
% User values are given in table 3.1 for this investigation.

function M = mutual(Smalla, LargeA, twoM1, twoM2, s, n1, n2)
% get the radii, lengths, separation distance and turn densities
digits(5)


% solves for various Grover parameters
M1 = twoM1/2;
M2 = twoM2/2;

x1 = s + (M1+M2);
x2 = s + (M1-M2);
x3 = s - (M1-M2);
x4 = s - (M1+M2);

r1 = sqrt(LargeA^2 + x1^2);
r2 = sqrt(LargeA^2 + x2^2);
r3 = sqrt(LargeA^2 + x3^2);
r4 = sqrt(LargeA^2 + x4^2);

alpha = (Smalla/LargeA);

% chooses which subprogram to call
if alpha == 1

    % solves for rhos and calls equal radii for Bn values
    rho1 = vpa((LargeA^2)/(r1^2));
    rho = double(rho1);
    equalradii(rho);
    B1 = (ans);

    rho2 = vpa((LargeA^2)/(r2^2));
    rho = double(rho2);
    equalradii(rho);
    B2 = (ans);

    rho3 = vpa((LargeA^2)/(r3^2));
    rho = double(rho3);
    equalradii(rho);
    B3 = vpa(ans);

    rho4 = vpa((LargeA^2)/(r4^2));
    rho = double(rho4);
```

```
    equalradii(rho);
    B4 = vpa(ans);


else
    %solves for rhos checks if rho and alpha are larger than 0.9
    % largergrover subprogram called if yes, returns values of Bn
    % induct4 subprogram called if no, returns values of Bn
    rho1 = vpa((LargeA^2)/(r1^2));
    rho = double(rho1);
    if rho >= .90 && alpha >= .90
        largegrover(rho, alpha);

    else
        induct4(rho, alpha);
    end
    B1 = vpa(ans)

    rho2 = vpa((LargeA^2)/(r2^2));
    rho = double(rho2);
    if rho >= .9 && alpha >= .9
        largegrover(rho, alpha);

    else
        induct4(rho, alpha);
    end
    B2 = vpa(ans)

    rho3 = vpa((LargeA^2)/(r3^2));
    rho = double(rho3);
    if rho >= .9 && alpha >= .9
        largegrover(rho, alpha);

    else
        induct4(rho, alpha);
    end
    B3 = vpa(ans)

    rho4 = vpa((LargeA^2)/(r4^2));
    rho = double(rho4);
    if rho >= .9 && alpha >= .9
        largegrover(rho, alpha);

    else
        induct4(rho, alpha);
    end
    B4 = vpa(ans)
end

digits(9)
r1;
r2;
r3;
r4;
% Grover's mutual inductance equation
```

```
M = vpa(.002*(pi^2)*(Smalla^2)*n1*n2*[(r1*B1)-(r2*B2)...
-(r3*B3)+(r4*B4)]);
```

B.4 induct4.m file

```
%induct4.m program file
% solves for the Bn values by interpolating
%the values of alpha and rho as needed

function B = induct(rho, alpha)

% interpolate if rho is mult of .05

%format long
digits(5)

% creates two matrices for the values of rho and alpha
rhos=vpa([1:-.05:0]);
alphas=vpa([1:-.05:0]);


% Grover's full table
A = [.84833 .87727 .89552 .91020 .92264 .93345 .94298 .95144 .959... .96576
.97180 .97718 .98194 .98612 .98974 .99282 .99535 .99735 .9988... .99969 1;
.86783 .88982 .90561 .91859 .92971 .93944 .94805 .95573... .96261 .96877
.97428 .97919 .98354 .98736 .99066 .99346 .99577...
.99759 .99891 .99972 1; .88418 .90175 .91531 .92666 .93655 .94524... .95298
.95990 .96612 .97169 .97668 .98114 .98509 .98855 .99155...
.99409 .99618 .99783 .99902 .99975 1; .89870 .91296 .92456 .93444... .94314
.95085 .95774 .96393 .96951 .97452.97901 .98302 .98658...
.98970 .99240 .99469 .99657 .99805 .99912 .99978 1; .91176 .92344... .93329
.94185 .94944 .95622 .96231 .96781 .97276 .97723 .98124...
.98483 .98801 .99080 .99322 .99526 .99695 .99827 .99922 .99980 1;...
.92356 .93318 .94150 .94885 .95542 .96132 .96668 .97151 .97588...
.97983 .98338 .98656 .98938 .99185 .99399 .99581 .99730 .99847...
.99931 .99983 1 ; .93426 .94217 .94917 .95543 .96107 .96618 .97082...
.97503 .97884 .98230 .98541 .98820 .99068 .99285 .99473 .99633...
.99764 .99866 .99940 .99985 1; .94394 .95045 .95629 .96157 .96637...
.97074 .97472 .97835 .98164 .98464 .98732 .98975 .99190 .99380...
.99543 .99682 .99795 .99884 .99948 .99987 1; .95270 .95803 .96286...
.96727 .97130 .97499 .97837 .98146 .98427 .98683 .98913 .99121...
.99306 .99467 .99608 .99727 .99825 .99901 .99956 .99989 1; .96060...
.96492 .96888 .97252 .97586 .97894 .98176 .98435 .98672 .98887 ...
.99082 .99257 .99413 .99550 .99669 .99770 .99852 .99916 .99963...
.99990 1; .96769 .97115 .97434 .97730 .98003 .98256 .98488 .98702...
.98897 .99076 .99237 .99383 .99512 .99626 .99725 .99809 .99877...
.99931 .99969 .99992 1; 97400 .97673 .97927 .98163 .98382 .98584 ...
.98772 .98945 .99103 .99248 .99379 .99498 .99603 .99696 .99776...
.99844 .99900 .99944 .99975 .99994 1; .97958 .98169 .98366 .98550...
.98721 .98880 .99028 .99164 .99289 .99404 .99508 .99601 .99685...
.99759 .99823 .99877 .99921 .99955 .99980 .99995 1; .98444 .98603...
.98751 .98890 .99020 .99142 .99254 .99358 .99454 .99542 .99622...
.99694 .99758 .99815 .99864 .99905 .99939 .99966 .99985 .99996 1;...
.98862 .98976 .99084 .99186 .99280 .99369 .99451 .99527 .99598...
.99662 .99721 .99774 .99822 .99864 .99900 .99930 .99955 .99975...
.99989 .99997 1; .99212 .99291 .99365 .99435 .99500 .99561 .99618...
.99671 .99720 .99765 .99806 .99843 .99876 .99905 .99930 .99952...
.99969 .99982 .99992 .99998 1; .99498 .99547 .99594 .99638 .99680...
.99719 .99755 .99789 .99821 .99849 .99875 .99899 .99920 .99939...
.99955 .99969 .99980 .99989 .99995 .99999 1; .99718 .99746 .99772...
.99797 .99820 .99842 .99862 .99881 .99899 .99915 .99930 .99943...
.99955 .99966 .99975 .99982 .99989 .99994 .99997 .99999 1; .99875... .99887
.99899 .99910 .99920 .99930 .99939 .99947 .99955 .99962...
.99969 .99975 .99980 .99985 .99989 .99992 .99995 .99997 .99999 1 1;...
.99969 .99972 .99975 .99977 .99980 .99982 .99985 .99987 .99989...
.99991 .99992 .99994 .99995 .99996 .99997 .99998 .99999 .99999 1 1...
```

```
1; 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 ];

% a garbage matrix to test with
   % A=zeros(21,21);
    %for i = 1:21;
     %   for j=1:21
      %       A(i,j)=i*j;
      % end
      % end
A;


% changing formats so that a comparison can be made
alpha = vpa(alpha);
alpha = double(alpha)
rho = vpa(rho);
rho = double(rho)
 % find row and column if both numbers are on table
if rem(rho, .05)==0 && rem(alpha, .05)==0

    %get row number of Grover's table
    X=double(alpha)>double(alphas);

    countr=0;
    countc=0;
    for i=1:length(alphas)
        if X(i) ==0
            countr= countr+1;
        end
    end
    row=countr;


    % get column number of Grover's table
    X=double(rho)>double(rhos);

    for i=1:length(rhos)
        if X(i) == 0
            countc = countc+1;
        end
    end
    column = countc;

    % B is value located in the Grover's table at rho and alpha
    B = A(row,column);



% interpolate if rho is mult of .05
% finds column if rho is a number on the table
elseif rem(rho,.05)==0

 % finds mults of .05 between known alpha
    X=alpha>double(alphas);

    countr=0;
    countc=0;
% increases counter to locate rows on both sides of alpha
    for i=1:length(alphas)
        if X(i) == 0
            countr = countr+1;
        end
```

```
    end
    countr = countr + 1;
    row = countr;

  % two rows values on both sides of alpha
    lowervalue = alphas(countr);
    uppervalue = alphas(countr-1);


    % get the column number for rho
    X=rho>double(rhos);

    for i=1:length(rhos)
        if X(i) == 0
            countc = countc+1;
        end
    end
     countc = countc;
    column = countc;


    row;
    column;
   % Finds the values in the table at the corresponding values
   % of the two alphas (rows) and the rho (column)
    fminus = A(row,column);
    fplus = A(row-1,column);

    % interpolates to find B
    rightalpha = ((alpha - lowervalue)/(uppervalue-lowervalue))*...
        (fplus-fminus)+fminus;

    B = rightalpha;



% interpolate if alpha is mult of .05
% finds row if alpha is a number on the table
elseif rem(alpha, .05)==0

   % finds mults of .05 between known rho
    X=rho>double(rhos);
    count=0;
   % increases counter to locate columns on both sides of rho
    for i=1:length(rhos)
        if X(i) == 0
            count = count+1;
        end
    end
    count = count + 1;
    column = count;

   % two column values on both sides of rho
    % also fixes a formatting problem
    lowervalue1 = vpa(rhos(count));
    lowervalue = double(lowervalue1);
    uppervalue1 = vpa(rhos(count-1));
    uppervalue = double(uppervalue1);
 % get the row number for alpha
    X=alpha>double(alphas);
    count=0;
    for i=1:length(alphas)
```

```
        if X(i) == 0
            count = count+1;
        end
    end

    row = count;

    row;
    column;

   % Finds the values in the table at the corresponding values
   % of the two alphas (rows) and the rho (column)
   % also fixes a formatting problem
    fminus1 = vpa(A(row,column))
    fminus = double(fminus1)
    fplus1 = vpa(A(row, column-1))
    fplus = double(fplus1)

    % interpolates to find B
    rightrho = ((rho - lowervalue)/(uppervalue-lowervalue))*...
        (fplus-fminus)+fminus;
    B = rightrho;

else
     % need to double interpolate

    % finds mults of .05 between known alpha
     X=alpha>double(alphas);

    countr=0;
    countc=0;
    % increases counter to locate rows on both sides of alpha
    for i=1:length(alphas)
        if X(i) == 0
            countr = countr+1;
        end
    end
    countr = countr + 1;
    row = countr;
    % two rows values on both sides of alpha
    alphalower = alphas(countr);
    alphaupper = alphas(countr-1);

    % finds mults of .05 between known rho
    X=rho>double(rhos);
    count=0;
  % increases counter to locate columns on both sides of rho
    for i=1:length(rhos)
        if X(i) == 0
            count = count+1;
        end
    end
    count = count + 1;
    column = count;
     % two column values on both sides of rho
    rholower = rhos(count);
    rhoupper = rhos(count-1);

    % gets the four values in Grover's Table to be interpolated
    firstminus = A(row,column);
    firstplus = A(row, column-1);
    secondminus = A(row-1, column);
    secondplus = A(row-1, column-1);
```

```
    % Double interpolation
    Z1 = ((rho - rholower)/(rhoupper-rholower))*(firstplus-firstminus)...
        +firstminus;

    Z2 = ((rho - rholower)/(rhoupper-rholower))*(secondplus-secondminus)...
        +secondminus;

    Z3 = (Z2-Z1)*[(alpha-alphalower)/(alphaupper-alphalower)]+Z1;

    % ouputs B
    B=Z3;
end
```

B.5 largegrover.m file

```
% largegrover.m program file
% inputs alpha and rho
% Solves for values of Bn when alpha and rho are greater than 0.9
% returns Bn

function W = Large(rho, alpha)
digits(6)

% sets up two 1D arrays for the values of rho and alpha
% located on the outside of Grover's aux table
rhos=vpa([1:-0.01:.9]);
alphas=vpa([1:-0.01:.9]);

% values in Grover's aux table
LargeA = [.84883 .85698 .86298 .86820 .87292 .87727 .88133 .88515...
.88877 .89222 .89552; .85294 .86035 .86606 .87107 .87562 .87982...
.88376 .88747 .89100 .89436 .89757; .85686 .86366 .86910 .87391...
.87829 .88236 .88617 .88978 .89320 .89647 .89960; .86063 .86693...
.87210 .87672 .88094 .88487 .88857 .89207 .89539 .89858 .90162;...
.86428 .87014 .87506 .87949 .88356 .88736 .89094 .89433 .89757...
.90066 .90362; .86783 .87329 .87798 .88223 .88615 .88982 .89329...
.89658 .89972 .90273 .90561; .87127 .87639 .88086 .88494 .88872...
.89226 .89562 .89881 .90186 .90478 .90759; .87462 .87944 .88370...
.88761 .89125 .89468 .89792 .90102 .90397 .90681 .90954; .87788...
.88242 .88649 .89024 .89375 .89706 .90020 .90320 .90607 .90883... .91148;
.88107 .88536 .88924 .89285 .89622 .89942 .90246 .90536...
.90815 .91082 .91340; .88418 .88824 .89195 .89541 .89866 .90175...
.90469 .90750 .91020 .91280 .91531];

LargeA;

% fixing formatting
alpha = vpa(alpha);
alpha = double(alpha);
rho = vpa(rho);
rho = double(rho);

% interpolate if rho is mult of .01
% finds column if rho is a number on the table
if rem(rho,.01)==0

    % finds mults of .01 between known alpha
    X=alpha>double(alphas);

    countr=0;
    countc=0;
    % increases counter to locate rows on both sides of alpha
    for i=1:length(alphas)
        if X(i) == 0
            countr = countr+1;
        end
    end
```

```
    countr = countr + 1;
    row = countr;
    % two rows values on both sides of alpha
    lowervalue = alphas(countr);
    uppervalue = alphas(countr-1);

    % get the column number for rho
    X=rho>double(rhos);

    for i=1:length(rhos)
        if X(i) == 0
            countc = countc+1;
        end
    end
    % countc = countc +1
    column = countc;
    row;
    column;

   % Finds the values in the table at the corresponding values
   % of the two alphas (rows) and the rho (column)
    fminus = LargeA(row,column);
    fplus = LargeA(row-1,column);

   % interpolates to find B
    rightalpha = ((alpha - lowervalue)/(uppervalue-lowervalue))*...
      (fplus-fminus)+fminus;

    W = rightalpha;



% interpolate if alpha is mult of .01
% finds column if alpha is a number on the table
elseif rem(alpha, .01)==0

    % finds mults of .01 between known rho
    X=rho>double(rhos);
    count=0;
   % increases counter to locate columns on both sides of rho
    for i=1:length(rhos)
        if X(i) == 0
            count = count+1;
        end
    end
    count = count + 1;
    column = count;
    % two column values on both sides of rho
    lowervalue = rhos(count);
    uppervalue = rhos(count-1);

 % get the row number for alpha
    X=alpha>double(alphas);
    count=0;
    for i=1:length(alphas)
```

```
        if X(i) == 0
            count = count+1;
        end
    end
    row = count;

    row;
    column;
   % Finds the values in the table at the corresponding values
   % of the two rhos (columns) and the alpha (row)
    fminus = LargeA(row,column);
    fplus = LargeA(row, column-1);

    % interpolates to find B
    rightrho = ((rho - lowervalue)/(uppervalue-lowervalue))*...
        (fplus-fminus)+fminus;
    W = rightrho;

else
     % need to double interpolate
% finds mults of .01 between known alpha
    X=alpha>double(alphas);
    countr=0;
    countc=0;
    % increases counter to locate rows on both sides of alpha
    for i=1:length(alphas)
        if X(i) == 0
            countr = countr+1;
        end
    end
    countr = countr + 1;
    row = countr;
    % lower value of alpha
    alphalowervalue = alphas(countr);

% finds mults of .01 between known rho
    X=rho>double(rhos);
    count=0;
    % increases counter to locate rows on both sides of rho
    for i=1:length(rhos)
        if X(i) == 0
            count = count+1;
        end
    end
    count = count + 1;
    column = count;
    % lower value of rho
    rholowervalue = rhos(count);


    %gets important part of matrix for double interpolation equation
    alpha0rho0 = LargeA(row, column);
    alpha1rho0 = LargeA(row-1, column);
    alpha2rho0 = LargeA(row-2, column);
    alpha0rho1 = LargeA(row, column-1);
    alpha1rho1 = LargeA(row-1, column-1);
```

```
    alpha0rho2 = LargeA(row, column-2);

    % double interpolation method found in [6]
    dx1 = alpha0rho1 - alpha0rho0;
    dx2 = (alpha0rho2 - alpha0rho1) - dx1;
    dxy2 = (alpha1rho1 - alpha1rho0) - dx1;

    dy1 = alpha1rho0 - alpha0rho0;
    dy2 = (alpha2rho0 - alpha1rho0) - dy1;
    dyx2 = (alpha1rho1 - alpha0rho1) - dy1;

    u = (rho - rholowervalue)/.05;
    v = (alpha - alphalowervalue)/.05;
    u1 = (1-u)/2;
    v1 = (1-v)/2;
    uv = u*v;


    rightrhoalpha = (alpha0rho0) + (u*(dx1-(u1*dx2))) +
      (v*(dy1-(v1*dy2))) + uv*dxy2;

    W=rightrhoalpha;
end
```

B.6 equalradii.m file

```
% equalradii.m program file
% program receives rho to find the correct Grover value for rho
% Solves
function EqR = equal(rho)
 %format long
 digits(6)

equalA = [1 .999987 .999950 .999889 .999804 .999695 .999562 .999407...
    .999228 .999026 .998802 .998556 .998287 .997996 .997684 .997349...
    .996992 .996614 .996214 .995793 .995351 .994886 .994401 .993894...
    .993366 .992815 .992815 .992244 .991650 .991035 .990399 .989742...
    .989062 .988360 .987637 .986891 .986123 .985332 .984520 .983684...
    .982826 .981944 .981039 .980110 .979158 .978182 .977181 .976156...
    .975106 .974031 .972930 .971802 .970649 .969469 .968262 .967027...
    .965763 .964471 .963149 .961798 .960416 .959002 .957558 .956080...
    .954570 .953024 .951443 .949826 .948172 .946480 .944748 .942975...
    .941161 .939302 .937398 .935448 .933448 .931397 .929294 .927135...
    .924918 .922639 .920297 .917886 .915403 .912843 .910202 .907472...
    .904648 .901721 .898683 .895522 .892225 .888774 .885151 .881327...
    .877266 .872917 .868201 .862983 .856980 .848826];

   % sets up a matrix for rho
    rhos=vpa([0:0.01:1]);

    row = 1;
    rho = vpa(rho);
    rho = double(rho);

