Be Careful of Self and Mutual Inductance Formulae

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ABSTRACT

Inductance has been identified as an emerging troublemaker of interconnects for the next generation high-speed digital VLSI design. As a result, accurate and fast inductance extraction is of crucial importance. Formulae-based partial inductance extraction method approaches have been widely adopted for inductance extraction due to its simplicity and efficiency. However, there are so many inductance formulae in the literature and there seems luck of detail comparisons of those formulae to check their accuracy and applicability.

In this paper, we carefully compare several well-known partial self and mutual inductance formulae in details to check their range of applicability and validity. Since some formulae are originally derived for other applications where the parameter dimensions are quite different from the VLSI application, they are not numerically stable and accurate for all the concerned range of VLSI applications. As a result, carefulness must be deployed while applying those formulae.

1. Introduction

As the clock frequency of VLSI technologies grows over 4 giga hertz, inductance effect emerges to be another signal integrity issues and hence becomes an important design factor for high performance VLSI design [6][7][8][9]. Excessive inductance will cause inaccurate delay estimation, signal ringing, inductive coupling noises, and L*di/dt* drop for the power delivery nets. For these reasons, accurate inductance extraction is of great importance for the next generation high-speed VLSI design.

Due to the difficulty of obtaining the current returning paths, Partial Element Equivalent Circuit (PEEC) approach has been proposed by Ruehli [1] to use partial inductance instead of loop inductance as basic circuit analysis elements. Since its introduction, PEEC has been widely used in the VLSI field due to its simplicity and compatibility with the general circuit simulation engines such as SPICE. While the capacitance extraction required matrix-solving techniques, partial self and mutual inductance can be easily obtained using precomputed and simplified integration formulae. For this reason, several formulae-based inductance extraction algorithms have been proposed. However, near each method uses different formulae since there are so many existing inductance formulae in the literature. Unfortunately, most of the existing inductance formulae are originally derived for other applications where the parameter dimensions are quite different from the VLSI application. Hence, the validity and accuracy of those formulae need to be carefully verified for the VLSI application. Unfortunately, there seems luck of comprehensive study of those formulae in the VLSI field.

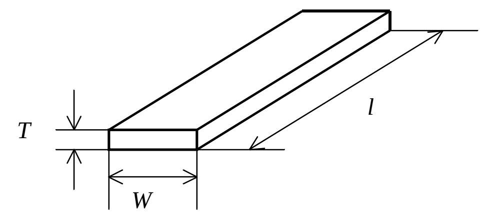

Fig. 1. Single Conductor

In this paper, we carefully study several most popular classic inductance formulae such as Ruehli [1], Grover [2], Hoer [3], and FastHenry [4]. Each formula is tested and compared in extensive VLSI parameter ranges. Although some formulae have quite simple forms, they produce inaccurate or unstable results for relatively short and long range of conductors. As a result, carefulness must be deployed while applying those formulae.

The organization of this paper is as follows: The various self inductance and mutual inductance formulae are introduced in section 2 and section 3, respectively. In section 4, extensive computational results and comparisons are provided. The conclusion of this paper is presented in section 5.

2. Self Inductance Extraction

In this section, we briefly summarize four popular inductance formulae for partial self inductance extraction including Ruehli [1], Grover [2], Hoer [3], and FastHenry [4]. Along with PEEC, Ruehli [1] provides inductance formulae to facilitate PEEC development. Grover [2] uses Geometric Mean Distance method and provides greatly simplified formulae for inductance calculation. Hoer [3] provides exact formulae for calculating self inductance. Inside the FastHenry [4] package, an inductance formula was used for multipole-accerated inductance extraction application.

2.1 Ruehli's Formula

Given a single conductor with width (W), thickness (T), and length (*l*) as shown in Fig. 1. Ruehli gives the following self inductance formula [1].

