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# INDUCTANCE CALCULATIONS

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*Working Formulas and Tables*

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FREDERICK W. GROVER, PH.D.

*Professor of Electrical Engineering, Union College;  
Formerly Consulting Physicist, National Bureau of Standards*



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## PREFACE

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The design of inductors to have a given inductance or the calculation of the inductance of existing circuits are problems of importance in electrical engineering and especially in the field of communication.

Collections of formulas for the calculation of inductance and mutual inductance for different types of coils and other inductors are to be found in various electrical engineering handbooks and notably in the publications of the National Bureau of Standards.

It has, however, been the observation of the author of the present work, who has participated in the preparation of the Bureau of Standards collections, that certain difficulties are experienced in the use of this material. The engineer who has occasion to calculate an inductance is likely to be overwhelmed by the very wealth of the formulas offered him, and especially is this true in the more common types of inductor. Furthermore, certain formulas require the use of elliptic integrals or allied functions, others zonal harmonic functions or hyperbolic functions. Other formulas appear in the form of infinite series and it is necessary to choose from among those offered that formula whose degree of convergence will best suit the problem in question. Undoubtedly these complexities discourage the computer in many cases and lead to the substitution of empirical formulas or rough approximations for the accurate formulas.

The present work has been prepared with the idea of providing for each special type of inductor a single simple formula that will involve only the parameters that naturally enter together with numerical factors that may be interpolated from tables computed for the purpose. It has been found possible to accomplish this end in all the more important cases, and, even in the more complex arrangements of conductors, to outline a straightforward procedure. For the accomplishment of this end extensive tables have had to be calculated. Fortunately, certain of the tables are useful in more than a single case, but even so the tables represent a vast amount of computation. The tabular intervals are chosen so that where possible linear interpolation or at worst the inclusion of second order differences suffice. An accuracy of a part in a thousand is aimed at in general, but for the most part the tables lead to a better precision.

Illustrative examples are included with each case and where possible the numerical values found have been checked by other known formulas or methods. Procedure for the design of the more usual types of inductor has been included.

It is believed that all the more important forms of inductor and circuit elements have been covered, but in any new case it is usual to build a formula or method from the basic formulas by the general methods that are explained in the introductory chapters.

F. W. G.

Union College,  
Schenectady, N. Y.  
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