    % makes a matrix for the threshold of rho
    X=rho>double(rhos);

    count=0;
  % increases counter to locate rows on both sides of rho
    for i=1:length(rhos)
        if X(i) == 1
            count = count+1;
        end
    end
    count = count + 1;
    column = count;

% two column values on both sides of rho
    uppervalue = rhos(count);
    lowervalue = rhos(count-1);

   % Finds the values in the table at the corresponding values
   % of the two alphas (rows) and the rho (column)
    fminus = equalA(row,column);
    fplus = equalA(row, column+1);

    % interpolates to find Bn
    rightrho = ((rho - lowervalue)/(uppervalue-lowervalue))*...
        (fplus-fminus)+fminus;
```

```
EqR = rightrho
```

B.7 MATLAB ELLIPTCAL model variables

Variable	Desciption	Units
largeA	Large coil radius	centimeters
smalla	Small coil radius	centimeters
d	Separation distance	centimeters
N1	Small turn density	Turns/centimeter
N2	Large turn density	Turns/centimeter
twoM1	Small coil length	centimeters
twoM2	Large Coil length	centimeters
Turns1	Small coil turns	Turns
Turns2	Large coil turns	Turns
r1	Diagonal distance from top end of the large radius to the bottom end of small radius with separation distance d	centimeters
r2	Diagonal distance from top end of the large radius to the top end of small radius with separation distance d	centimeters
m	Parameter of the modulus	-
k	Modulus of elliptical integrals	-
F	Complete elliptical integral of the first kind	-
E	Complete elliptical integral of the second kind	-
M	Elliptical integral mutual inductance	Henry

B.8 mutualchange.m file

```
% mutualchange.m program file
% S is set to either .04 to -.04 with steps of -.002 or
% .4 to -.4 with steps of .02.
% Program calls ellmutual function with the various values
% of s, then returns a value of M

digits(6)
 for sdistance = .40 :-.02 :-.40
    array1 = ellmutual(sdistance)
    fprintf('\n');
    sdistance

end
```

B.9 ellmutual.m file

```
% ellmutual.m program file
% Receives sdistance, from mutualchange.m
% User changes largeA, smalla, small turn density,
% large turn density, small length and large length
% for examples in table 3.1.
% Program uses various inputs to use a matlab function
% to solve the elliptical integrals then the mutual inductance
% is solved using (3.8) and (3.9) found in [8].
function M = ellmutual(sdistance)
%clear;
digits(6)


largeA = .05            %large radius
smalla = .05;           %small radius
d = sdistance;          %s separation

N1 = 10;                %small turn density
N2 = 10;                %large turn density
twoM1 = .01;            %small length
twoM2 = .01;            %large length

Turns1 = N1*twoM1;      %small coil turns
Turns2 = N2*twoM2;      %large coil turns


r1 = sqrt((largeA+smalla)^2 + d^2);
r2 = sqrt((largeA-smalla)^2 + d^2);

m = 1-((r2/r1)^2);
k = sqrt(m);

% solves elliptical integrals
[F,E] = ellipke(m);


Mo = 4*pi*(sqrt(largeA*smalla))*((((2/k)-k)*F) - ((2/k)*E))

 M = vpa(Turns1*Turns2*(Mo)*.001);