$$
\begin{aligned}
\frac{L_{ii}}{l_i} = \frac{2\mu}{\pi}\Bigg\{ & \frac{\omega^2}{24u}\left[\ln\left(\frac{1+A_2}{\omega}\right)-A_5\right]+\frac{1}{24u\omega}\left[\ln(\omega+A_2)-A_6\right] \\
& +\frac{\omega^2}{60u}(A_4-A_3)+\frac{\omega^2}{24}\left[\ln\left(\frac{u+A_3}{\omega}\right)-A_7\right]+\frac{\omega^2}{60u}(\omega-A_2) \\
& +\frac{1}{20u}(A_2-A_4)+\frac{u}{4}A_5-\frac{u^2}{6\omega}\tan^{-1}\left(\frac{\omega}{uA_4}\right) \\
& +\frac{u}{4\omega}A_6-\frac{\omega}{6}\tan^{-1}\left(\frac{u}{\omega A_4}\right)+\frac{A_7}{4}-\frac{1}{6\omega}\tan^{-1}\left(\frac{u\omega}{A_4}\right) \\
& +\frac{1}{24\omega^2}\left[\ln(u+A_1)-A_7\right]+\frac{u}{20\omega^2}(A_1-A_4)+\frac{1}{60\omega^2 u}(1-A_2) \\
& +\frac{1}{60u\omega^2}(A_4-A_1)+\frac{u}{20}(A_3-A_4)+\frac{u^3}{24\omega^2}\left[\ln\left(\frac{1+A_1}{u}\right)-A_5\right] \\
& +\frac{u^3}{24\omega}\left[\ln\left(\frac{\omega+A_3}{u}\right)-A_6\right]+\frac{u^3}{60\omega^2}\left[(A_4-A_1)+(u-A_3)\right]\Bigg\}
\end{aligned}
$$

, *where*

$$
\begin{aligned}
& u \equiv \frac{l}{W} && \omega \equiv \frac{T}{W} \\
& A_1 \equiv \sqrt{1+u^2} && A_2 \equiv \sqrt{1+\omega^2} \\
& A_3 \equiv \sqrt{\omega^2+u^2} && A_4 \equiv \sqrt{1+\omega^2+u^2} \\
& A_5 \equiv \ln\left(\frac{1+A_4}{A_3}\right) && A_6 \equiv \ln\left(\frac{\omega+A_4}{A_1}\right) \qquad A_7 \equiv \ln\left(\frac{u+A_4}{A_2}\right)
\end{aligned}
$$

Since the errors of the above formula become large for very large values of u and very small values of ω, the second formula is given for infinitely thin conductors which is accurate for the case when $\omega < 0.01$. The following formula is given based on the assumption that $\omega = 0$ (zero thickness)

$$
\begin{aligned}
\frac{L_{ii}}{l_i} = \frac{\mu}{6\pi}\Bigg\{ & 3\ln\left[u+\sqrt{u^2+1}\right]+u^2+\frac{1}{u} \\
& +3u\ln\left[\frac{1}{u}+\sqrt{\frac{1}{u^2}+1}\right]-\left[u^{\frac{4}{3}}+\left(\frac{1}{u}\right)^{\frac{2}{3}}\right]^{\frac{3}{2}}\Bigg\}.
\end{aligned}
$$

2.2 Grover's Formula

Grover greatly simplified the self inductance formulae by Geometric Mean Distance Method as follows: (All lengths in this formula are in centimeters).

$$
\frac{L_{ii}}{l_i} = 0.002\left[\ln\frac{2l}{W+T}+\frac{1}{2}-\log_e e\right]
$$

The last term, $\log_e e$, is obtained from Table 1 for the given value of T/W *or* W/T. It is observed that $\log_e e$ is relative small terms.

The above formula gives negative values when the length of conductor is very short, in this situation, the following modified version of Grover's formula can be used in this range.

$$
\frac{L_{ii}}{l_i} = 0.002\left[\ln\frac{2l}{W+T}+\frac{1}{2}-0.2235\times\frac{(W+T)}{l}\right]
$$

Table 1. Constants for the Geometric Mean Distance

T/W *or* W/T	$\log_e e$	T/W *or* W/T	$\log_e e$
0	0	0.50	0.00211
0.025	0.00089	0.55	0.00203
0.05	0.00146	0.60	0.00197
0.10	0.00210	0.65	0.00192
0.15	0.00239	0.70	0.00187
0.20	0.00249	0.75	0.00184
0.25	0.00249	0.80	0.00181
0.30	0.00244	0.85	0.00179
0.35	0.00236	0.90	0.00178
0.40	0.00228	0.95	0.00177
0.45	0.00219	1.00	0.00177

2.3 FastHenry's Formula

In the widely used FastHenry package [4], the following self inductance formula is used.