end
```

APPENDIX C MUTUAL INDUCTANCE VALUES

C.1 Mutual Inductance Values for Equal Length Coils

α = .1	MagNet	Elliptical	Grover
S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	1.571806E-07	1.581518E-07	1.579687E-07
3.8	1.601731E-07	1.614301E-07	1.611913E-07
3.6	1.639463E-07	1.646472E-07	1.644109E-07
3.4	1.662269E-07	1.677908E-07	1.675135E-07
3.2	1.694382E-07	1.708482E-07	1.706110E-07
3	1.731204E-07	1.738066E-07	1.735642E-07
2.8	1.751194E-07	1.766529E-07	1.764461E-07
2.6	1.776366E-07	1.793742E-07	1.791403E-07
2.4	1.810581E-07	1.819573E-07	1.817185E-07
2.2	1.835369E-07	1.843895E-07	1.841269E-07
2	1.857551E-07	1.866583E-07	1.863744E-07
1.8	1.877990E-07	1.887519E-07	1.884109E-07
1.6	1.890891E-07	1.906587E-07	1.902860E-07
1.4	1.910244E-07	1.923684E-07	1.919701E-07
1.2	1.921363E-07	1.938710E-07	1.934339E-07
1	1.934254E-07	1.951580E-07	1.946696E-07
0.8	1.946511E-07	1.962217E-07	1.957303E-07
0.6	1.954373E-07	1.970558E-07	1.965307E-07
0.4	1.958306E-07	1.976552E-07	1.971252E-07
0.2	1.957658E-07	1.980163E-07	1.974716E-07
0	1.955463E-07	1.981370E-07	1.976071E-07
-0.2	1.967974E-07	1.980163E-07	1.974716E-07
-0.4	1.964135E-07	1.976552E-07	1.971252E-07
-0.6	1.957690E-07	1.970558E-07	1.965307E-07
-0.8	1.949873E-07	1.962217E-07	1.957303E-07
-1	1.935615E-07	1.951580E-07	1.946696E-07
-1.2	1.920870E-07	1.938710E-07	1.934339E-07
-1.4	1.907706E-07	1.923684E-07	1.919701E-07
-1.6	1.889510E-07	1.906587E-07	1.902860E-07
-1.8	1.880475E-07	1.887519E-07	1.884109E-07
-2	1.857581E-07	1.866583E-07	1.863744E-07
-2.2	1.835276E-07	1.843895E-07	1.841269E-07
-2.4	1.809148E-07	1.819573E-07	1.817185E-07
-2.6	1.786677E-07	1.793742E-07	1.791403E-07
-2.8	1.759447E-07	1.766529E-07	1.764461E-07
-3	1.731085E-07	1.738066E-07	1.735642E-07
-3.2	1.692699E-07	1.708482E-07	1.706110E-07
-3.4	1.663494E-07	1.677908E-07	1.675135E-07
-3.6	1.640134E-07	1.646472E-07	1.644109E-07
-3.8	1.607448E-07	1.614301E-07	1.611913E-07
-4	1.573968E-07	1.581518E-07	1.579688E-07

α = .2 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	6.327446E-07	6.344169E-07	6.345813E-07
3.8	6.454928E-07	6.479809E-07	6.478298E-07
3.6	6.580732E-07	6.613168E-07	6.606380E-07
3.4	6.685198E-07	6.743722E-07	6.733018E-07
3.2	6.807092E-07	6.870933E-07	6.862770E-07
3	6.933118E-07	6.994248E-07	6.985752E-07
2.8	7.058172E-07	7.113102E-07	7.104160E-07
2.6	7.132957E-07	7.226927E-07	7.217597E-07
2.4	7.258795E-07	7.335154E-07	7.323890E-07
2.2	7.406949E-07	7.437220E-07	7.424972E-07
2	7.494319E-07	7.532575E-07	7.518875E-07
1.8	7.578517E-07	7.620685E-07	7.605239E-07
1.6	7.644758E-07	7.701045E-07	7.685270E-07
1.4	7.717942E-07	7.773176E-07	7.758623E-07
1.2	7.772622E-07	7.836643E-07	7.819340E-07
1	7.810052E-07	7.891050E-07	7.871158E-07
0.8	7.886930E-07	7.936054E-07	7.914640E-07
0.6	7.902472E-07	7.971366E-07	7.947962E-07
0.4	7.886708E-07	7.996756E-07	7.970200E-07
0.2	7.921386E-07	8.012057E-07	7.985187E-07
0	7.932406E-07	8.017169E-07	7.990440E-07
-0.2	7.968567E-07	8.012057E-07	7.985187E-07
-0.4	7.942495E-07	7.996756E-07	7.970200E-07
-0.6	7.912907E-07	7.971366E-07	7.947962E-07
-0.8	7.872855E-07	7.936054E-07	7.914640E-07
-1	7.796925E-07	7.891050E-07	7.871158E-07
-1.2	7.784402E-07	7.836643E-07	7.819340E-07
-1.4	7.696142E-07	7.773176E-07	7.758623E-07
-1.6	7.651808E-07	7.701045E-07	7.685270E-07
-1.8	7.580577E-07	7.620685E-07	7.605239E-07
-2	7.471782E-07	7.532575E-07	7.518875E-07
-2.2	7.401096E-07	7.437220E-07	7.424972E-07
-2.4	7.270671E-07	7.335154E-07	7.323890E-07
-2.6	7.195557E-07	7.226927E-07	7.217597E-07
-2.8	7.083547E-07	7.113102E-07	7.104160E-07
-3	6.971836E-07	6.994248E-07	6.985752E-07
-3.2	6.862355E-07	6.870933E-07	6.862770E-07
-3.4	6.713766E-07	6.743722E-07	6.733018E-07
-3.6	6.603772E-07	6.613168E-07	6.606380E-07
-3.8	6.440042E-07	6.479809E-07	6.478298E-07
-4	6.295408E-07	6.344169E-07	6.345813E-07

α = .3 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	1.427780E-06	1.433598E-06	1.431161E-06
3.8	1.462269E-06	1.465885E-06	1.463792E-06
3.6	1.493146E-06	1.497735E-06	1.495748E-06
3.4	1.523788E-06	1.529021E-06	1.527128E-06
3.2	1.554193E-06	1.559607E-06	1.560452E-06
3	1.580364E-06	1.589355E-06	1.589355E-06
2.8	1.612695E-06	1.618120E-06	1.617208E-06
2.6	1.642016E-06	1.645755E-06	1.643748E-06
2.4	1.665977E-06	1.672111E-06	1.669418E-06
2.2	1.689760E-06	1.697041E-06	1.693363E-06
2	1.710508E-06	1.720396E-06	1.717167E-06
1.8	1.735231E-06	1.742034E-06	1.738947E-06
1.6	1.751106E-06	1.761817E-06	1.759349E-06
1.4	1.770266E-06	1.779614E-06	1.777221E-06
1.2	1.776894E-06	1.795304E-06	1.792326E-06
1	1.795574E-06	1.808778E-06	1.804427E-06
0.8	1.806094E-06	1.819941E-06	1.814580E-06
0.6	1.814746E-06	1.828709E-06	1.822200E-06
0.4	1.818186E-06	1.835020E-06	1.828151E-06
0.2	1.827691E-06	1.838826E-06	1.831519E-06
0	1.816211E-06	1.840098E-06	1.832642E-06
-0.2	1.823139E-06	1.838826E-06	1.831519E-06
-0.4	1.821675E-06	1.835020E-06	1.828151E-06
-0.6	1.814899E-06	1.828709E-06	1.822200E-06
-0.8	1.805289E-06	1.819941E-06	1.814580E-06
-1	1.788851E-06	1.808778E-06	1.804427E-06
-1.2	1.784508E-06	1.795304E-06	1.792326E-06
-1.4	1.771211E-06	1.779614E-06	1.777221E-06
-1.6	1.745227E-06	1.761817E-06	1.759349E-06
-1.8	1.728550E-06	1.742034E-06	1.738947E-06
-2	1.713832E-06	1.720396E-06	1.717167E-06
-2.2	1.689755E-06	1.697041E-06	1.693363E-06
-2.4	1.659565E-06	1.672111E-06	1.669418E-06
-2.6	1.639569E-06	1.645755E-06	1.643748E-06
-2.8	1.612059E-06	1.618120E-06	1.617208E-06
-3	1.571105E-06	1.589355E-06	1.589355E-06
-3.2	1.547432E-06	1.559607E-06	1.560452E-06
-3.4	1.509112E-06	1.529021E-06	1.527128E-06
-3.6	1.490556E-06	1.497735E-06	1.495748E-06
-3.8	1.458127E-06	1.465885E-06	1.463792E-06
-4	1.426845E-06	1.433598E-06	1.431162E-06

α = .4 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	2.553465E-06	2.561168E-06	2.562213E-06
3.8	2.608434E-06	2.623240E-06	2.622868E-06
3.6	2.674605E-06	2.684793E-06	2.682519E-06
3.4	2.733692E-06	2.745575E-06	2.741031E-06
3.2	2.784985E-06	2.805313E-06	2.806621E-06
3	2.849497E-06	2.863720E-06	2.862833E-06
2.8	2.895380E-06	2.920493E-06	2.916778E-06
2.6	2.964103E-06	2.975317E-06	2.968672E-06
2.4	3.015116E-06	3.027868E-06	3.018679E-06
2.2	3.064054E-06	3.077815E-06	3.067503E-06
2	3.101574E-06	3.124827E-06	3.117355E-06
1.8	3.154056E-06	3.168573E-06	3.163686E-06
1.6	3.178789E-06	3.208734E-06	3.206396E-06
1.4	3.216643E-06	3.245000E-06	3.244763E-06
1.2	3.246942E-06	3.277083E-06	3.275475E-06
1	3.284218E-06	3.304717E-06	3.298201E-06
0.8	3.294610E-06	3.327669E-06	3.317404E-06
0.6	3.318757E-06	3.345737E-06	3.332082E-06
0.4	3.335385E-06	3.358761E-06	3.342820E-06
0.2	3.345277E-06	3.366623E-06	3.348962E-06
0	3.337867E-06	3.369251E-06	3.351433E-06
-0.2	3.337657E-06	3.366623E-06	3.348962E-06
-0.4	3.334492E-06	3.358761E-06	3.342820E-06
-0.6	3.324234E-06	3.345737E-06	3.332082E-06
-0.8	3.286293E-06	3.327669E-06	3.317404E-06
-1	3.274243E-06	3.304717E-06	3.298201E-06
-1.2	3.252616E-06	3.277083E-06	3.275475E-06
-1.4	3.216658E-06	3.245000E-06	3.244763E-06
-1.6	3.170234E-06	3.208734E-06	3.206396E-06
-1.8	3.144257E-06	3.168573E-06	3.163686E-06
-2	3.106905E-06	3.124827E-06	3.117355E-06
-2.2	3.063821E-06	3.077815E-06	3.067503E-06
-2.4	3.016165E-06	3.027868E-06	3.018679E-06
-2.6	2.964694E-06	2.975317E-06	2.968672E-06
-2.8	2.905920E-06	2.920493E-06	2.916778E-06
-3	2.841499E-06	2.863720E-06	2.862833E-06
-3.2	2.785637E-06	2.805313E-06	2.806621E-06
-3.4	2.735074E-06	2.745575E-06	2.741031E-06
-3.6	2.677497E-06	2.684793E-06	2.682519E-06
-3.8	2.616242E-06	2.623240E-06	2.622868E-06
-4	2.552365E-06	2.561168E-06	2.562215E-06

α = .5 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	3.989934E-06	4.017680E-06	4.024862E-06
3.8	4.097668E-06	4.124683E-06	4.128710E-06
3.6	4.223227E-06	4.231611E-06	4.232250E-06
3.4	4.281282E-06	4.338028E-06	4.333681E-06
3.2	4.428799E-06	4.443454E-06	4.456420E-06
3	4.522287E-06	4.547363E-06	4.552029E-06
2.8	4.618225E-06	4.649183E-06	4.645506E-06
2.6	4.710238E-06	4.748302E-06	4.735161E-06
2.4	4.823787E-06	4.844070E-06	4.822341E-06
2.2	4.907573E-06	4.935806E-06	4.910047E-06
2	4.997822E-06	5.022804E-06	5.007434E-06
1.8	5.071452E-06	5.104350E-06	5.097918E-06
1.6	5.139881E-06	5.179725E-06	5.181962E-06
1.4	5.213984E-06	5.248225E-06	5.258011E-06
1.2	5.261970E-06	5.309175E-06	5.315425E-06
1	5.306896E-06	5.361945E-06	5.353815E-06
0.8	5.331409E-06	5.405966E-06	5.385790E-06
0.6	5.382031E-06	5.440746E-06	5.410816E-06
0.4	5.396292E-06	5.465886E-06	5.427878E-06
0.2	5.434336E-06	5.481091E-06	5.438909E-06
0	5.444621E-06	5.486179E-06	5.442918E-06
-0.2	5.429365E-06	5.481091E-06	5.438909E-06
-0.4	5.432605E-06	5.465886E-06	5.427878E-06
-0.6	5.402751E-06	5.440746E-06	5.410816E-06
-0.8	5.348434E-06	5.405966E-06	5.385790E-06
-1	5.307464E-06	5.361945E-06	5.353815E-06
-1.2	5.273389E-06	5.309175E-06	5.315425E-06
-1.4	5.212691E-06	5.248225E-06	5.258011E-06
-1.6	5.130702E-06	5.179725E-06	5.181962E-06
-1.8	5.072466E-06	5.104350E-06	5.097918E-06
-2	5.000621E-06	5.022804E-06	5.007434E-06
-2.2	4.905562E-06	4.935806E-06	4.910047E-06
-2.4	4.821495E-06	4.844070E-06	4.822341E-06
-2.6	4.729050E-06	4.748302E-06	4.735161E-06
-2.8	4.621382E-06	4.649183E-06	4.645506E-06
-3	4.516218E-06	4.547363E-06	4.552029E-06
-3.2	4.428027E-06	4.443454E-06	4.456420E-06
-3.4	4.315040E-06	4.338028E-06	4.333681E-06
-3.6	4.217194E-06	4.231611E-06	4.232250E-06
-3.8	4.116927E-06	4.124683E-06	4.128710E-06
-4	4.005355E-06	4.017680E-06	4.024861E-06

α = .6 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	5.783396E-06	5.786603E-06	5.807256E-06
3.8	5.892709E-06	5.959126E-06	5.971081E-06
3.6	6.118799E-06	6.133401E-06	6.133050E-06
3.4	6.263215E-06	6.308809E-06	6.293752E-06
3.2	6.447366E-06	6.484621E-06	6.513300E-06
3	6.639632E-06	6.659999E-06	6.672411E-06
2.8	6.794853E-06	6.833979E-06	6.827300E-06
2.6	6.969568E-06	7.005477E-06	6.980045E-06
2.4	7.139513E-06	7.173283E-06	7.129060E-06
2.2	7.272666E-06	7.336066E-06	7.284761E-06
2	7.449638E-06	7.492385E-06	7.470212E-06
1.8	7.591191E-06	7.640704E-06	7.645164E-06
1.6	7.732096E-06	7.779419E-06	7.804660E-06
1.4	7.841678E-06	7.906891E-06	7.951712E-06
1.2	7.958057E-06	8.021485E-06	8.056620E-06
1	8.028004E-06	8.121623E-06	8.118424E-06
0.8	8.127399E-06	8.205834E-06	8.168560E-06
0.6	8.172665E-06	8.272815E-06	8.209220E-06
0.4	8.219109E-06	8.321480E-06	8.237520E-06
0.2	8.201648E-06	8.351017E-06	8.254225E-06
0	8.285598E-06	8.360919E-06	8.260020E-06
-0.2	8.226454E-06	8.351017E-06	8.254225E-06
-0.4	8.235648E-06	8.321480E-06	8.237520E-06
-0.6	8.156699E-06	8.272815E-06	8.209220E-06
-0.8	8.087865E-06	8.205834E-06	8.168560E-06
-1	8.045326E-06	8.121623E-06	8.118424E-06
-1.2	7.948384E-06	8.021485E-06	8.056620E-06
-1.4	7.823059E-06	7.906891E-06	7.951712E-06
-1.6	7.711855E-06	7.779419E-06	7.804660E-06
-1.8	7.598485E-06	7.640704E-06	7.645164E-06
-2	7.451345E-06	7.492385E-06	7.470212E-06
-2.2	7.304038E-06	7.336066E-06	7.284761E-06
-2.4	7.128021E-06	7.173283E-06	7.129060E-06
-2.6	6.961379E-06	7.005477E-06	6.980045E-06
-2.8	6.794431E-06	6.833979E-06	6.827300E-06
-3	6.639321E-06	6.659999E-06	6.672411E-06
-3.2	6.441015E-06	6.484621E-06	6.513300E-06
-3.4	6.287941E-06	6.308809E-06	6.293752E-06
-3.6	6.114295E-06	6.133401E-06	6.133050E-06
-3.8	5.928092E-06	5.959126E-06	5.971081E-06
-4	5.752358E-06	5.786603E-06	5.807258E-06

α = .7 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	7.798693E-06	7.810327E-06	7.853506E-06
3.8	8.044382E-06	8.073917E-06	8.094944E-06
3.6	8.285118E-06	8.344058E-06	8.340990E-06
3.4	8.607748E-06	8.620181E-06	8.585137E-06
3.2	8.880039E-06	8.901524E-06	8.972170E-06
3	9.138279E-06	9.187101E-06	9.226745E-06
2.8	9.450837E-06	9.475659E-06	9.481700E-06
2.6	9.719875E-06	9.765640E-06	9.731940E-06
2.4	1.000358E-05	1.005514E-05	9.979530E-06
2.2	1.028325E-05	1.034186E-05	1.025554E-05
2	1.058682E-05	1.062310E-05	1.061393E-05
1.8	1.084553E-05	1.089573E-05	1.095110E-05
1.6	1.107351E-05	1.115620E-05	1.126371E-05
1.4	1.131279E-05	1.140059E-05	1.155224E-05
1.2	1.147058E-05	1.162470E-05	1.174203E-05
1	1.168820E-05	1.182418E-05	1.183930E-05
0.8	1.185442E-05	1.199471E-05	1.191698E-05
0.6	1.197823E-05	1.213226E-05	1.198015E-05
0.4	1.207096E-05	1.223329E-05	1.202210E-05
0.2	1.218358E-05	1.229507E-05	1.205073E-05
0	1.219123E-05	1.231587E-05	1.205832E-05
-0.2	1.216891E-05	1.229507E-05	1.205073E-05
-0.4	1.209818E-05	1.223329E-05	1.202210E-05
-0.6	1.196785E-05	1.213226E-05	1.198015E-05
-0.8	1.186400E-05	1.199471E-05	1.191698E-05
-1	1.168740E-05	1.182418E-05	1.183930E-05
-1.2	1.152826E-05	1.162470E-05	1.174203E-05
-1.4	1.131925E-05	1.140059E-05	1.155224E-05
-1.6	1.108312E-05	1.115620E-05	1.126371E-05
-1.8	1.084413E-05	1.089573E-05	1.095110E-05
-2	1.058032E-05	1.062310E-05	1.061393E-05
-2.2	1.029947E-05	1.034186E-05	1.025554E-05
-2.4	1.001151E-05	1.005514E-05	9.979530E-06
-2.6	9.738235E-06	9.765640E-06	9.731940E-06
-2.8	9.426937E-06	9.475659E-06	9.481700E-06
-3	9.160832E-06	9.187101E-06	9.226745E-06
-3.2	8.880177E-06	8.901524E-06	8.972170E-06
-3.4	8.607209E-06	8.620181E-06	8.585140E-06
-3.6	8.314091E-06	8.344058E-06	8.340990E-06
-3.8	8.060689E-06	8.073917E-06	8.094940E-06
-4	7.792790E-06	7.810327E-06	7.853510E-06

α = .8 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	9.907451E-06	9.951494E-06	1.004036E-05
3.8	1.033047E-05	1.032958E-05	1.037275E-05
3.6	1.071317E-05	1.072377E-05	1.071194E-05
3.4	1.113019E-05	1.113439E-05	1.105626E-05
3.2	1.156901E-05	1.156161E-05	1.170706E-05
3	1.200695E-05	1.200540E-05	1.207896E-05
2.8	1.244708E-05	1.246536E-05	1.245467E-05
2.6	1.290781E-05	1.294067E-05	1.283698E-05
2.4	1.341984E-05	1.342990E-05	1.322225E-05
2.2	1.391582E-05	1.393082E-05	1.373031E-05
2	1.441214E-05	1.444013E-05	1.449202E-05
1.8	1.489907E-05	1.495319E-05	1.521077E-05
1.6	1.538664E-05	1.546369E-05	1.587785E-05
1.4	1.586539E-05	1.596330E-05	1.649601E-05
1.2	1.630882E-05	1.644150E-05	1.687671E-05
1	1.671071E-05	1.688547E-05	1.702355E-05
0.8	1.705519E-05	1.728046E-05	1.714246E-05
0.6	1.738158E-05	1.761059E-05	1.723907E-05
0.4	1.760264E-05	1.786029E-05	1.730740E-05
0.2	1.776886E-05	1.801621E-05	1.735032E-05
0	1.781452E-05	1.806926E-05	1.736291E-05
-0.2	1.773824E-05	1.801621E-05	1.735032E-05
-0.4	1.761924E-05	1.786029E-05	1.730740E-05
-0.6	1.738859E-05	1.761059E-05	1.723907E-05
-0.8	1.707548E-05	1.728046E-05	1.714246E-05
-1	1.673948E-05	1.688547E-05	1.702354E-05
-1.2	1.632367E-05	1.644150E-05	1.687671E-05
-1.4	1.585824E-05	1.596330E-05	1.649601E-05
-1.6	1.538227E-05	1.546369E-05	1.587785E-05
-1.8	1.490726E-05	1.495319E-05	1.521077E-05
-2	1.440885E-05	1.444013E-05	1.449202E-05
-2.2	1.391239E-05	1.393082E-05	1.373031E-05
-2.4	1.342143E-05	1.342990E-05	1.322225E-05
-2.6	1.293624E-05	1.294067E-05	1.283698E-05
-2.8	1.245984E-05	1.246536E-05	1.245467E-05
-3	1.201211E-05	1.200540E-05	1.207896E-05
-3.2	1.155962E-05	1.156161E-05	1.170706E-05
-3.4	1.111951E-05	1.113439E-05	1.105626E-05
-3.6	1.070065E-05	1.072377E-05	1.071194E-05
-3.8	1.033394E-05	1.032958E-05	1.037275E-05
-4	9.952270E-06	9.951494E-06	1.004036E-05

α = .9 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	1.197258E-05	1.195889E-05	1.213020E-05
3.8	1.242873E-05	1.245287E-05	1.251958E-05
3.6	1.299556E-05	1.297594E-05	1.293351E-05
3.4	1.357284E-05	1.353060E-05	1.336463E-05
3.2	1.415556E-05	1.411963E-05	1.436255E-05
3	1.471571E-05	1.474615E-05	1.473053E-05
2.8	1.538843E-05	1.541356E-05	1.514352E-05
2.6	1.615468E-05	1.612563E-05	1.559317E-05
2.4	1.692673E-05	1.688638E-05	1.608159E-05
2.2	1.773684E-05	1.770005E-05	1.708095E-05
2	1.858143E-05	1.857082E-05	1.889080E-05
1.8	1.950819E-05	1.950239E-05	2.059657E-05
1.6	2.051780E-05	2.049705E-05	2.218938E-05
1.4	2.155889E-05	2.155414E-05	2.366674E-05
1.2	2.264479E-05	2.266711E-05	2.452498E-05
1	2.368542E-05	2.381866E-05	2.475280E-05
0.8	2.472104E-05	2.497307E-05	2.494024E-05
0.6	2.562737E-05	2.606613E-05	2.509010E-05
0.4	2.633145E-05	2.699752E-05	2.519691E-05
0.2	2.680865E-05	2.763817E-05	2.525975E-05
0	2.697172E-05	2.786849E-05	2.528223E-05
-0.2	2.682529E-05	2.763817E-05	2.525975E-05
-0.4	2.632886E-05	2.699752E-05	2.519691E-05
-0.6	2.562939E-05	2.606613E-05	2.509010E-05
-0.8	2.470880E-05	2.497307E-05	2.494024E-05
-1	2.372376E-05	2.381866E-05	2.475280E-05
-1.2	2.264056E-05	2.266711E-05	2.452498E-05
-1.4	2.158597E-05	2.155414E-05	2.366674E-05
-1.6	2.051745E-05	2.049705E-05	2.218938E-05
-1.8	1.954219E-05	1.950239E-05	2.059657E-05
-2	1.858006E-05	1.857082E-05	1.889080E-05
-2.2	1.772286E-05	1.770005E-05	1.708095E-05
-2.4	1.689331E-05	1.688638E-05	1.608159E-05
-2.6	1.616602E-05	1.612563E-05	1.559317E-05
-2.8	1.533713E-05	1.541356E-05	1.514352E-05
-3	1.465371E-05	1.474615E-05	1.473053E-05
-3.2	1.411162E-05	1.411963E-05	1.436255E-05
-3.4	1.354107E-05	1.353060E-05	1.336463E-05
-3.6	1.298373E-05	1.297594E-05	1.293351E-05
-3.8	1.246870E-05	1.245287E-05	1.251958E-05
-4	1.193624E-05	1.195889E-05	1.213021E-05

α = .95 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	1.283215E-05	1.280968E-05	1.360138E-05
3.8	1.325573E-05	1.334812E-05	1.415077E-05
3.6	1.390975E-05	1.392094E-05	1.447752E-05
3.4	1.454385E-05	1.453175E-05	1.459219E-05
3.2	1.519401E-05	1.518471E-05	1.526923E-05
3	1.593758E-05	1.588473E-05	1.563603E-05
2.8	1.667755E-05	1.663759E-05	1.627055E-05
2.6	1.748288E-05	1.745017E-05	1.713204E-05
2.4	1.841137E-05	1.833076E-05	1.840033E-05
2.2	1.937899E-05	1.928940E-05	1.944874E-05
2	2.049286E-05	2.033838E-05	2.060269E-05
1.8	2.169463E-05	2.149290E-05	2.157591E-05
1.6	2.298912E-05	2.277181E-05	2.299180E-05
1.4	2.447285E-05	2.419848E-05	2.445580E-05
1.2	2.617139E-05	2.580135E-05	2.637543E-05
1	2.796725E-05	2.761292E-05	2.802536E-05
0.8	2.986005E-05	2.966279E-05	3.030104E-05
0.6	3.167546E-05	3.195143E-05	3.202039E-05
0.4	3.325014E-05	3.436588E-05	3.337348E-05
0.2	3.434532E-05	3.647477E-05	3.416824E-05
0	3.471710E-05	3.737966E-05	3.477887E-05
-0.2	3.436607E-05	3.647477E-05	3.416824E-05
-0.4	3.327657E-05	3.436588E-05	3.337348E-05
-0.6	3.170419E-05	3.195143E-05	3.202039E-05
-0.8	2.987556E-05	2.966279E-05	3.030104E-05
-1	2.790862E-05	2.761292E-05	2.802536E-05
-1.2	2.618255E-05	2.580135E-05	2.637543E-05
-1.4	2.445960E-05	2.419848E-05	2.445580E-05
-1.6	2.300404E-05	2.277181E-05	2.299180E-05
-1.8	2.165730E-05	2.149290E-05	2.157591E-05
-2	2.043855E-05	2.033838E-05	2.060269E-05
-2.2	1.941730E-05	1.928940E-05	1.944874E-05
-2.4	1.839264E-05	1.833076E-05	1.840033E-05
-2.6	1.749529E-05	1.745017E-05	1.713204E-05
-2.8	1.658201E-05	1.663759E-05	1.627055E-05
-3	1.586267E-05	1.588473E-05	1.563603E-05
-3.2	1.521299E-05	1.518471E-05	1.526923E-05
-3.4	1.451835E-05	1.453175E-05	1.459219E-05
-3.6	1.393286E-05	1.392094E-05	1.447752E-05
-3.8	1.333818E-05	1.334812E-05	1.415077E-05
-4	1.282223E-05	1.280968E-05	1.360138E-05

α = .96 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	1.290373E-05	1.296246E-05	1.386651E-05
3.8	1.346729E-05	1.350788E-05	1.442382E-05
3.6	1.409162E-05	1.408845E-05	1.474577E-05
3.4	1.474473E-05	1.470795E-05	1.484153E-05
3.2	1.538707E-05	1.537075E-05	1.542590E-05
3	1.608102E-05	1.608202E-05	1.575801E-05
2.8	1.689635E-05	1.684791E-05	1.640737E-05
2.6	1.774445E-05	1.767582E-05	1.728281E-05
2.4	1.865349E-05	1.857471E-05	1.858459E-05
2.2	1.966237E-05	1.955563E-05	1.970551E-05
2	2.079802E-05	2.063233E-05	2.092415E-05
1.8	2.203038E-05	2.182221E-05	2.186948E-05
1.6	2.343081E-05	2.314757E-05	2.334346E-05
1.4	2.503552E-05	2.463740E-05	2.490978E-05
1.2	2.679521E-05	2.632968E-05	2.701929E-05
1	2.881841E-05	2.827385E-05	2.884535E-05
0.8	3.096733E-05	3.053067E-05	3.151654E-05
0.6	3.315954E-05	3.315751E-05	3.355209E-05
0.4	3.503988E-05	3.612735E-05	3.516910E-05
0.2	3.635124E-05	3.900867E-05	3.614107E-05
0	3.680565E-05	4.037817E-05	3.689844E-05
-0.2	3.632814E-05	3.900867E-05	3.614107E-05
-0.4	3.504255E-05	3.612735E-05	3.516910E-05
-0.6	3.317841E-05	3.315751E-05	3.355209E-05
-0.8	3.097460E-05	3.053067E-05	3.151654E-05
-1	2.879722E-05	2.827385E-05	2.884535E-05
-1.2	2.679004E-05	2.632968E-05	2.701929E-05
-1.4	2.501855E-05	2.463740E-05	2.490978E-05
-1.6	2.335100E-05	2.314757E-05	2.334346E-05
-1.8	2.200617E-05	2.182221E-05	2.186948E-05
-2	2.081911E-05	2.063233E-05	2.092415E-05
-2.2	1.968191E-05	1.955563E-05	1.970551E-05
-2.4	1.868300E-05	1.857471E-05	1.858459E-05
-2.6	1.777769E-05	1.767582E-05	1.728281E-05
-2.8	1.688953E-05	1.684791E-05	1.640737E-05
-3	1.611665E-05	1.608202E-05	1.575801E-05
-3.2	1.539178E-05	1.537075E-05	1.542590E-05
-3.4	1.471962E-05	1.470795E-05	1.484153E-05
-3.6	1.409439E-05	1.408845E-05	1.474577E-05
-3.8	1.347075E-05	1.350788E-05	1.442382E-05
-4	1.295860E-05	1.296246E-05	1.386651E-05

α = .97 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	1.302408E-05	1.310883E-05	1.408524E-05
3.8	1.358891E-05	1.366048E-05	1.466596E-05
3.6	1.416437E-05	1.424796E-05	1.498495E-05
3.4	1.484619E-05	1.487513E-05	1.504788E-05
3.2	1.543697E-05	1.554657E-05	1.555893E-05
3	1.620234E-05	1.626768E-05	1.591487E-05
2.8	1.703645E-05	1.704490E-05	1.654420E-05
2.6	1.789731E-05	1.788604E-05	1.742220E-05
2.4	1.884383E-05	1.880067E-05	1.878294E-05
2.2	1.992521E-05	1.980067E-05	1.995930E-05
2	2.101163E-05	2.090104E-05	2.119856E-05
1.8	2.228520E-05	2.212109E-05	2.210546E-05
1.6	2.377126E-05	2.348617E-05	2.363927E-05
1.4	2.543193E-05	2.503034E-05	2.529969E-05
1.2	2.734725E-05	2.680058E-05	2.762427E-05
1	2.962091E-05	2.886321E-05	2.966250E-05
0.8	3.213488E-05	3.131317E-05	3.279989E-05
0.6	3.468340E-05	3.428157E-05	3.520424E-05
0.4	3.695489E-05	3.789948E-05	3.712838E-05
0.2	3.852778E-05	4.193357E-05	3.830463E-05
0	3.916007E-05	4.193357E-05	3.923115E-05
-0.2	3.858487E-05	4.193357E-05	3.830463E-05
-0.4	3.697471E-05	3.789948E-05	3.712838E-05
-0.6	3.463084E-05	3.428157E-05	3.520424E-05
-0.8	3.209060E-05	3.131317E-05	3.279989E-05
-1	2.964778E-05	2.886321E-05	2.966250E-05
-1.2	2.738819E-05	2.680058E-05	2.762427E-05
-1.4	2.545555E-05	2.503034E-05	2.529969E-05
-1.6	2.376300E-05	2.348617E-05	2.363927E-05
-1.8	2.236400E-05	2.212109E-05	2.210546E-05
-2	2.105126E-05	2.090104E-05	2.119856E-05
-2.2	1.992270E-05	1.980067E-05	1.995930E-05
-2.4	1.888275E-05	1.880067E-05	1.878294E-05
-2.6	1.798133E-05	1.788604E-05	1.742220E-05
-2.8	1.707408E-05	1.704490E-05	1.654420E-05
-3	1.631842E-05	1.626768E-05	1.591487E-05
-3.2	1.556492E-05	1.554657E-05	1.555893E-05
-3.4	1.485104E-05	1.487513E-05	1.504788E-05
-3.6	1.422640E-05	1.424796E-05	1.498495E-05
-3.8	1.360595E-05	1.366048E-05	1.466596E-05
-4	1.307983E-05	1.310883E-05	1.408524E-05

α = .98 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	1.325719E-05	1.324856E-05	1.427234E-05
3.8	1.375771E-05	1.380569E-05	1.486362E-05
3.6	1.439364E-05	1.439915E-05	1.516374E-05
3.4	1.507646E-05	1.503295E-05	1.520029E-05
3.2	1.576276E-05	1.571177E-05	1.575232E-05
3	1.646556E-05	1.644119E-05	1.606940E-05
2.8	1.722960E-05	1.722790E-05	1.669544E-05
2.6	1.806902E-05	1.808004E-05	1.759887E-05
2.4	1.897430E-05	1.900761E-05	1.899453E-05
2.2	2.008187E-05	2.002314E-05	2.014894E-05
2	2.126681E-05	2.114262E-05	2.144655E-05
1.8	2.265299E-05	2.238685E-05	2.228406E-05
1.6	2.410240E-05	2.378360E-05	2.383895E-05
1.4	2.583938E-05	2.537104E-05	2.559100E-05
1.2	2.785647E-05	2.720363E-05	2.816313E-05
1	3.033878E-05	2.936238E-05	3.043780E-05
0.8	3.327172E-05	3.197421E-05	3.412898E-05
0.6	3.631221E-05	3.524908E-05	3.699357E-05
0.4	3.910835E-05	3.954074E-05	3.929690E-05
0.2	4.111337E-05	4.522714E-05	4.072852E-05
0	4.181968E-05	4.953382E-05	4.185908E-05
-0.2	4.110456E-05	4.522714E-05	4.072852E-05
-0.4	3.908946E-05	3.954074E-05	3.929690E-05
-0.6	3.632048E-05	3.524908E-05	3.699357E-05
-0.8	3.328449E-05	3.197421E-05	3.412898E-05
-1	3.034648E-05	2.936238E-05	3.043780E-05
-1.2	2.791504E-05	2.720363E-05	2.816313E-05
-1.4	2.587241E-05	2.537104E-05	2.559100E-05
-1.6	2.416286E-05	2.378360E-05	2.383895E-05
-1.8	2.260418E-05	2.238685E-05	2.228406E-05
-2	2.134184E-05	2.114262E-05	2.144655E-05
-2.2	2.017299E-05	2.002314E-05	2.014894E-05
-2.4	1.911723E-05	1.900761E-05	1.899453E-05
-2.6	1.815668E-05	1.808004E-05	1.759887E-05
-2.8	1.729247E-05	1.722790E-05	1.669544E-05
-3	1.648262E-05	1.644119E-05	1.606940E-05
-3.2	1.575045E-05	1.571177E-05	1.575232E-05
-3.4	1.505962E-05	1.503295E-05	1.520029E-05
-3.6	1.442659E-05	1.439915E-05	1.516374E-05
-3.8	1.379029E-05	1.380569E-05	1.486362E-05
-4	1.325006E-05	1.324856E-05	1.427234E-05

α = .99 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	1.340996E-05	1.338147E-05	1.444399E-05
3.8	1.393848E-05	1.394326E-05	1.502373E-05
3.6	1.455723E-05	1.454178E-05	1.531432E-05
3.4	1.518339E-05	1.518109E-05	1.531376E-05
3.2	1.591200E-05	1.586597E-05	1.590031E-05
3	1.667202E-05	1.660213E-05	1.620699E-05
2.8	1.742486E-05	1.739640E-05	1.685357E-05
2.6	1.833966E-05	1.825713E-05	1.775701E-05
2.4	1.931142E-05	1.919464E-05	1.920615E-05
2.2	2.037983E-05	2.022190E-05	2.035465E-05
2	2.154169E-05	2.135551E-05	2.168021E-05
1.8	2.288984E-05	2.261728E-05	2.238050E-05
1.6	2.440808E-05	2.403657E-05	2.392773E-05
1.4	2.619963E-05	2.565431E-05	2.573515E-05
1.2	2.825888E-05	2.753000E-05	2.853025E-05
1	3.094801E-05	2.975488E-05	3.105381E-05
0.8	3.436775E-05	3.247921E-05	3.546268E-05
0.6	3.805100E-05	3.597609E-05	3.893416E-05
0.4	4.140264E-05	4.081812E-05	4.175437E-05
0.2	4.395641E-05	4.845551E-05	4.355728E-05
0	4.495971E-05	5.851216E-05	4.499075E-05
-0.2	4.395811E-05	4.845551E-05	4.355728E-05
-0.4	4.140725E-05	4.081812E-05	4.175437E-05
-0.6	3.803363E-05	3.597609E-05	3.893416E-05
-0.8	3.437237E-05	3.247921E-05	3.546268E-05
-1	3.098133E-05	2.975488E-05	3.105381E-05
-1.2	2.831011E-05	2.753000E-05	2.853025E-05
-1.4	2.620101E-05	2.565431E-05	2.573515E-05
-1.6	2.440503E-05	2.403657E-05	2.392773E-05
-1.8	2.291530E-05	2.261728E-05	2.238050E-05
-2	2.158594E-05	2.135551E-05	2.168021E-05
-2.2	2.037560E-05	2.022190E-05	2.035465E-05
-2.4	1.928318E-05	1.919464E-05	1.920615E-05
-2.6	1.835012E-05	1.825713E-05	1.775701E-05
-2.8	1.746078E-05	1.739640E-05	1.685357E-05
-3	1.665563E-05	1.660213E-05	1.620699E-05
-3.2	1.588636E-05	1.586597E-05	1.590031E-05
-3.4	1.521653E-05	1.518109E-05	1.531376E-05
-3.6	1.454151E-05	1.454178E-05	1.531432E-05
-3.8	1.397461E-05	1.394326E-05	1.502373E-05
-4	1.341704E-05	1.338147E-05	1.444399E-05

α = 1 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
4	1.343899E-05	1.350739E-05	1.357775E-05
3.8	1.402403E-05	1.407302E-05	1.414969E-05
3.6	1.464296E-05	1.467563E-05	1.475763E-05
3.4	1.525569E-05	1.531931E-05	1.539798E-05
3.2	1.594654E-05	1.600889E-05	1.610597E-05
3	1.681098E-05	1.675013E-05	1.686985E-05
2.8	1.754722E-05	1.754995E-05	1.769114E-05
2.6	1.849327E-05	1.841680E-05	1.849698E-05
2.4	1.948316E-05	1.936113E-05	1.958909E-05
2.2	2.052850E-05	2.039608E-05	2.049392E-05
2	2.174649E-05	2.153856E-05	2.186428E-05
1.8	2.304630E-05	2.281076E-05	2.233869E-05
1.6	2.460829E-05	2.424271E-05	2.380772E-05
1.4	2.633285E-05	2.587642E-05	2.564428E-05
1.2	2.850374E-05	2.777337E-05	2.867987E-05
1	3.133486E-05	3.002876E-05	3.147442E-05
0.8			3.683153E-05
0.6			4.110412E-05
0.4			4.460223E-05
0.2			4.690257E-05
0			4.874336E-05
-0.2			4.690257E-05
-0.4			4.460223E-05
-0.6			4.110412E-05
-0.8			3.683153E-05
-1	3.135016E-05	3.002876E-05	3.147442E-05
-1.2	2.843535E-05	2.777337E-05	2.867987E-05
-1.4	2.636926E-05	2.587642E-05	2.564428E-05
-1.6	2.458564E-05	2.424271E-05	2.380772E-05
-1.8	2.309541E-05	2.281076E-05	2.233869E-05
-2	2.177052E-05	2.153856E-05	2.186428E-05
-2.2	2.053526E-05	2.039608E-05	2.049392E-05
-2.4	1.948005E-05	1.936113E-05	1.958909E-05
-2.6	1.845224E-05	1.841680E-05	1.849698E-05
-2.8	1.760853E-05	1.754995E-05	1.769114E-05
-3	1.678358E-05	1.675013E-05	1.686985E-05
-3.2	1.604902E-05	1.600889E-05	1.610597E-05
-3.4	1.532478E-05	1.531931E-05	1.539798E-05
-3.6	1.466439E-05	1.467563E-05	1.475763E-05
-3.8	1.404917E-05	1.407302E-05	1.414969E-05
-4	1.341158E-05	1.350739E-05	1.357775E-05

C.2 Mutual Inductance Values for Not Equal Length Coils

	MagNet	Grover	Elliptical
α = .1 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	1.198418E-05	1.206351E-05	1.127399E-05
38	1.351237E-05	1.366964E-05	1.273294E-05
36	1.557008E-05	1.554916E-05	1.443370E-05
34	1.773775E-05	1.775107E-05	1.642286E-05
32	2.028478E-05	2.033489E-05	1.875630E-05
30	2.319233E-05	2.337010E-05	2.150061E-05
28	2.685447E-05	2.693453E-05	2.473432E-05
26	3.092595E-05	3.110976E-05	2.854853E-05
24	3.585916E-05	3.597410E-05	3.304616E-05
22	4.140218E-05	4.160583E-05	3.833877E-05
20	4.785354E-05	4.804528E-05	4.453892E-05
18	5.513813E-05	5.529302E-05	5.174562E-05
16	6.313616E-05	6.327812E-05	6.001966E-05
14	7.176115E-05	7.183707E-05	6.934568E-05
13	7.610116E-05	7.624771E-05	7.436507E-05
12	8.049848E-05	8.068872E-05	7.958055E-05
10	8.934389E-05	8.944742E-05	9.039375E-05
8	9.756430E-05	9.764153E-05	1.012172E-04
6	1.046567E-04	1.047444E-04	1.112362E-04
4	1.101442E-04	1.102518E-04	1.194613E-04
2	1.135947E-04	1.137459E-04	1.249011E-04
0	1.148601E-04	1.149455E-04	1.268077E-04
-2	1.136022E-04	1.137459E-04	1.249011E-04
-4	1.100897E-04	1.102518E-04	1.194613E-04
-6	1.044910E-04	1.047444E-04	1.112362E-04
-8	9.757017E-05	9.764153E-05	1.012172E-04
-10	8.925557E-05	8.944742E-05	9.039375E-05
-12	8.043050E-05	8.068872E-05	7.958055E-05
-13	7.600652E-05	7.624771E-05	7.436507E-05
-14	7.171649E-05	7.183707E-05	6.934568E-05
-16	6.310543E-05	6.327812E-05	6.001966E-05
-18	5.516328E-05	5.529302E-05	5.174562E-05
-20	4.789271E-05	4.804528E-05	4.453892E-05
-22	4.148053E-05	4.160583E-05	3.833877E-05
-24	3.586938E-05	3.597410E-05	3.304616E-05
-26	3.092246E-05	3.110976E-05	2.854853E-05
-28	2.686416E-05	2.693453E-05	2.473432E-05
-30	2.331153E-05	2.337010E-05	2.150061E-05
-32	2.020639E-05	2.033489E-05	1.875630E-05
-34	1.759200E-05	1.775106E-05	1.642286E-05
-36	1.557536E-05	1.554916E-05	1.443370E-05
-38	1.361231E-05	1.366964E-05	1.273294E-05
-40	1.201083E-05	1.206351E-05	1.127399E-05

α = .2 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	4.761195E-05	4.790450E-05	4.479294E-05
38	5.383146E-05	5.426068E-05	5.057098E-05
36	6.112117E-05	6.171718E-05	5.730450E-05
34	7.014380E-05	7.044060E-05	6.517771E-05
32	8.037025E-05	8.066373E-05	7.441190E-05
30	9.223720E-05	9.270004E-05	8.527152E-05
28	1.064613E-04	1.068372E-04	9.806960E-05
26	1.230207E-04	1.234099E-04	1.131713E-04
24	1.416194E-04	1.427593E-04	1.309927E-04
22	1.645846E-04	1.652141E-04	1.519908E-04
20	1.902626E-04	1.909487E-04	1.766368E-04
18	2.193098E-04	2.200443E-04	2.053631E-04
16	2.513207E-04	2.522330E-04	2.384691E-04
14	2.866015E-04	2.868779E-04	2.759723E-04
13	3.039191E-04	3.047824E-04	2.962496E-04
12	3.220275E-04	3.228756E-04	3.173937E-04
10	3.581976E-04	3.586366E-04	3.614946E-04
8	3.916024E-04	3.921956E-04	4.060268E-04
6	4.210412E-04	4.213812E-04	4.476318E-04
4	4.437613E-04	4.440628E-04	4.820848E-04