$$
\begin{aligned}
\frac{L_{ii}}{l_i} = \frac{2\mu}{\pi}\Bigg[& \frac{1}{4}\left\{\frac{1}{w}\sinh^{-1}\left(\frac{w}{at}\right)+\frac{1}{t}\sinh^{-1}\left(\frac{t}{aw}\right)+\sinh^{-1}\left(\frac{1}{r}\right)\right\} \\
& +\frac{1}{24}\Bigg\{\frac{t^2}{w}\sinh^{-1}\left(\frac{w}{t\times at\times(r+ar)}\right)+\frac{w^2}{t}\sinh^{-1}\left(\frac{t}{w\times aw\times(r+ar)}\right) \\
& +\frac{t^2}{w^2}\sinh^{-1}\left(\frac{w^2}{t\times r\times(at+ar)}\right)+\frac{w^2}{t^2}\sinh^{-1}\left(\frac{t^2}{w\times r\times(aw+ar)}\right) \\
& +\frac{1}{w\times t^2}\sinh^{-1}\left(\frac{w\times t^2}{at\times(aw+ar)}\right)+\frac{1}{t\times w^2}\sinh^{-1}\left(\frac{t\times w^2}{aw\times(at+ar)}\right)\Bigg\} \\
& -\frac{1}{6}\left\{\frac{1}{w\times t}\tan^{-1}\left(\frac{w\times t}{ar}\right)+\frac{t}{w}\tan^{-1}\left(\frac{w}{t\times ar}\right)+\frac{w}{t}\tan^{-1}\left(\frac{t}{w\times ar}\right)\right\} \\
& -\frac{1}{60}\Bigg\{\frac{(ar+r+t+at)\times t^2}{(ar+r)(r+t)(t+at)(at+ar)} \\
& \qquad +\frac{(ar+r+w+aw)\times w^2}{(ar+r)(r+w)(w+aw)(aw+ar)} \\
& \qquad +\frac{(ar+aw+1+at)}{(ar+aw)(aw+1)(1+at)(at+ar)}\Bigg\} \\
& -\frac{1}{20}\left\{\frac{1}{r+ar}+\frac{1}{aw+ar}+\frac{1}{at+ar}\right\}\Bigg]
\end{aligned}
$$

, *where*

$$
\begin{aligned}
& w \equiv \frac{W}{l} && t \equiv \frac{T}{l} && r \equiv \sqrt{w^2+t^2} \\
& aw \equiv \sqrt{w^2+1} && at \equiv \sqrt{t^2+1} && ar \equiv \sqrt{w^2+t^2+1}
\end{aligned}
$$

2.4 Hoer's Formula

In [3], Hoer obtained a self inductance formula with the assumption that conductor consists of the multiple filaments and self inductance can be obtained from mutual inductance by volume self-integrals. Therefore this formula is similar to the formula of Hoer's mutual inductance in section 3.3. This formula is exact when the current flows through the full conductor are uniform.

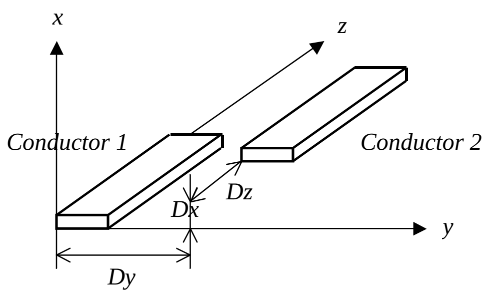

Fig. 2. Geometry of Conductors

$$
\begin{aligned}
L_{ii} = \frac{0.008}{W^2T^2}\Bigg[\Bigg[\Bigg[& \left(\frac{y^2z^2}{4}-\frac{y^4}{24}-\frac{z^4}{24}\right)\cdot x\cdot\ln\left(\frac{x+\sqrt{x^2+y^2+z^2}}{\sqrt{y^2+z^2}}\right) \\
& +\left(\frac{z^2x^2}{4}-\frac{x^4}{24}-\frac{z^4}{24}\right)\cdot y\cdot\ln\left(\frac{y+\sqrt{x^2+y^2+z^2}}{\sqrt{z^2+x^2}}\right) \\
& +\left(\frac{x^2y^2}{4}-\frac{x^4}{24}-\frac{y^4}{24}\right)\cdot z\cdot\ln\left(\frac{z+\sqrt{x^2+y^2+z^2}}{\sqrt{x^2+y^2}}\right) \\
& +\frac{1}{60}\left(x^4+y^4+z^4-3x^2y^2-3y^2z^2-3z^2x^2\right)\sqrt{x^2+y^2+z^2} \\
& -\frac{xyz^3}{6}\tan^{-1}\left(\frac{xy}{z\sqrt{x^2+y^2+z^2}}\right)-\frac{xy^3z}{6}\tan^{-1}\left(\frac{xz}{y\sqrt{x^2+y^2+z^2}}\right) \\
& -\frac{x^3yz}{6}\tan^{-1}\left(\frac{yz}{x\sqrt{x^2+y^2+z^2}}\right)\Bigg]_0^W(x)\Bigg]_0^T(y)\Bigg]_0^l(z)
\end{aligned}
$$