2	4.582185E-04	4.584662E-04	5.050272E-04
0	4.629909E-04	4.634047E-04	5.130988E-04
-2	4.572501E-04	4.584662E-04	5.050272E-04
-4	4.435421E-04	4.440628E-04	4.820848E-04
-6	4.203338E-04	4.213812E-04	4.476318E-04
-8	3.914401E-04	3.921956E-04	4.060268E-04
-10	3.579764E-04	3.586366E-04	3.614946E-04
-12	3.218253E-04	3.228756E-04	3.173937E-04
-13	3.037765E-04	3.047824E-04	2.962496E-04
-14	2.866100E-04	2.868779E-04	2.759723E-04
-16	2.515559E-04	2.522330E-04	2.384691E-04
-18	2.195403E-04	2.200443E-04	2.053631E-04
-20	1.902424E-04	1.909487E-04	1.766368E-04
-22	1.645475E-04	1.652141E-04	1.519908E-04
-24	1.422478E-04	1.427593E-04	1.309927E-04
-26	1.226677E-04	1.234099E-04	1.131713E-04
-28	1.065760E-04	1.068372E-04	9.806960E-05
-30	9.234616E-05	9.270004E-05	8.527152E-05
-32	8.045205E-05	8.066373E-05	7.441190E-05
-34	7.008193E-05	7.044060E-05	6.517771E-05
-36	6.020277E-05	6.171718E-05	5.730450E-05
-38	5.391277E-05	5.426068E-05	5.057098E-05
-40	4.763180E-05	4.790450E-05	4.479294E-05

α = .3 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	1.057758E-04	1.065968E-04	9.966003E-05
38	1.201325E-04	1.206323E-04	1.124466E-04
36	1.365922E-04	1.371035E-04	1.273390E-04
34	1.547040E-04	1.563706E-04	1.447435E-04
32	1.782875E-04	1.789559E-04	1.651497E-04
30	2.049589E-04	2.055637E-04	1.891443E-04
28	2.359665E-04	2.368781E-04	2.174265E-04
26	2.726195E-04	2.737332E-04	2.508185E-04
24	3.159608E-04	3.167874E-04	2.902700E-04
22	3.649001E-04	3.670224E-04	3.368443E-04
20	4.238856E-04	4.248858E-04	3.916740E-04
18	4.893965E-04	4.907000E-04	4.558609E-04
16	5.627200E-04	5.639654E-04	5.302872E-04
14	6.426408E-04	6.435175E-04	6.152940E-04
13	6.835177E-04	6.848258E-04	6.616053E-04
12	7.245145E-04	7.267311E-04	7.101844E-04
10	8.075833E-04	8.099529E-04	8.125472E-04
8	8.869425E-04	8.886055E-04	9.175028E-04
6	9.552953E-04	9.573176E-04	1.017187E-03
4	1.009899E-03	1.010901E-03	1.101053E-03
2	1.042949E-03	1.045010E-03	1.157618E-03
0	1.054717E-03	1.056778E-03	1.177662E-03
-2	1.042958E-03	1.045010E-03	1.157618E-03
-4	1.008585E-03	1.010901E-03	1.101053E-03
-6	9.544883E-04	9.573176E-04	1.017187E-03
-8	8.867374E-04	8.886055E-04	9.175028E-04
-10	8.074129E-04	8.099529E-04	8.125472E-04
-12	7.253566E-04	7.267311E-04	7.101844E-04
-13	6.836140E-04	6.848258E-04	6.616053E-04
-14	6.426135E-04	6.435175E-04	6.152940E-04
-16	5.630923E-04	5.639654E-04	5.302872E-04
-18	4.897719E-04	4.907000E-04	4.558609E-04
-20	4.232192E-04	4.248858E-04	3.916740E-04
-22	3.652272E-04	3.670224E-04	3.368443E-04
-24	3.145398E-04	3.167874E-04	2.902700E-04
-26	2.729142E-04	2.737332E-04	2.508185E-04
-28	2.364747E-04	2.368781E-04	2.174265E-04
-30	2.037110E-04	2.055637E-04	1.891443E-04
-32	1.769079E-04	1.789559E-04	1.651497E-04
-34	1.549932E-04	1.563706E-04	1.447435E-04
-36	1.361672E-04	1.371035E-04	1.273390E-04
-38	1.199317E-04	1.206323E-04	1.124466E-04
-40	1.067779E-04	1.065968E-04	9.966E-05

α = .4 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	1.839597E-04	1.862924E-04	1.744210E-04
38	2.088784E-04	2.105381E-04	1.966290E-04
36	2.380687E-04	2.391819E-04	2.224724E-04
34	2.721806E-04	2.726855E-04	2.526529E-04
32	3.112095E-04	3.118966E-04	2.880178E-04
30	3.558123E-04	3.581253E-04	3.295884E-04
28	4.110525E-04	4.126832E-04	3.785897E-04
26	4.745288E-04	4.770055E-04	4.364785E-04
24	5.498011E-04	5.522008E-04	5.049635E-04
22	6.382131E-04	6.405926E-04	5.860049E-04
20	7.409289E-04	7.430970E-04	6.817709E-04
18	8.581891E-04	8.606590E-04	7.945122E-04
16	9.909153E-04	9.928127E-04	9.262933E-04
14	1.135057E-03	1.138201E-03	1.078484E-03
13	1.211958E-03	1.214191E-03	1.162266E-03
12	1.289203E-03	1.291815E-03	1.250888E-03
10	1.444062E-03	1.447030E-03	1.440390E-03
8	1.592256E-03	1.595200E-03	1.639148E-03
6	1.723300E-03	1.725490E-03	1.832781E-03
4	1.826479E-03	1.827439E-03	1.999889E-03
2	1.889089E-03	1.892626E-03	2.115019E-03
0	1.910395E-03	1.915276E-03	2.156321E-03
-2	1.889189E-03	1.892626E-03	2.115019E-03
-4	1.822357E-03	1.827439E-03	1.999889E-03
-6	1.722651E-03	1.725490E-03	1.832781E-03
-8	1.591394E-03	1.595200E-03	1.639148E-03
-10	1.443495E-03	1.447030E-03	1.440390E-03
-12	1.289476E-03	1.291815E-03	1.250888E-03
-13	1.212499E-03	1.214191E-03	1.162266E-03
-14	1.135295E-03	1.138201E-03	1.078484E-03
-16	9.906614E-04	9.928127E-04	9.262933E-04
-18	8.584473E-04	8.606590E-04	7.945122E-04
-20	7.406547E-04	7.430970E-04	6.817709E-04
-22	6.374825E-04	6.405926E-04	5.860049E-04
-24	5.506019E-04	5.522008E-04	5.049635E-04
-26	4.753325E-04	4.770055E-04	4.364785E-04
-28	4.101200E-04	4.126832E-04	3.785897E-04
-30	3.547007E-04	3.581253E-04	3.295884E-04
-32	3.112429E-04	3.118966E-04	2.880178E-04
-34	2.719541E-04	2.726855E-04	2.526529E-04
-36	2.380913E-04	2.391819E-04	2.224724E-04
-38	2.102203E-04	2.105381E-04	1.966290E-04
-40	1.836246E-04	1.862924E-04	0.000174421

α = .5 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	2.846596E-04	2.850573E-04	2.671236E-04
38	3.223018E-04	3.217271E-04	3.007961E-04
36	3.606005E-04	3.653291E-04	3.399351E-04
34	4.138058E-04	4.160240E-04	3.855936E-04
32	4.724396E-04	4.751455E-04	4.390477E-04
30	5.446610E-04	5.451110E-04	5.018435E-04
28	6.262234E-04	6.278571E-04	5.758490E-04
26	7.229406E-04	7.256294E-04	6.633118E-04
24	8.370252E-04	8.403558E-04	7.669133E-04
22	9.707163E-04	9.762237E-04	8.898089E-04
20	1.128768E-03	1.134941E-03	1.035627E-03
18	1.313975E-03	1.319293E-03	1.208382E-03
16	1.524551E-03	1.529300E-03	1.412205E-03
14	1.761110E-03	1.764153E-03	1.650743E-03
13	1.886444E-03	1.888218E-03	1.783761E-03
12	2.011947E-03	2.016080E-03	1.925960E-03
10	2.268961E-03	2.274249E-03	2.235944E-03
8	2.516154E-03	2.523668E-03	2.571315E-03
6	2.741607E-03	2.745065E-03	2.910312E-03
4	2.917106E-03	2.919304E-03	3.214595E-03
2	3.024252E-03	3.031271E-03	3.431645E-03
0	3.063631E-03	3.070344E-03	3.511155E-03
-2	3.027182E-03	3.031271E-03	3.431645E-03
-4	2.914280E-03	2.919304E-03	3.214595E-03
-6	2.739514E-03	2.745065E-03	2.910312E-03
-8	2.518175E-03	2.523668E-03	2.571315E-03
-10	2.271363E-03	2.274249E-03	2.235944E-03
-12	2.011907E-03	2.016080E-03	1.925960E-03
-13	1.885648E-03	1.888218E-03	1.783761E-03
-14	1.761443E-03	1.764153E-03	1.650743E-03
-16	1.524520E-03	1.529300E-03	1.412205E-03
-18	1.314914E-03	1.319293E-03	1.208382E-03
-20	1.129971E-03	1.134941E-03	1.035627E-03
-22	9.727611E-04	9.762237E-04	8.898089E-04
-24	8.381343E-04	8.403558E-04	7.669133E-04
-26	7.231297E-04	7.256294E-04	6.633118E-04
-28	6.258752E-04	6.278571E-04	5.758490E-04
-30	5.442162E-04	5.451110E-04	5.018435E-04
-32	4.743991E-04	4.751455E-04	4.390477E-04
-34	4.128810E-04	4.160240E-04	3.855936E-04
-36	3.628181E-04	3.653291E-04	3.399351E-04
-38	3.167291E-04	3.217271E-04	3.007961E-04
-40	2.830640E-04	2.850573E-04	0.000267124

α = .6 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	3.958569E-04	4.000269E-04	3.753974E-04
38	4.463508E-04	4.507500E-04	4.221319E-04
36	5.035651E-04	5.111924E-04	4.763698E-04
34	5.737627E-04	5.813308E-04	5.395506E-04
32	6.562849E-04	6.630618E-04	6.134232E-04
30	7.567585E-04	7.598964E-04	7.001162E-04
28	8.695731E-04	8.744068E-04	8.022207E-04
26	1.005725E-03	1.010395E-03	9.228880E-04
24	1.166484E-03	1.169935E-03	1.065940E-03
22	1.356071E-03	1.360590E-03	1.235985E-03
20	1.579982E-03	1.585657E-03	1.438520E-03
18	1.844842E-03	1.850774E-03	1.679978E-03
16	2.153221E-03	2.157620E-03	1.967632E-03
14	2.506433E-03	2.508887E-03	2.309160E-03
13	2.694941E-03	2.696991E-03	2.502377E-03
12	2.889635E-03	2.893411E-03	2.711485E-03
10	3.292029E-03	3.294650E-03	3.178094E-03
8	3.686242E-03	3.689223E-03	3.703426E-03
6	4.037608E-03	4.043408E-03	4.262399E-03
4	4.318442E-03	4.323730E-03	4.795126E-03
2	4.497682E-03	4.504841E-03	5.197839E-03
0	4.562923E-03	4.568777E-03	5.350988E-03
-2	4.495511E-03	4.504841E-03	5.197839E-03
-4	4.316156E-03	4.323730E-03	4.795126E-03
-6	4.031985E-03	4.043408E-03	4.262399E-03
-8	3.681177E-03	3.689223E-03	3.703426E-03
-10	3.291990E-03	3.294650E-03	3.178094E-03
-12	2.888422E-03	2.893411E-03	2.711485E-03
-13	2.693621E-03	2.696991E-03	2.502377E-03
-14	2.506107E-03	2.508887E-03	2.309160E-03
-16	2.152724E-03	2.157620E-03	1.967632E-03
-18	1.848253E-03	1.850774E-03	1.679978E-03
-20	1.580953E-03	1.585657E-03	1.438520E-03
-22	1.356270E-03	1.360590E-03	1.235985E-03
-24	1.166525E-03	1.169935E-03	1.065940E-03
-26	1.005694E-03	1.010395E-03	9.228880E-04
-28	8.679817E-04	8.744068E-04	8.022207E-04
-30	7.560303E-04	7.598964E-04	7.001162E-04
-32	6.600015E-04	6.630618E-04	6.134232E-04
-34	5.741727E-04	5.813308E-04	5.395506E-04
-36	5.088506E-04	5.111924E-04	4.763698E-04
-38	4.516840E-04	4.507500E-04	4.221319E-04
-40	3.991871E-04	4.000269E-04	0.000375397

α = .7 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	5.242527E-04	5.284131E-04	4.965510E-04
38	5.883188E-04	5.937565E-04	5.574452E-04
36	6.671639E-04	6.725183E-04	6.279785E-04
34	7.605549E-04	7.636056E-04	7.099850E-04
32	8.654493E-04	8.689830E-04	8.056977E-04
30	9.913694E-04	9.941689E-04	9.178435E-04
28	1.139504E-03	1.142906E-03	1.049762E-03
26	1.312773E-03	1.319720E-03	1.205552E-03
24	1.519521E-03	1.526722E-03	1.390255E-03
22	1.770485E-03	1.777030E-03	1.610078E-03
20	2.068589E-03	2.075093E-03	1.872659E-03
18	2.424926E-03	2.431332E-03	2.187365E-03
16	2.847378E-03	2.852456E-03	2.565585E-03
14	3.337706E-03	3.350249E-03	3.020889E-03
13	3.607836E-03	3.621641E-03	3.282227E-03
12	3.884106E-03	3.909656E-03	3.568707E-03
10	4.502282E-03	4.508040E-03	4.224594E-03
8	5.102435E-03	5.110240E-03	4.998609E-03
6	5.647963E-03	5.658009E-03	5.879745E-03
4	6.086510E-03	6.093684E-03	6.798782E-03
2	6.369257E-03	6.377194E-03	7.567476E-03
0	6.464352E-03	6.478718E-03	7.882156E-03
-2	6.370863E-03	6.377194E-03	7.567476E-03
-4	6.086175E-03	6.093684E-03	6.798782E-03
-6	5.651898E-03	5.658009E-03	5.879745E-03
-8	5.102242E-03	5.110240E-03	4.998609E-03
-10	4.501953E-03	4.508040E-03	4.224594E-03
-12	3.889348E-03	3.909656E-03	3.568707E-03
-13	3.613954E-03	3.621641E-03	3.282227E-03
-14	3.327286E-03	3.350249E-03	3.020889E-03
-16	2.850241E-03	2.852456E-03	2.565585E-03
-18	2.424642E-03	2.431332E-03	2.187365E-03
-20	2.070314E-03	2.075093E-03	1.872659E-03
-22	1.770672E-03	1.777030E-03	1.610078E-03
-24	1.516270E-03	1.526722E-03	1.390255E-03
-26	1.311193E-03	1.319720E-03	1.205552E-03
-28	1.140000E-03	1.142906E-03	1.049762E-03
-30	9.828859E-04	9.941689E-04	9.178435E-04
-32	8.628720E-04	8.689830E-04	8.056977E-04
-34	7.599038E-04	7.636056E-04	7.099850E-04
-36	6.704647E-04	6.725183E-04	6.279785E-04
-38	5.933197E-04	5.937565E-04	0.000557445
-40	5.266780E-04	5.284131E-04	0.000496551

α = .8 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	6.674947E-04	6.669866E-04	6.276816E-04
38	7.494631E-04	7.473958E-04	7.033056E-04
36	8.403973E-04	8.451726E-04	7.906895E-04
34	9.537184E-04	9.574831E-04	8.920423E-04
32	1.085548E-03	1.086751E-03	1.010054E-03
30	1.238369E-03	1.240484E-03	1.148017E-03
28	1.418246E-03	1.423229E-03	1.309977E-03
26	1.634425E-03	1.640911E-03	1.500931E-03
24	1.889134E-03	1.895090E-03	1.727090E-03
22	2.195655E-03	2.205113E-03	1.996221E-03
20	2.570389E-03	2.577420E-03	2.318105E-03
18	3.018771E-03	3.029000E-03	2.705163E-03
16	3.565831E-03	3.573702E-03	3.173310E-03
14	4.236874E-03	4.247157E-03	3.743112E-03
13	4.615868E-03	4.624579E-03	4.074199E-03
12	5.023645E-03	5.034213E-03	4.441299E-03
10	5.901470E-03	5.904974E-03	5.302391E-03
8	6.794080E-03	6.807252E-03	6.368956E-03
6	7.624398E-03	7.640930E-03	7.683454E-03
4	8.296113E-03	8.305770E-03	9.241682E-03
2	8.730732E-03	8.740623E-03	1.080670E-02
0	8.879980E-03	8.899455E-03	1.156432E-02
-2	8.728585E-03	8.740623E-03	1.080670E-02
-4	8.295480E-03	8.305770E-03	9.241682E-03
-6	7.627790E-03	7.640930E-03	7.683454E-03
-8	6.799951E-03	6.807252E-03	6.368956E-03
-10	5.897879E-03	5.904974E-03	5.302391E-03
-12	5.017253E-03	5.034213E-03	4.441299E-03
-13	4.596524E-03	4.624579E-03	4.074199E-03
-14	4.230697E-03	4.247157E-03	3.743112E-03
-16	3.554942E-03	3.573702E-03	3.173310E-03
-18	3.023517E-03	3.029000E-03	2.705163E-03
-20	2.568108E-03	2.577420E-03	2.318105E-03
-22	2.187501E-03	2.205113E-03	1.996221E-03
-24	1.889649E-03	1.895090E-03	1.727090E-03
-26	1.625465E-03	1.640911E-03	1.500931E-03
-28	1.420381E-03	1.423229E-03	1.309977E-03
-30	1.238542E-03	1.240484E-03	1.148017E-03
-32	1.085721E-03	1.086751E-03	1.010054E-03
-34	9.567307E-04	9.574831E-04	8.920423E-04
-36	8.406469E-04	8.451726E-04	7.906895E-04
-38	7.467578E-04	7.473958E-04	7.033056E-04
-40	6.649847E-04	6.669866E-04	6.276816E-04

α = .9 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	8.037797E-04	8.124223E-04	7.657916E-04
38	9.023657E-04	9.071631E-04	8.561872E-04
36	1.015892E-03	1.023830E-03	9.603379E-04
34	1.148716E-03	1.156816E-03	1.080779E-03
32	1.297299E-03	1.308524E-03	1.220593E-03
30	1.484539E-03	1.489699E-03	1.383552E-03
28	1.697050E-03	1.704668E-03	1.574292E-03
26	1.950562E-03	1.961353E-03	1.798560E-03
24	2.244693E-03	2.258029E-03	2.063532E-03
22	2.598742E-03	2.622570E-03	2.378260E-03
20	3.051089E-03	3.062348E-03	2.754304E-03
18	3.590157E-03	3.601191E-03	3.206657E-03
16	4.259080E-03	4.255947E-03	3.755139E-03
14	5.088717E-03	5.126149E-03	4.426624E-03
13	5.600240E-03	5.635126E-03	4.819681E-03
12	6.191127E-03	6.205483E-03	5.258735E-03
10	7.466076E-03	7.456080E-03	6.306348E-03
8	8.795066E-03	8.802384E-03	7.653688E-03
6	1.004486E-02	1.006720E-02	9.437535E-03
4	1.106849E-02	1.107348E-02	1.188533E-02
2	1.172169E-02	1.173078E-02	1.524394E-02
0	1.193850E-02	1.198082E-02	1.783584E-02
-2	1.172194E-02	1.173078E-02	1.524394E-02
-4	1.106578E-02	1.107348E-02	1.188533E-02
-6	1.004484E-02	1.006720E-02	9.437535E-03
-8	8.793934E-03	8.802384E-03	7.653688E-03
-10	7.466144E-03	7.456080E-03	6.306348E-03
-12	6.194529E-03	6.205483E-03	5.258735E-03
-13	5.600470E-03	5.635126E-03	4.819681E-03
-14	5.090366E-03	5.126149E-03	4.426624E-03
-16	4.253314E-03	4.255947E-03	3.755139E-03
-18	3.589529E-03	3.601191E-03	3.206657E-03
-20	3.052316E-03	3.062348E-03	2.754304E-03
-22	2.609416E-03	2.622570E-03	2.378260E-03
-24	2.251341E-03	2.258029E-03	2.063532E-03
-26	1.947313E-03	1.961353E-03	1.798560E-03
-28	1.691136E-03	1.704668E-03	1.574292E-03
-30	1.486078E-03	1.489699E-03	1.383552E-03
-32	1.301637E-03	1.308524E-03	1.220593E-03
-34	1.148080E-03	1.156816E-03	1.080779E-03
-36	1.014244E-03	1.023830E-03	9.603379E-04
-38	9.025174E-04	9.071631E-04	8.561872E-04
-40	7.995406E-04	8.124223E-04	0.000765792

α = .95 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	8.807222E-04	8.867646E-04	8.365295E-04
38	9.880113E-04	9.880530E-04	9.341686E-04
36	1.108977E-03	1.113950E-03	1.046481E-03
34	1.245520E-03	1.257096E-03	1.176139E-03
32	1.414973E-03	1.419339E-03	1.326387E-03
30	1.609379E-03	1.612920E-03	1.501187E-03
28	1.835286E-03	1.843173E-03	1.705411E-03
26	2.101178E-03	2.117458E-03	1.945091E-03
24	2.427025E-03	2.432332E-03	2.227762E-03
22	2.814963E-03	2.821097E-03	2.562947E-03
20	3.284551E-03	3.288816E-03	2.962838E-03
18	3.857971E-03	3.870452E-03	3.443326E-03
16	4.567823E-03	4.587739E-03	4.025582E-03
14	5.499956E-03	5.523709E-03	4.738654E-03
13	6.082528E-03	6.100461E-03	5.156518E-03
12	6.761920E-03	6.775809E-03	5.623961E-03
10	8.286853E-03	8.305881E-03	6.743708E-03
8	9.905972E-03	9.918156E-03	8.198198E-03
6	1.143843E-02	1.144319E-02	1.016623E-02
4	1.270940E-02	1.271394E-02	1.301656E-02
2	1.350750E-02	1.350944E-02	1.767227E-02
0	1.375893E-02	1.376425E-02	2.392299E-02
-2	1.350915E-02	1.350942E-02	1.767227E-02
-4	1.271010E-02	1.271394E-02	1.301656E-02
-6	1.143895E-02	1.144319E-02	1.016623E-02
-8	9.906274E-03	9.918156E-03	8.198198E-03
-10	8.293313E-03	8.305881E-03	6.743708E-03
-12	6.760968E-03	6.775809E-03	5.623961E-03
-13	6.080461E-03	6.100461E-03	5.156518E-03
-14	5.499661E-03	5.523709E-03	4.738654E-03
-16	4.576983E-03	4.587739E-03	4.025582E-03
-18	3.857860E-03	3.870452E-03	3.443326E-03
-20	3.283828E-03	3.288816E-03	2.962838E-03
-22	2.811438E-03	2.821097E-03	2.562947E-03
-24	2.420567E-03	2.432332E-03	2.227762E-03
-26	2.097581E-03	2.117458E-03	1.945091E-03
-28	1.835277E-03	1.843173E-03	1.705411E-03
-30	1.612999E-03	1.612920E-03	1.501187E-03
-32	1.418048E-03	1.419339E-03	1.326387E-03
-34	1.252234E-03	1.257096E-03	1.176139E-03
-36	1.109562E-03	1.113950E-03	1.046481E-03
-38	9.765474E-04	9.880530E-04	9.341686E-04
-40	8.666347E-04	8.867646E-04	0.00083653

α = .96 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	8.937737E-04	9.023040E-04	8.507649E-04
38	1.006385E-03	1.004466E-03	9.498336E-04
36	1.125663E-03	1.131992E-03	1.063752E-03
34	1.265200E-03	1.277564E-03	1.195215E-03
32	1.429195E-03	1.441538E-03	1.347499E-03
30	1.632495E-03	1.637180E-03	1.524600E-03
28	1.864402E-03	1.870597E-03	1.731428E-03
26	2.134983E-03	2.147964E-03	1.974065E-03
24	2.458869E-03	2.465957E-03	2.260107E-03
22	2.860312E-03	2.858990E-03	2.599152E-03
20	3.328395E-03	3.332028E-03	3.003496E-03
18	3.909234E-03	3.920937E-03	3.489166E-03
16	4.638023E-03	4.646877E-03	4.077540E-03
14	5.556217E-03	5.595954E-03	4.797988E-03
13	6.165558E-03	6.186683E-03	5.220170E-03
12	6.868805E-03	6.884946E-03	5.692482E-03
10	8.461327E-03	8.476383E-03	6.824276E-03
8	1.013956E-02	1.015127E-02	8.295975E-03
6	1.173423E-02	1.173835E-02	1.029249E-02
4	1.306317E-02	1.306770E-02	1.320469E-02
2	1.389328E-02	1.389438E-02	1.809526E-02
0	1.414919E-02	1.415414E-02	2.584203E-02
-2	1.389079E-02	1.389435E-02	1.809526E-02
-4	1.306355E-02	1.306770E-02	1.320469E-02
-6	1.173452E-02	1.173835E-02	1.029249E-02
-8	1.013405E-02	1.015127E-02	8.295975E-03
-10	8.464339E-03	8.476383E-03	6.824276E-03
-12	6.871689E-03	6.884946E-03	5.692482E-03
-13	6.163227E-03	6.186683E-03	5.220170E-03
-14	5.560627E-03	5.595954E-03	4.797988E-03
-16	4.627204E-03	4.646877E-03	4.077540E-03
-18	3.900113E-03	3.920937E-03	3.489166E-03
-20	3.329165E-03	3.332028E-03	3.003496E-03
-22	2.854412E-03	2.858990E-03	2.599152E-03
-24	2.456615E-03	2.465957E-03	2.260107E-03
-26	2.126959E-03	2.147964E-03	1.974065E-03
-28	1.856609E-03	1.870597E-03	1.731428E-03
-30	1.635237E-03	1.637180E-03	1.524600E-03
-32	1.440651E-03	1.441538E-03	1.347499E-03
-34	1.274695E-03	1.277564E-03	1.195215E-03
-36	1.130081E-03	1.131992E-03	1.063752E-03
-38	1.004948E-03	1.004466E-03	9.498336E-04
-40	8.909623E-04	9.023040E-04	0.000850765

α = .97 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	8.948488E-04	9.160348E-04	8.650227E-04
38	1.025810E-03	1.020585E-03	9.655138E-04
36	1.145431E-03	1.149470E-03	1.081027E-03
34	1.292033E-03	1.296702E-03	1.214283E-03
32	1.458750E-03	1.463495E-03	1.368584E-03
30	1.654530E-03	1.661983E-03	1.547959E-03
28	1.889917E-03	1.897107E-03	1.757358E-03
26	2.155238E-03	2.179207E-03	2.002907E-03
24	2.494269E-03	2.500648E-03	2.292260E-03
22	2.892218E-03	2.897003E-03	2.635084E-03
20	3.371163E-03	3.374699E-03	3.043766E-03
18	3.958152E-03	3.970896E-03	3.534460E-03
16	4.693808E-03	4.703568E-03	4.128725E-03
14	5.651374E-03	5.664039E-03	4.856211E-03
13	6.239481E-03	6.269852E-03	5.282479E-03
12	6.977847E-03	6.991849E-03	5.759365E-03
10	8.631880E-03	8.646880E-03	6.902334E-03
8	1.037256E-02	1.038754E-02	8.389650E-03
6	1.203011E-02	1.203953E-02	1.041131E-02
4	1.342286E-02	1.343046E-02	1.337667E-02
2	1.428762E-02	1.428981E-02	1.847246E-02
0	1.454916E-02	1.455385E-02	2.829100E-02
-2	1.428838E-02	1.428981E-02	1.847246E-02
-4	1.342536E-02	1.343046E-02	1.337667E-02
-6	1.203269E-02	1.203953E-02	1.041131E-02
-8	1.037446E-02	1.038754E-02	8.389650E-03
-10	8.633813E-03	8.646880E-03	6.902334E-03
-12	6.979100E-03	6.991849E-03	5.759365E-03
-13	6.257592E-03	6.269852E-03	5.282479E-03
-14	5.648766E-03	5.664039E-03	4.856211E-03
-16	4.685170E-03	4.703568E-03	4.128725E-03
-18	3.957167E-03	3.970896E-03	3.534460E-03
-20	3.371872E-03	3.374699E-03	3.043766E-03
-22	2.887028E-03	2.897003E-03	2.635084E-03
-24	2.488262E-03	2.500648E-03	2.292260E-03
-26	2.168769E-03	2.179207E-03	2.002907E-03
-28	1.891296E-03	1.897107E-03	1.757358E-03
-30	1.661493E-03	1.661983E-03	1.547959E-03
-32	1.461525E-03	1.463495E-03	1.368584E-03
-34	1.294326E-03	1.296702E-03	1.214283E-03
-36	1.144745E-03	1.149470E-03	1.081027E-03
-38	1.012327E-03	1.020585E-03	9.655138E-04
-40	9.117829E-04	9.160348E-04	0.000865023

α = .98 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	9.100701E-04	9.310337E-04	8.793004E-04
38	1.019019E-03	1.036152E-03	9.812058E-04
36	1.162331E-03	1.168405E-03	1.098304E-03
34	1.308277E-03	1.316998E-03	1.233338E-03
32	1.486492E-03	1.485501E-03	1.389637E-03
30	1.683190E-03	1.686560E-03	1.571261E-03
28	1.918714E-03	1.924856E-03	1.783195E-03
26	2.185270E-03	2.208784E-03	2.031610E-03
24	2.524986E-03	2.533679E-03	2.324211E-03
22	2.929601E-03	2.934604E-03	2.670729E-03
20	3.412537E-03	3.417048E-03	3.083633E-03
18	4.007125E-03	4.019318E-03	3.579189E-03
16	4.750097E-03	4.759253E-03	4.179112E-03
14	5.707891E-03	5.729336E-03	4.913289E-03
13	6.327962E-03	6.348462E-03	5.343404E-03
12	7.086151E-03	7.095375E-03	5.824563E-03
10	8.804117E-03	8.818382E-03	6.977804E-03
8	1.060986E-02	1.062734E-02	8.479080E-03
6	1.233765E-02	1.234645E-02	1.052236E-02
4	1.379482E-02	1.380290E-02	1.353128E-02
2	1.469158E-02	1.469597E-02	1.879192E-02
0	1.495801E-02	1.496265E-02	3.170165E-02
-2	1.469098E-02	1.469594E-02	1.879192E-02
-4	1.379412E-02	1.380290E-02	1.353128E-02
-6	1.233823E-02	1.234645E-02	1.052236E-02
-8	1.061325E-02	1.062734E-02	8.479080E-03
-10	8.805043E-03	8.818382E-03	6.977804E-03
-12	7.084385E-03	7.095375E-03	5.824563E-03
-13	6.336622E-03	6.348462E-03	5.343404E-03
-14	5.705137E-03	5.729336E-03	4.913289E-03
-16	4.742600E-03	4.759253E-03	4.179112E-03
-18	4.003865E-03	4.019318E-03	3.579189E-03
-20	3.413534E-03	3.417048E-03	3.083633E-03
-22	2.927092E-03	2.934604E-03	2.670729E-03
-24	2.512355E-03	2.533679E-03	2.324211E-03
-26	2.189695E-03	2.208784E-03	2.031610E-03
-28	1.923325E-03	1.924856E-03	1.783195E-03
-30	1.677499E-03	1.686560E-03	1.571261E-03
-32	1.484438E-03	1.485501E-03	1.389637E-03
-34	1.306980E-03	1.316998E-03	1.233338E-03
-36	1.163482E-03	1.168405E-03	1.098304E-03
-38	1.019869E-03	1.036152E-03	0.000981206
-40	9.265695E-04	9.310337E-04	0.0008793

α = .99 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	9.295118E-04	9.447513E-04	8.935949E-04
38	1.044652E-03	1.052284E-03	9.969065E-04
36	1.176190E-03	1.185886E-03	1.115578E-03
34	1.331047E-03	1.336078E-03	1.252375E-03
32	1.507447E-03	1.507399E-03	1.410651E-03
30	1.707935E-03	1.711278E-03	1.594497E-03
28	1.947880E-03	1.951147E-03	1.808932E-03
26	2.225768E-03	2.239778E-03	2.060165E-03
24	2.563633E-03	2.567994E-03	2.355949E-03
22	2.955094E-03	2.972029E-03	2.706075E-03
20	3.433219E-03	3.458903E-03	3.123084E-03
18	4.054152E-03	4.066995E-03	3.623335E-03
16	4.789111E-03	4.812119E-03	4.228676E-03
14	5.766369E-03	5.788640E-03	4.969191E-03
13	6.402465E-03	6.422001E-03	5.402908E-03
12	7.184747E-03	7.195017E-03	5.888029E-03
10	8.978083E-03	8.989520E-03	7.050617E-03
8	1.085555E-02	1.087000E-02	8.564142E-03
6	1.265416E-02	1.265980E-02	1.062536E-02
4	1.417140E-02	1.418591E-02	1.366753E-02
2	1.510698E-02	1.511460E-02	1.904312E-02
0	1.537402E-02	1.538206E-02	3.744778E-02
-2	1.510566E-02	1.511457E-02	1.904312E-02
-4	1.417187E-02	1.418591E-02	1.366753E-02
-6	1.265262E-02	1.265980E-02	1.062536E-02
-8	1.085644E-02	1.087000E-02	8.564142E-03
-10	8.973307E-03	8.989520E-03	7.050617E-03
-12	7.188103E-03	7.195017E-03	5.888029E-03
-13	6.408015E-03	6.422001E-03	5.402908E-03
-14	5.780309E-03	5.788640E-03	4.969191E-03
-16	4.800470E-03	4.812119E-03	4.228676E-03
-18	4.055700E-03	4.066995E-03	3.623335E-03
-20	3.439131E-03	3.458903E-03	3.123084E-03
-22	2.959422E-03	2.972029E-03	2.706075E-03
-24	2.541634E-03	2.567994E-03	2.355949E-03
-26	2.222498E-03	2.239778E-03	2.060165E-03
-28	1.945339E-03	1.951147E-03	1.808932E-03
-30	1.706927E-03	1.711278E-03	1.594497E-03
-32	1.504754E-03	1.507399E-03	1.410651E-03
-34	1.331362E-03	1.336078E-03	1.252375E-03
-36	1.171044E-03	1.185886E-03	1.115578E-03
-38	1.046460E-03	1.052284E-03	0.000996906
-40	9.363690E-04	9.447513E-04	0.000893595

α = 1 S Distance	Mutual Inductance	Mutual Inductance	Mutual Inductance
40	9.439264E-04	9.975947E-04	9.079035E-04
38	1.060670E-03	1.090726E-03	1.012613E-03
36	1.196176E-03	1.208599E-03	1.132845E-03
34	1.337836E-03	1.360740E-03	1.271389E-03
32	1.528219E-03	1.515257E-03	1.431623E-03
30	1.734263E-03	1.707141E-03	1.617663E-03
28	1.973248E-03	1.948071E-03	1.834561E-03
26	2.261701E-03	2.232452E-03	2.088564E-03
24	2.598018E-03	2.570772E-03	2.387467E-03
22	3.007432E-03	2.998263E-03	2.741111E-03
20	3.496388E-03	3.503801E-03	3.162102E-03
18	4.097637E-03	4.107540E-03	3.666879E-03
16	4.852545E-03	4.863692E-03	4.277396E-03
14	5.835867E-03	5.834899E-03	5.023885E-03
13	6.435390E-03	6.488996E-03	5.460955E-03
12			
10			
8			
6			
4			
2			
0			
-2			
-4			
-6			
-8			
-10			
-12			
-13	6.472722E-03	6.488996E-03	5.460955E-03
-14	5.837453E-03	5.834772E-03	5.023885E-03
-16	4.850338E-03	4.863628E-03	4.277396E-03
-18	4.102500E-03	4.107507E-03	3.666879E-03
-20	3.490836E-03	3.503801E-03	3.162102E-03
-22	3.000238E-03	2.998263E-03	2.741111E-03
-24	2.598718E-03	2.570808E-03	2.387467E-03
-26	2.257006E-03	2.232452E-03	2.088564E-03
-28	1.972386E-03	1.948071E-03	1.834561E-03
-30	1.735235E-03	1.707141E-03	1.617663E-03
-32	1.527378E-03	1.515301E-03	1.431623E-03
-34	1.340124E-03	1.360786E-03	1.271389E-03
-36	1.200792E-03	1.208599E-03	1.132845E-03
-38	1.071054E-03	1.090776E-03	0.001012613
-40	9.487094E-04	9.975947E-04	0.000907904

APPENDIX D PERCENT ERROR VALUES

D.1 Percent Error for Equal Coil Lengths

	Grover vs. Elliptical	Grover vs. MagNet
S Distance	% Error	% Error
α = .1		
4	0.115785572	0.501427166
3.8	0.147927989	0.635703492
3.6	0.143505524	0.283392412
3.4	0.165236575	0.773977153
3.2	0.138828533	0.692156668
3	0.139463	0.25636904
2.8	0.117092636	0.757619637
2.6	0.130380569	0.846477002
2.4	0.131222725	0.364766004
2.2	0.142397138	0.321438253
2	0.152104177	0.333405828
1.8	0.180636027	0.325832927
1.6	0.195496881	0.632971043
1.4	0.207025151	0.495093092
1.2	0.225457254	0.675357145
1	0.250239518	0.643219807
0.8	0.250414865	0.554431962
0.6	0.266452788	0.559477924
0.4	0.268144701	0.661079038
0.2	0.275099441	0.871323425
0	0.267425476	1.053860991
-0.2	0.275099441	0.342598612
-0.4	0.268144701	0.362338517
-0.6	0.266452788	0.38909069
-0.8	0.250414865	0.381046442
-1	0.250239518	0.572464341
-1.2	0.225457254	0.701169497
-1.4	0.207025151	0.628754841
-1.6	0.195496881	0.706545638
-1.8	0.180636027	0.193233044
-2	0.152104177	0.331769606
-2.2	0.142397138	0.326533986
-2.4	0.131222725	0.444225892
-2.6	0.130380569	0.264523465
-2.8	0.117092636	0.284982661
-3	0.139463	0.263256106
-3.2	0.138828533	0.792259041
-3.4	0.165236575	0.699789532
-3.6	0.143505524	0.242347586
-3.8	0.147927989	0.277797582
-4	0.115722341	0.363418055

α = .2 S Distance	% Error	% Error
4	0.025917256	0.290271628
3.8	0.023325464	0.362052849
3.6	0.102636383	0.389749817
3.4	0.158722219	0.715310203
3.2	0.118804751	0.817942758
3	0.121464747	0.759164982
2.8	0.125708614	0.651559208
2.6	0.12909818	1.18660186
2.4	0.153561	0.896771673
2.2	0.164688284	0.243331545
2	0.181875115	0.327667505
1.8	0.202691408	0.352605412
1.6	0.20483691	0.529930471
1.4	0.187224576	0.527092573
1.2	0.220791925	0.601059067
1	0.252081551	0.782407368
0.8	0.269830712	0.35133667
0.6	0.293597433	0.575647935
0.4	0.332080589	1.05864424
0.2	0.335370423	0.805428868
0	0.333394841	0.73160233
-0.2	0.335370423	0.2085706
-0.4	0.332080589	0.348815094
-0.6	0.293597433	0.443011409
-0.8	0.269830712	0.530753268
-1	0.252081551	0.952080317
-1.2	0.220791925	0.448822139
-1.4	0.187224576	0.811843514
-1.6	0.20483691	0.437313407
-1.8	0.202691408	0.325328716
-2	0.181875115	0.630273932
-2.2	0.164688284	0.32260086
-2.4	0.153561	0.7319696
-2.6	0.12909818	0.30629456
-2.8	0.125708614	0.291003546
-3	0.121464747	0.199599487
-3.2	0.118804751	0.006047821
-3.4	0.158722219	0.286746877
-3.6	0.102636383	0.039496424
-3.8	0.023325464	0.594030738
-4	0.025917256	0.800660817

α = .3 S Distance	% Error	% Error
4	0.170001334	0.236822747
3.8	0.142776982	0.104152747
3.6	0.132686013	0.174273785
3.4	0.123797453	0.219170566
3.2	0.054158229	0.402696298
3	3.64487E-06	0.568911911
2.8	0.05634419	0.279860434
2.6	0.121930237	0.105497445
2.4	0.161065669	0.206534455
2.2	0.216709003	0.213245307
2	0.187692124	0.389322101
1.8	0.177208065	0.214173886
1.6	0.140064552	0.470731194
1.4	0.134446646	0.392872753
1.2	0.165876944	0.868476794
1	0.240568233	0.493021602
0.8	0.294545076	0.469876084
0.6	0.355957081	0.410718996
0.4	0.374340413	0.548048238
0.2	0.397362917	0.209469544
0	0.405171174	0.904664957
-0.2	0.397362917	0.459632908
-0.4	0.374340413	0.355518659
-0.6	0.355957081	0.40229596
-0.8	0.294545076	0.514667392
-1	0.240568233	0.870734886
-1.2	0.165876944	0.438083209
-1.4	0.134446646	0.339306338
-1.6	0.140064552	0.809165384
-1.8	0.177208065	0.601491183
-2	0.187692124	0.194616813
-2.2	0.216709003	0.213510658
-2.4	0.161065669	0.593721337
-2.6	0.121930237	0.254910292
-2.8	0.05634419	0.319422126
-3	3.64487E-06	1.161605696
-3.2	0.054158229	0.841384093
-3.4	0.123797453	1.193814681
-3.6	0.132686013	0.348332828
-3.8	0.142776982	0.38850041
-4	0.169931579	0.30255259

α = .4 S Distance	% Error	% Error
4	0.04079529	0.342601966
3.8	0.014172423	0.553358992
3.6	0.084711841	0.295908521
3.4	0.165505026	0.268477702
3.2	0.046610517	0.776878151
3	0.030988973	0.468027884
2.8	0.12721925	0.739030818
2.6	0.223351907	0.154139354
2.4	0.303483164	0.118157791
2.2	0.335046866	0.112556935
2	0.239103584	0.508805624
1.8	0.154240465	0.305314136
1.6	0.072855702	0.868490971
1.4	0.007305556	0.874193603
1.2	0.049065798	0.878762459
1	0.197187612	0.425770219
0.8	0.308467416	0.691850245
0.6	0.408121742	0.401512323
0.4	0.474594838	0.222908444
0.2	0.524580541	0.110148968
0	0.528853232	0.406426264
-0.2	0.524580541	0.338724601
-0.4	0.474594838	0.249756448
-0.6	0.408121742	0.236082013
-0.8	0.308467416	0.946675266
-1	0.197187612	0.731724527
-1.2	0.049065798	0.702778707
-1.4	0.007305556	0.873740363
-1.6	0.072855702	1.140669051
-1.8	0.154240465	0.617907527
-2	0.239103584	0.336333665
-2.2	0.335046866	0.120190714
-2.4	0.303483164	0.083349908
-2.6	0.223351907	0.134183075
-2.8	0.12721925	0.373646534
-3	0.030988973	0.750802507
-3.2	0.046610517	0.753309423
-3.4	0.165505026	0.217802432
-3.6	0.084711841	0.187562086
-3.8	0.014172423	0.253282964
-4	0.04087338	0.385904005

α = .5 S Distance	% Error	% Error
4	0.17875962	0.875400232
3.8	0.097626565	0.757564655
3.6	0.015109676	0.213662718
3.4	0.100199998	1.223898401
3.2	0.29180279	0.623666538
3	0.102616574	0.657678622
2.8	0.079089455	0.590728667
2.6	0.276755747	0.52912151
2.4	0.448571376	0.029986519
2.2	0.521871375	0.050417512
2	0.306012999	0.19232068
1.8	0.126009226	0.521870308
1.6	0.043191993	0.81872359
1.4	0.186464209	0.844404062
1.2	0.117717254	1.01586862
1	0.151627407	0.884105207
0.8	0.373217378	1.020008281
0.6	0.550110996	0.534842989
0.4	0.695369449	0.585322151
0.2	0.769589096	0.084152438
0	0.788553741	0.031287223
-0.2	0.769589096	0.175785152
-0.4	0.695369449	0.087011853
-0.6	0.550110996	0.149277169
-0.8	0.373217378	0.698455454
-1	0.151627407	0.873308518
-1.2	0.117717254	0.797131687
-1.4	0.186464209	0.86940695
-1.6	0.043191993	0.999091362
-1.8	0.126009226	0.50176489
-2	0.306012999	0.136245161
-2.2	0.521871375	0.091427989
-2.4	0.448571376	0.017544424
-2.6	0.276755747	0.129223254
-2.8	0.079089455	0.521998322
-3	0.102616574	0.79293304
-3.2	0.29180279	0.64122099
-3.4	0.100199998	0.432009182
-3.6	0.015109676	0.35700802
-3.8	0.097626565	0.286215651
-4	0.17873473	0.487002001

α = .6 S Distance	% Error	% Error
4	0.356902406	0.412567934
3.8	0.20062495	1.329975642
3.6	0.005722119	0.232900043
3.4	0.238660074	0.487565316
3.2	0.442255159	1.022644546
3	0.186372731	0.493689109
2.8	0.097731665	0.477516594
2.6	0.363033479	0.150330525
2.4	0.616499717	0.146412831
2.2	0.699357643	0.166304708
2	0.295940321	0.276173808
1.8	0.058371596	0.710994855
1.6	0.32445561	0.938473679
1.4	0.566860974	1.403191103
1.2	0.438008828	1.238533486
1	0.039388369	1.12631327
0.8	0.454240762	0.506452205
0.6	0.768716978	0.447281238
0.4	1.008955135	0.224007807
0.2	1.159040293	0.641056601
0	1.206796609	0.308704488
-0.2	1.159040293	0.337581214
-0.4	1.008955135	0.022725738
-0.6	0.768716978	0.643896215
-0.8	0.454240762	0.997732806
-1	0.039388369	0.908571819
-1.2	0.438008828	1.361731118
-1.4	0.566860974	1.644530258
-1.6	0.32445561	1.20341203
-1.8	0.058371596	0.61431708
-2	0.295940321	0.253202878
-2.2	0.699357643	0.263919385
-2.4	0.616499717	0.014580771
-2.6	0.363033479	0.268141544
-2.8	0.097731665	0.483768314
-3	0.186372731	0.498390825
-3.2	0.442255159	1.122253565
-3.4	0.238660074	0.09241766
-3.6	0.005722119	0.30673973
-3.8	0.20062495	0.725180492
-4	0.356936969	0.954389023

α = .7 S Distance	% Error	% Error
4	0.552841145	0.702847823
3.8	0.260434409	0.628535354
3.6	0.036767187	0.674370391
3.4	0.406529339	0.262681894
3.2	0.793639404	1.037504717
3	0.431517613	0.968080173
2.8	0.063750558	0.326568864
2.6	0.345083354	0.12412528
2.4	0.751912543	0.24039091
2.2	0.834625017	0.269421843
2	0.086336163	0.256078399
1.8	0.508199608	0.973346787
1.6	0.963701631	1.717640694
1.4	1.330164493	2.116607082
1.2	1.009299156	2.366457193
1	0.127835256	1.292709468
0.8	0.648060469	0.52776346
0.6	1.253768816	0.015983801
0.4	1.726347855	0.404768263
0.2	1.9873207	1.09041024
0	2.091197394	1.090236453
-0.2	1.9873207	0.971161268
-0.4	1.726347855	0.62884506
-0.6	1.253768816	0.102695409
-0.8	0.648060469	0.446545989
-1	0.127835256	1.299628238
-1.2	1.009299156	1.854307559
-1.4	1.330164493	2.058367112
-1.6	0.963701631	1.629436482
-1.8	0.508199608	0.986406011
-2	0.086336163	0.317623339
-2.2	0.834625017	0.426471026
-2.4	0.751912543	0.319390111
-2.6	0.345083354	0.064645947
-2.8	0.063750558	0.580922068
-3	0.431517613	0.719505518
-3.2	0.793639404	1.035940149
-3.4	0.406494537	0.256401899
-3.6	0.036767187	0.323533009
-3.8	0.260384867	0.424917673
-4	0.552892359	0.779175738

α = .8 S Distance	% Error	% Error
4	0.892987878	1.341509046
3.8	0.417903624	0.409321791
3.6	0.11029116	0.011493283
3.4	0.701666161	0.664235286
3.2	1.258002443	1.193296095
3	0.61274479	0.599735774
2.8	0.085729109	0.060987845
2.6	0.801246477	0.548761944
2.4	1.546167395	1.472379002
2.2	1.439302278	1.333108815
2	0.359359391	0.554271363
1.8	1.722573167	2.092087114
1.6	2.67830667	3.192440192
1.4	3.337084192	3.974788572
1.2	2.647028577	3.482078856
1	0.817737142	1.872098692
0.8	0.798596299	0.511711127
0.6	2.109644921	0.81991119
0.4	3.095633043	1.677243467
0.2	3.696039876	2.35544376
0	3.90910153	2.535056345
-0.2	3.696039876	2.186911623
-0.4	3.095633043	1.769885936
-0.6	2.109644921	0.859852528
-0.8	0.798596299	0.392259669
-1	0.81767792	1.69696331
-1.2	2.647028577	3.387933291
-1.4	3.337084192	4.021707736
-1.6	2.67830667	3.221751654
-1.8	1.722573167	2.035990457
-2	0.359359391	0.577242003
-2.2	1.439302278	1.308770569
-2.4	1.546167395	1.484008692
-2.6	0.801246477	0.767272526
-2.8	0.085729109	0.041483735
-3	0.61274479	0.556532232
-3.2	1.258002443	1.275514914
-3.4	0.701666161	0.568825801
-3.6	0.11029116	0.105475171
-3.8	0.417903624	0.37555025
-4	0.892987878	0.885123873

α = .9 S Distance	% Error	% Error
4	1.432511095	1.316535361
3.8	0.535664159	0.730930087
3.6	0.327001987	0.47749568
3.4	1.226612366	1.533987377
3.2	1.720410413	1.462261696
3	0.105916627	0.100713697
2.8	1.751988707	1.591501571
2.6	3.301941025	3.475829451
2.4	4.76592555	4.992919676
2.2	3.497742033	3.69788669
2	1.723000412	1.664943514
1.8	5.610511587	5.579078386
1.6	8.25646379	8.146987991
1.4	9.801390438	9.777192931
1.2	8.196341331	8.302986726
1	3.921872924	4.506498393
0.8	0.131454058	0.886710735
0.6	3.744452212	2.096472459
0.4	6.669527063	4.308693417
0.2	8.60554784	5.77760191
0	9.280238343	6.263934048
-0.2	8.60554784	5.836044038
-0.4	6.669527063	4.299278042
-0.6	3.744452212	2.104189096
-0.8	0.131454058	0.936658607
-1	3.921872924	4.337608618
-1.2	8.196341331	8.323212976
-1.4	9.801390438	9.639441152
-1.6	8.25646379	8.148793804
-1.8	5.610511587	5.395390059
-2	1.723000412	1.672454559
-2.2	3.497742033	3.621907635
-2.4	4.76592555	4.804951558
-2.6	3.301941025	3.54354694
-2.8	1.751988707	1.262336466
-3	0.105916627	0.524204185
-3.2	1.720410413	1.778157811
-3.4	1.226612366	1.302998912
-3.6	0.327001987	0.386815941
-3.8	0.535664159	0.408033915
-4	1.432594715	1.625082207

α = .95 S Distance	% Error	% Error
4	6.180451887	5.994587972
3.8	6.013204969	6.752080513
3.6	3.998123213	4.081775862
3.4	0.415931108	0.332357441
3.2	0.556595897	0.495070939
3	1.565673369	1.89207876
2.8	2.206096545	2.440411218
2.6	1.823102555	2.006785725
2.4	0.379518387	0.05998811
2.2	0.826068481	0.359905531
2	1.299563273	0.535941545
1.8	0.386224029	0.547220943
1.6	0.966069821	0.011654304
1.4	1.063378856	0.069665447
1.2	2.224984536	0.779624104
1	1.493658081	0.207793522
0.8	2.151675518	1.476848874
0.6	0.215821815	1.088961964
0.4	2.887752271	0.3709554
0.2	6.32361867	0.515598386
0	6.957778426	0.177923926
-0.2	6.32361867	0.575664143
-0.4	2.887752271	0.291211474
-0.6	0.215821815	0.997346145
-0.8	2.151675518	1.424186606
-1	1.493658081	0.418288182
-1.2	2.224984536	0.736690801
-1.4	1.063378856	0.015547
-1.6	0.966069821	0.053204251
-1.8	0.386224029	0.375806053
-2	1.299563273	0.803098828
-2.2	0.826068481	0.161910994
-2.4	0.379518387	0.04180714
-2.6	1.823102555	2.076276137
-2.8	2.206096545	1.87828553
-3	1.565673369	1.428769804
-3.2	0.556595897	0.369666509
-3.4	0.415931108	0.508610186
-3.6	3.998123213	3.909177847
-3.8	6.013204969	6.09224612
-4	6.180451887	6.076525508

α = .96 S Distance	% Error	% Error
4	6.974359609	7.461273021
3.8	6.780805036	7.10263836
3.6	4.665631468	4.642096809
3.4	0.908224911	0.656479766
3.2	0.358817774	0.252385002
3	2.014730928	2.008662188
2.8	2.614831726	2.893972921
2.6	2.223433902	2.601629128
2.4	0.053180722	0.369362237
2.2	0.766434054	0.219421147
2	1.414379505	0.606475232
1.8	0.216619765	0.73035932
1.6	0.84627103	0.3728081
1.4	1.105560849	0.502241361
1.2	2.61911908	0.836270531
1	2.021311336	0.093490592
0.8	3.229123583	1.773502473
0.6	1.190012079	1.183829485
0.4	2.652413471	0.368776173
0.2	7.351180537	0.578157231
0	8.617859035	0.252118802
-0.2	7.351180537	0.514947799
-0.4	2.652413471	0.361133861
-0.6	1.190012079	1.126282324
-0.8	3.229123583	1.749616605
-1	2.021311336	0.167117427
-1.2	2.61911908	0.85571299
-1.4	1.105560849	0.434753565
-1.6	0.84627103	0.032306643
-1.8	0.216619765	0.621148057
-2	1.414379505	0.504535202
-2.2	0.766434054	0.119894565
-2.4	0.053180722	0.526735413
-2.6	2.223433902	2.783696606
-2.8	2.614831726	2.854779986
-3	2.014730928	2.22524824
-3.2	0.358817774	0.221685204
-3.4	0.908224911	0.828206037
-3.6	4.665631468	4.621564829
-3.8	6.780805036	7.075126361
-4	6.974359609	7.006249217

α = .97 S Distance	% Error	% Error
4	7.44851082	8.147697704
3.8	7.360469042	7.925936119
3.6	5.172627	5.793257987
3.4	1.161317566	1.35852237
3.2	0.079481875	0.790041683
3	2.168760251	1.77425663
2.8	2.937524329	2.88939558
2.6	2.593323602	2.654644258
2.4	0.094325347	0.323150133
2.2	0.801122487	0.171070724
2	1.423448236	0.889661177
1.8	0.070667544	0.806524033
1.6	0.65188806	0.555241299
1.4	1.076087205	0.519990051
1.2	3.073388508	1.01295468
1	2.769218015	0.140408619
0.8	4.747894747	2.069440891
0.6	2.691451422	1.501690507
0.4	2.034591415	0.469462576
0.2	8.654024053	0.579185285
0	6.44452944	0.181510121
-0.2	8.654024053	0.72630483
-0.4	2.034591415	0.415621425
-0.6	2.691451422	1.655737646
-0.8	4.747894747	2.210262025
-1	2.769218015	0.049658191
-1.2	3.073388508	0.861976398
-1.4	1.076087205	0.612288529
-1.6	0.65188806	0.520678579
-1.8	0.070667544	1.156075522
-2	1.423448236	0.699721414
-2.2	0.801122487	0.183734535
-2.4	0.094325347	0.528559939
-2.6	2.593323602	3.109493873
-2.8	2.937524329	3.103426377
-3	2.168760251	2.472944759
-3.2	0.079481875	0.03846367
-3.4	1.161317566	1.325455156
-3.6	5.172627	5.331976998
-3.8	7.360469042	7.790807966
-4	7.44851082	7.686759836

α = .98 S Distance	% Error	% Error
4	7.727462546	7.657378345
3.8	7.663023353	8.038461574
3.6	5.309957139	5.350285096
3.4	1.113173727	0.821379551
3.2	0.258091402	0.066205504
3	2.261342233	2.405974669
2.8	3.090711084	3.100265519
2.6	2.661321499	2.601965249
2.4	0.068792735	0.106637963
2.2	0.628259503	0.333986346
2	1.437499999	0.845171463
1.8	0.459166638	1.628632945
1.6	0.232741145	1.093029505
1.4	0.866958807	0.961255756
1.2	3.527092006	1.10087164
1	3.662569915	0.326370454
0.8	6.739087186	2.576547749
0.6	4.949028755	1.87640627
0.4	0.616670082	0.4821266
0.2	9.946724882	0.936061392
0	15.49394394	0.094216043
-0.2	9.946724882	0.914832247
-0.4	0.616670082	0.530684311
-0.6	4.949028755	1.8531933
-0.8	6.739087186	2.537173089
-1	3.662569915	0.300932324
-1.2	3.527092006	0.888739957
-1.4	0.866958807	1.087674189
-1.6	0.232741145	1.340512668
-1.8	0.459166638	1.41617668
-2	1.437499999	0.490627111
-2.2	0.628259503	0.119195279
-2.4	0.068792735	0.641821573
-2.6	2.661321499	3.072226549
-2.8	3.090711084	3.452532438
-3	2.261342233	2.506997351
-3.2	0.258091402	0.011875003
-3.4	1.113173727	0.934070796
-3.6	5.309957139	5.109635156
-3.8	7.663023353	7.783228492
-4	7.727462546	7.715280863

α = .99 S Distance	% Error	% Error
4	7.940221452	7.710909091
3.8	7.749029809	7.785967399
3.6	5.312549503	5.20079324
3.4	0.873920552	0.858660292
3.2	0.216413516	0.073481394
3	2.380040608	2.789270582
2.8	3.120351116	3.278610321
2.6	2.739327226	3.176983709
2.4	0.05994221	0.545099156
2.2	0.656468952	0.123541739
2	1.520427019	0.643014373
1.8	1.046901408	2.225180811
1.6	0.452793412	1.968011255
1.4	0.315125239	1.772851525
1.2	3.633314545	0.960316108
1	4.365435112	0.341852875
0.8	9.185799511	3.185915257
0.6	8.222327991	2.32098091
0.4	2.293703145	0.849525722
0.2	10.10871006	0.908002469
0	23.10871265	0.069029197
-0.2	10.10871006	0.911844379
-0.4	2.293703145	0.838310341
-0.6	8.222327991	2.367720046
-0.8	9.185799511	3.172041679
-1	4.365435112	0.23396228
-1.2	3.633314545	0.77760096
-1.4	0.315125239	1.778040801
-1.6	0.452793412	1.955751799
-1.8	1.046901408	2.33381465
-2	1.520427019	0.43670179
-2.2	0.656468952	0.102811378
-2.4	0.05994221	0.399492743
-2.6	2.739327226	3.232187645
-2.8	3.120351116	3.477557645
-3	2.380040608	2.693612272
-3.2	0.216413516	0.087833033
-3.4	0.873920552	0.639003106
-3.6	5.312549503	5.314497911
-3.8	7.749029809	7.507295075
-4	7.940221452	7.654109103

α = 1 S Distance	% Error	% Error
4	0.520909421	1.032526595
3.8	0.544790518	0.896054038
3.6	0.55876432	0.783131994
3.4	0.513551414	0.932685473
3.2	0.606400577	0.999776009
3	0.714723775	0.350194023
2.8	0.80447745	0.820181904
2.6	0.435352361	0.020050441
2.4	1.177430691	0.543704672
2.2	0.479689731	0.168468307
2	1.512264142	0.541639163
1.8	2.069511541	3.070387099
1.6	1.794292891	3.253234319
1.4	0.897101977	2.61486209
1.2	3.263912215	0.61790582
1	4.814240804	0.445381919
0.8		
0.6		
0.4		
0.2		
0		
-0.2		
-0.4		
-0.6		
-0.8		
-1	4.814240804	0.396352109
-1.2	3.263912215	0.859907627
-1.4	0.897101977	2.74934291
-1.6	1.794292891	3.164140464
-1.8	2.069511541	3.276487187
-2	1.512264142	0.430685845
-2.2	0.479689731	0.201313775
-2.4	1.177430691	0.559730432
-2.6	0.435352361	0.242482009
-2.8	0.80447745	0.469170754
-3	0.714723775	0.514021707
-3.2	0.606400577	0.354830506
-3.4	0.513551414	0.477641543
-3.6	0.55876432	0.635834816
-3.8	0.544790518	0.715483841
-4	0.520909421	1.238980481

D.2 Percent Error for Not Equal Coil Lengths

S Distance	Grover vs. Elliptical	Grover vs. MagNet
α = .1	**% Error**	**% Error**
40	7.002981615	0.661909365
38	7.35652527	1.16388852
36	7.728202038	0.134328696
34	8.087550833	0.07509823
32	8.416296226	0.247000338
30	8.695060577	0.766504514
28	8.895390277	0.298146807
26	8.971518505	0.594363132
24	8.860156205	0.320540102
22	8.521554059	0.491887597
20	7.872575647	0.40067897
18	6.855455543	0.280916742
16	5.428989372	0.224848837
14	3.592714154	0.105793251
13	2.531610285	0.192564846
12	1.39251196	0.236324999
10	1.046891665	0.115887607
8	3.532634458	0.079156589
6	5.836106919	0.083757211
4	7.709222909	0.097692527
2	8.931214541	0.133149397
0	9.354434733	0.074328941
-2	8.931214541	0.126514103
-4	7.709222909	0.147266523
-6	5.836106919	0.242455303
-8	3.532634458	0.073140605
-10	1.046891665	0.214954316
-12	1.39251196	0.321043765
-13	2.531610285	0.317322914
-14	3.592714154	0.168132416
-16	5.428989372	0.273648747
-18	6.855455543	0.23518489
-20	7.872575647	0.318555052
-22	8.521554059	0.302069864
-24	8.860156205	0.291964575
-26	8.971518505	0.605701167
-28	8.895390277	0.26197589
-30	8.695060577	0.251235728
-32	8.416296226	0.635902483
-34	8.087540385	0.904184516
-36	7.728202038	0.168233779
-38	7.35652527	0.42118309
-40	7.002981615	0.438559453

α = .2 S Distance	% Error	% Error
40	6.946541247	0.614458093
38	7.296081292	0.797341365
36	7.700405219	0.975129068
34	8.074682863	0.423132882
32	8.401641808	0.365151147
30	8.711605908	0.501788884
28	8.940157953	0.353086363
26	9.04700706	0.316341422
24	8.982600056	0.804869745
22	8.700044936	0.382504011
20	8.102429559	0.36059098
18	7.148937195	0.334910129
16	5.77176323	0.363022325
14	3.951716527	0.096447257
13	2.880266394	0.284050818
12	1.727178452	0.263362068
10	0.790626993	0.122551093
8	3.406478331	0.151488816
6	5.864343948	0.080745778
4	7.886983966	0.06794464
2	9.219497303	0.054053366
0	9.685103818	0.08936896
-2	9.219497303	0.265965658
-4	7.886983966	0.117402317
-6	5.864343948	0.249174501
-8	3.406478331	0.192991371
-10	0.790626993	0.184407735
-12	1.727178452	0.326352363
-13	2.880266394	0.331100774
-14	3.951716527	0.093488323
-16	5.77176323	0.269178036
-18	7.148937195	0.229566831
-20	8.102429559	0.371252266
-22	8.700044936	0.405139951
-24	8.982600056	0.359556355
-26	9.04700706	0.605071207
-28	8.940157953	0.245034566
-30	8.711605908	0.383214032
-32	8.401641808	0.263109808
-34	8.074682863	0.511791949
-36	7.700405219	2.515523085
-38	7.296081292	0.64533118
-40	6.946541247	0.572517589

α = .3 S Distance	% Error	% Error
40	6.96045327	0.776141686
38	7.27969888	0.416061759
36	7.668118328	0.374321495
34	8.032862731	1.077263528
32	8.359858429	0.374935931
30	8.680852745	0.295076425
28	8.946320172	0.386339961
26	9.135967656	0.408532638
24	9.135395467	0.261611854
22	8.959036196	0.581588156
20	8.479447362	0.235965692
18	7.642499767	0.26635973
16	6.350939652	0.221319298
14	4.586988701	0.136420955
13	3.509721346	0.191383412
12	2.329920502	0.305944086
10	0.319273959	0.293420927
8	3.149559957	0.187495458
6	5.885794189	0.211693686
4	8.187872165	0.099132974
2	9.727542915	0.197648982
0	10.26475973	0.195469053
-2	9.727542915	0.1967673
-4	8.187872165	0.229553503
-6	5.885794189	0.296425397
-8	3.149559957	0.210672904
-10	0.319273959	0.314580783
-12	2.329920502	0.189497816
-13	3.509721346	0.177269896
-14	4.586988701	0.1406702
-16	6.350939652	0.155059892
-18	7.642499767	0.189494482
-20	8.479447362	0.393778691
-22	8.959036196	0.491522586
-24	9.135395467	0.714551607
-26	9.135967656	0.30011912
-28	8.946320172	0.170617112
-30	8.680852745	0.909436938
-32	8.359858429	1.157692823
-34	8.032862731	0.88870847
-36	7.668118328	0.68756101
-38	7.27969888	0.584225043
-40	6.96045327	0.169602715

α = .4 S Distance	% Error	% Error
40	6.806184943	1.26805414
38	7.073785108	0.794587341
36	7.510782932	0.467595899
34	7.928904319	0.185495424
32	8.290756572	0.220770542
30	8.658352329	0.650058659
28	9.005405254	0.396725691
26	9.285002902	0.521925696
24	9.354606193	0.436471587
22	9.315227014	0.372830261
20	8.995120491	0.292618194
18	8.325461932	0.287802027
16	7.181237918	0.19147374
14	5.537141177	0.276969578
13	4.467571652	0.184236615
12	3.271869351	0.202588363
10	0.460982823	0.205492387
8	2.681150514	0.184844632
6	5.854000306	0.127092433
4	8.622959362	0.052581183
2	10.51495827	0.187200697
0	11.17851585	0.255499227
-2	10.51495827	0.181927421
-4	8.622959362	0.278895567
-6	5.854000306	0.164803556
-8	2.681150514	0.239164459
-10	0.460982823	0.244873631
-12	3.271869351	0.1814103
-13	4.467571652	0.139562402
-14	5.537141177	0.256006675
-16	7.181237918	0.217157818
-18	8.325461932	0.257643859
-20	8.995120491	0.329739369
-22	9.315227014	0.487875122
-24	9.354606193	0.290390034
-26	9.285002902	0.351980178
-28	9.005405254	0.624986033
-30	8.658352329	0.96549333
-32	8.290756572	0.210022902
-34	7.928904319	0.268939074
-36	7.510782932	0.458039762
-38	7.073785108	0.151180173
-40	6.806184943	1.452874676

α = .5 S Distance	% Error	% Error
40	6.713632069	0.139713359
38	6.958517422	0.178308141
36	7.470260188	1.311327259
34	7.891835117	0.536035856
32	8.221822803	0.57275495
30	8.621711296	0.082604602
28	9.031548558	0.260882417
26	9.394909458	0.371915926
24	9.57637696	0.397919838
22	9.711612689	0.567353222
20	9.589723093	0.546903733
18	9.178419286	0.404676729
16	8.291606915	0.311500338
14	6.870209282	0.172771636
13	5.855993282	0.094027105
12	4.67924356	0.205441937
10	1.713135894	0.233026254
8	1.853015009	0.298647795
6	5.677974454	0.126140672
4	9.18594451	0.075355828
2	11.66710467	0.232111699
0	12.55459643	0.219101812
-2	11.66710467	0.135101419
-4	9.18594451	0.172401307
-6	5.677974454	0.202654205
-8	1.853015009	0.218137356
-10	1.713135894	0.127027786
-12	4.67924356	0.207446161
-13	5.855993282	0.136304279
-14	6.870209282	0.153874344
-16	8.291606915	0.313539093
-18	9.178419286	0.333002637
-20	9.589723093	0.439864173
-22	9.711612689	0.355951902
-24	9.57637696	0.265054897
-26	9.394909458	0.345669868
-28	9.031548558	0.316652301
-30	8.621711296	0.164410403
-32	8.221822803	0.157328904
-34	7.891835117	0.761229944
-36	7.470260188	0.692083956
-38	6.958517422	1.57800014
-40	6.713632069	0.704184547

α = .6 S Distance	% Error	% Error
40	6.560906828	1.053403605
38	6.779424625	0.985591368
36	7.309973604	1.514647735
34	7.743523082	1.319029481
32	8.092060078	1.032620897
30	8.538605228	0.414646515
28	8.99829108	0.55587169
26	9.481821691	0.4643128
24	9.756144493	0.295859176
22	10.0814448	0.333268795
20	10.22836046	0.35916025
18	10.16657437	0.321547564
16	9.655718463	0.204318666
14	8.649356223	0.097911715
13	7.777141315	0.076057289
12	6.709469876	0.130651274
10	3.667485625	0.079605851
8	0.383514396	0.080879289
6	5.137751669	0.143639816
4	9.830747386	0.122445504
2	13.33241667	0.159176482
0	14.61807512	0.1282933
-2	13.33241667	0.207549001
-4	9.830747386	0.175476368
-6	5.137751669	0.283292341
-8	0.383514396	0.218581457
-10	3.667485625	0.080799412
-12	6.709469876	0.172717996
-13	7.777141315	0.125115069
-14	8.649356223	0.110934904
-16	9.655718463	0.227473266
-18	10.16657437	0.136397006
-20	10.22836046	0.297562912
-22	10.0814448	0.318550814
-24	9.756144493	0.292289808
-26	9.481821691	0.467444781
-28	8.99829108	0.740231688
-30	8.538605228	0.511366476
-32	8.092060078	0.463676315
-34	7.743523082	1.246673823
-36	7.309973604	0.460200989
-38	6.779424625	0.206781151
-40	6.560906828	0.210366614

α = .7 S Distance	% Error	% Error
40	6.416691805	0.793588141
38	6.513870778	0.924271408
36	7.09256953	0.802558099
34	7.552367573	0.401123318
32	7.854725295	0.408309068
30	8.315723463	0.282384981
28	8.872888804	0.298563286
26	9.470222658	0.529203119
24	9.816003853	0.473945595
22	10.36917307	0.369680521
20	10.80996744	0.314424335
18	11.15346285	0.26418539
16	11.18146762	0.178324899
14	10.90274562	0.375791333
13	10.34096041	0.382619956
12	9.553869458	0.657825806
10	6.709422389	0.127875632
8	2.233223069	0.152950523
6	3.771171178	0.177880626
4	10.37094646	0.117873734
2	15.72890777	0.124616848
0	17.80525699	0.222241757
-2	15.72890777	0.099377929
-4	10.37094646	0.123384498
-6	3.771171178	0.108136261
-8	2.233223069	0.156748206
-10	6.709422389	0.135195956
-12	9.553869458	0.522155146
-13	10.34096041	0.212694476
-14	10.90274562	0.690145652
-16	11.18146762	0.077709322
-18	11.15346285	0.275957141
-20	10.80996744	0.230847715
-22	10.36917307	0.359065243
-24	9.816003853	0.689364782
-26	9.470222658	0.650320715
-28	8.872888804	0.254891868
-30	8.315723463	1.147944773
-32	7.854725295	0.708212648
-34	7.552367573	0.487149394
-36	7.09256953	0.306291703
-38	6.513870778	0.073610014
-40	6.416691805	0.329452742

α = .8 S Distance	% Error	% Error
40	6.26193323	0.076112568
38	6.268991035	0.27583841
36	6.890581562	0.568217461
34	7.336070938	0.394745367
32	7.593282737	0.110760548
30	8.054442698	0.170786207
28	8.645362221	0.351344947
26	9.326226113	0.396844158
24	9.727361163	0.315297398
22	10.46437903	0.430763103
20	11.1865196	0.273558854
18	11.97106902	0.338839811
16	12.61748541	0.220744286
14	13.46593491	0.242711999
13	13.50889554	0.188701507
12	13.35003216	0.210375993
10	11.36436127	0.059376142
8	6.881745479	0.19386437
6	0.553459171	0.216829089
4	10.12706933	0.116406371
2	19.11849514	0.113285157
0	23.04387591	0.219316588
-2	19.11849514	0.137916634
-4	10.12706933	0.124045823
-6	0.553459171	0.172261447
-8	6.881745479	0.107364301
-10	11.36436127	0.120288067
-12	13.35003216	0.338035893
-13	13.50889554	0.610338193
-14	13.46593491	0.389080464
-16	12.61748541	0.527707133
-18	11.97106902	0.18136703
-20	11.1865196	0.362584217
-22	10.46437903	0.805103023
-24	9.727361163	0.28793497
-26	9.326226113	0.950214244
-28	8.645362221	0.200493701
-30	8.054442698	0.156731543
-32	7.593282737	0.09486706
-34	7.336070938	0.078646567
-36	6.890581562	0.538361562
-38	6.268991035	0.085436062
-40	6.26193323	0.301055343

α = .9 S Distance	% Error	% Error
40	6.089216267	1.075247E+00
38	5.953818524	5.316410E-01
36	6.611475553	7.814414E-01
34	7.035367613	7.050778E-01
32	7.203960915	8.652433E-01
30	7.672107174	3.476013E-01
28	8.281594292	4.489408E-01
26	9.051311257	5.532313E-01
24	9.425486467	5.941343E-01
22	10.27265072	9.168913E-01
20	11.18406856	3.689904E-01
18	12.30361106	3.073364E-01
16	13.33660645	7.356205E-02
14	15.80267919	7.355917E-01
13	16.91906741	6.229264E-01
12	18.00334312	2.318831E-01
10	18.23133393	1.338875E-01
8	15.00839805	8.320961E-02
6	6.671913516	2.223789E-01
4	6.830697231	4.507405E-02
2	23.04631739	7.751942E-02
0	32.82724889	3.544758E-01
-2	23.04631739	7.537300E-02
-4	6.830697231	6.952717E-02
-6	6.671913516	2.225571E-01
-8	15.00839805	9.608830E-02
-10	18.23133393	1.347933E-01
-12	18.00334312	1.768378E-01
-13	16.91906741	6.188023E-01
-14	15.80267919	7.029556E-01
-16	13.33660645	6.189966E-02
-18	12.30361106	3.248984E-01
-20	11.18406856	3.286493E-01
-22	10.27265072	5.040813E-01
-24	9.425486467	2.970774E-01
-26	9.051311257	7.210035E-01
-28	8.281594292	8.002058E-01
-30	7.672107174	2.436378E-01
-32	7.203960915	5.290934E-01
-34	7.035367613	7.608570E-01
-36	6.611475553	9.451660E-01
-38	5.953818524	5.147456E-01
-40	6.089216267	1.611136E+00

α = .95 S Distance	% Error	% Error
40	6.005173071	0.686074752
38	5.768167944	0.004227275
36	6.4472302	0.448428205
34	6.8833218	0.929405805
32	7.007961176	0.308536519
30	7.442987257	0.220068442
28	8.077906188	0.429718305
26	8.861636428	0.774795139
24	9.18272936	0.218662946
22	10.07240527	0.217924634
20	11.00223004	0.129874147
18	12.40447233	0.323518136
16	13.96461559	0.435995187
14	16.56704178	0.431881553
13	18.30582012	0.294828615
12	20.48106961	0.205404328
10	23.16490515	0.229625572
8	20.97971689	0.123002559
6	12.56078178	0.041610508
4	2.324901921	0.035710949
2	23.55568879	0.014401116
0	42.46434153	0.038632094
-2	23.55584554	0.001994123
-4	2.324901921	0.030222104
-6	12.56078178	0.037046084
-8	20.97971701	0.119949717
-10	23.16490515	0.151550489
-12	20.48106961	0.219515678
-13	18.30582012	0.328928165
-14	16.56704178	0.437260948
-16	13.96461559	0.234999421
-18	12.40447233	0.32638568
-20	11.00223004	0.151902978
-22	10.07240527	0.343582011
-24	9.18272936	0.486021154
-26	8.861636428	0.947585804
-28	8.077906188	0.430242309
-30	7.442987257	0.004884734
-32	7.007961176	0.091082448
-34	6.8833218	0.38827667
-36	6.4472302	0.395514294
-38	5.768167944	1.178193503
-40	6.005173071	2.32276149

α = .96 S Distance	% Error	% Error
40	6.057977086	0.954410208
38	5.751806539	0.1906582
36	6.415066421	0.562260448
34	6.889808028	0.97719204
32	6.978726203	0.863605988
30	7.384253858	0.286980093
28	8.037820557	0.332287237
26	8.809216249	0.6080468
24	9.107978326	0.288277423
22	9.997004179	0.046221102
20	10.93832255	0.109140106
18	12.37462136	0.299368731
16	13.9627439	0.19088328
14	16.63126417	0.715180416
13	18.5149752	0.342626872
12	20.94805488	0.234981574
10	24.20926569	0.17793835
8	22.36384742	0.115556999
6	14.0477079	0.035143899
4	1.037456639	0.034666676
2	23.21535612	0.007945399
0	45.22823474	0.034985605
-2	23.21551252	0.025692717
-4	1.037456639	0.031731396
-6	14.0477079	0.032618532
-8	22.36384742	0.169946515
-10	24.20926569	0.142296964
-12	20.94805488	0.19292471
-13	18.5149752	0.380580717
-14	16.63126417	0.63531994
-16	13.9627439	0.425154044
-18	12.37462136	0.533926499
-20	10.93832255	0.085991679
-22	9.997004179	0.160360958
-24	9.107978326	0.38026374
-26	8.809216249	0.987572403
-28	8.037820557	0.753395077
-30	7.384253858	0.118836422
-32	6.978726203	0.061522006
-34	6.889808028	0.225055689
-36	6.415066421	0.16905904
-38	5.751806539	0.047910321
-40	6.057977086	1.272976758

α = .97 S Distance	% Error	% Error
40	5.897191835	2.367552537
38	5.703837979	0.509316829
36	6.33133315	0.352618988
34	6.787415243	0.361308025
32	6.934955158	0.32526989
30	7.366053344	0.450436528
28	7.952211056	0.380472381
26	8.80216381	1.112095485
24	9.090963628	0.255752587
22	9.939682057	0.16543707
20	10.87250834	0.104902456
18	12.34801791	0.321966069
16	13.92301329	0.207934642
14	16.63492753	0.224103031
13	18.69147403	0.486748344
12	21.39964595	0.200666466
10	25.27473045	0.173775156
8	23.81377825	0.144446621
6	15.638879	0.078287155
4	0.40215657	0.056663901
2	22.64263923	0.015306497
0	48.55660549	0.032227649
-2	22.64262949	0.009991586
-4	0.40215657	0.038030103
-6	15.638879	0.056816829
-8	23.81377825	0.126058906
-10	25.27473045	0.151351802
-12	21.39964595	0.182670672
-13	18.69147403	0.195914468
-14	16.63492753	0.270363871
-16	13.92301329	0.392688923
-18	12.34801791	0.346943421
-20	10.87250834	0.083866683
-22	9.939682057	0.345510733
-24	9.090963628	0.49779524
-26	8.80216381	0.481269019
-28	7.952211056	0.307257634
-30	7.366053344	0.029503698
-32	6.934955158	0.134798934
-34	6.787415243	0.183511463
-36	6.33133315	0.412798232
-38	5.703837979	0.815760029
-40	5.897191835	0.466330288

α = .98 S Distance	% Error	% Error
40	5.883466103	2.303517014
38	5.599824714	1.681270834
36	6.38265317	0.522510231
34	6.78323296	0.666595856
32	6.898537155	0.066664729
30	7.338016596	0.200185147
28	7.944177142	0.320103974
26	8.720839216	1.076029844
24	9.012448145	0.344294632
22	9.880271	0.170775632
20	10.81239826	0.1321993
18	12.29687426	0.304271771
16	13.88192032	0.192747002
14	16.60897161	0.37572031
13	18.80931754	0.323957287
12	21.81815747	0.130175802
10	26.37760727	0.16203152
8	25.33601231	0.164746817
6	17.3353213	0.071337825
4	2.007350012	0.058544717
2	21.79633648	0.029909599
0	52.80165502	0.031025278
-2	21.79648868	0.033819887
-4	2.007350012	0.063631865
-6	17.3353213	0.066619724
-8	25.33601231	0.13272671
-10	26.37760727	0.151496607
-12	21.81815747	0.155127981
-13	18.80931754	0.186836607
-14	16.60897161	0.42415987
-16	13.88192032	0.351130228
-18	12.29687426	0.385946175
-20	10.81239826	0.102957184
-22	9.880271	0.256629163
-24	9.012448145	0.848742481
-26	8.720839216	0.871759075
-28	7.944177142	0.079582579
-30	7.338016596	0.540120398
-32	6.898537155	0.071633003
-34	6.78323296	0.766457906
-36	6.38265317	0.42309593
-38	5.599824714	1.596509247
-40	5.883466103	0.481797523

α = .99 S Distance	% Error	% Error
40	5.724788308	1.639514814
38	5.554947226	0.730552549
36	6.302375524	0.824341371
34	6.683614415	0.377990257
32	6.858353271	0.003229938
30	7.32398252	0.195737441
28	7.861850984	0.167752795
26	8.718352978	0.629413473
24	9.000383952	0.17011197
22	9.828024828	0.57308323
20	10.75282084	0.748108681
18	12.2445145	0.316789915
16	13.79729344	0.480429173
14	16.49060661	0.386231676
13	18.86194334	0.305129274
12	22.19737431	0.142950918
10	27.49976078	0.127385033
8	26.92456153	0.133070877
6	19.14701705	0.044589591
4	3.792793053	0.102418177
2	20.62963396	0.050402178
0	58.92397395	0.052267117
-2	20.62978782	0.0589435
-4	3.792793053	0.099108725
-6	19.14701705	0.056745706
-8	26.92456153	0.124865359
-10	27.49976078	0.18067822
-12	22.19737431	0.096193873
-13	18.86194334	0.218251992
-14	16.49060661	0.14412744
-16	13.79729344	0.242675281
-18	12.2445145	0.278497299
-20	10.75282084	0.574901418
-22	9.828024828	0.425973703
-24	9.000383952	1.03711359
-26	8.718352978	0.777491409
-28	7.861850984	0.298585064
-30	7.32398252	0.254898846
-32	6.858353271	0.175710691
-34	6.683614415	0.354241658
-36	6.302375524	1.267400707
-38	5.554947226	0.556567246
-40	5.724788308	0.895189301

α = 1 S Distance	% Error	% Error
40	9.878932018	5.685648978
38	7.714038953	2.833666382
36	6.687054196	1.038553204
34	7.027831105	1.712069261
32	5.841953437	0.848128124
30	5.531321606	1.56390187
28	6.187285106	1.275935488
26	6.88932428	1.293247411
24	7.677797899	1.048748562
22	9.381301868	0.304856422
20	10.80607199	0.212031731
18	12.01731432	0.24167742
16	13.7068401	0.229715697
14	16.14315821	0.01660009
13	18.82530738	0.832993643
12		
10		
8		
6		
4		
2		
0		
-2		
-4		
-6		
-8		
-10		
-12		
-13	18.82530738	0.251433287
-14	16.14064024	0.045927321
-16	13.7053462	0.274010587
-18	12.01642528	0.122058737
-20	10.80607199	0.371399393
-22	9.381301868	0.065807337
-24	7.679309962	1.074020911
-26	6.88932428	1.087909249
-28	6.187285106	1.232791113
-30	5.531321606	1.619048495
-32	5.844991947	0.790700146
-34	7.031433463	1.541838924
-36	6.687054196	0.650190392
-38	7.719026053	1.841420042
-40	9.878932018	5.152819348