, *where*

$$
\Big[\Big[\Big[\, f(x,y,z)\,\Big]_{q_2}^{q_1}(x)\,\Big]_{r_2}^{r_1}(y)\,\Big]_{s_2}^{s_1}(z) \equiv \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}(-1)^{i+j+k+1}\cdot f(q_i,r_j,s_k)
$$

Note that the above function definition needs to add 8 terms since it invokes three levels of the summation. The runtime and complexity can be much larger than other formulae.

3. Mutual Inductance Extraction

In this section, we briefly summarize the above-mentioned popular inductance formulae for mutual inductance extraction. Since FastHenry also uses Grover's mutual inductance formulae, we only present Grover's formula.

3.1 Ruehli's Formula

Ruehli considered the case when two parallel conductors are positioned in parallel, the directional distances of the conductor, *Dx*, *Dy* and *Dz*, are shown in Fig. 2. The lengths of these two conductors are l_i and l_j, respectively. It was assumed that the directions of current in two conductors are the same (along its length). Ruehli obtains the following formula:

$$
\frac{L_{ij}}{l_i}=\frac{\mu}{4\pi}\sum_{m=1}^{4}\left\{(-1)^{m+1}g_m\ln\left(g_m+\sqrt{g_m^2+r^2}\right)-\sqrt{g_m^2+r^2}\right\}
$$

, *where*

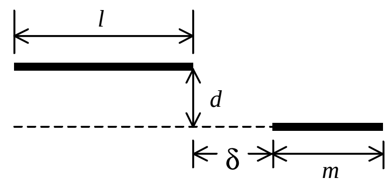

Fig. 3. (a) General position of two conductors (filaments)

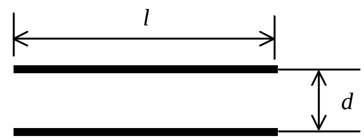

Fig. 3. (b) Two equal-length parallel conductors (filaments)

$$
v\equiv\frac{l_j}{l_i}\qquad p\equiv\frac{Dz}{l_i}\qquad r\equiv\frac{\sqrt{Dx^2+Dy^2}}{l_i}
$$

$$
g_1=1+p\quad g_2=1+p-v\quad g_3=p-v\quad g_4=p
$$

3.2 Grover's Formula

As shown in Fig. 3 (a), given two conductors with length (*l*) and (*m*), distance (*d*), and spacing (δ), respectively, Grover gives the following simplified mutual inductance formula [2]. (All lengths in this formula are in centimeters).

$$
\begin{aligned}
L_{ij}=0.001\Big[\, & \alpha\sinh^{-1}\left(\frac{\alpha}{d}\right)-\beta\sinh^{-1}\left(\frac{\beta}{d}\right)-\gamma\sinh^{-1}\left(\frac{\gamma}{d}\right)+\delta\sinh^{-1}\left(\frac{\delta}{d}\right) \\
& -\sqrt{\alpha^2+d^2}+\sqrt{\beta^2+d^2}+\sqrt{\gamma^2+d^2}-\sqrt{\delta^2+d^2}\,\Big]
\end{aligned}
$$

, *where*

$$
\sinh^{-1}(x)=\ln\left(x+\sqrt{x^2+1}\right)
$$

$$
\alpha=l+m+\delta\qquad \beta=l+\delta\qquad \gamma=m+\delta
$$

Note that in the case when the two conductors are overlapped then δ is negative.

The above formula can also be obtained from the more special case of equal-length parallel conductors by applying the laws of summation of mutual inductance. Therefore the total mutual inductance can be obtained by the summation and subtraction of the mutual inductance for four parallel equal-length conductors.

$$
M=0.5\left\{\left(M_{l+m+\delta}+M_{|\delta|}\right)-\left(M_{l+\delta}+M_{m+\delta}\right)\right\}
$$

where M_l is the mutual inductance of two equal-length and parallel conductors with length *l* and distance *d* as shown in Fig. 3 (b).

In this case, the mutual inductance can be calculated by

$$
M_l=0.002\,l\left[\ln\left(\frac{l}{d}+\sqrt{1+\frac{l^2}{d^2}}\right)-\sqrt{1+\frac{d^2}{l^2}}+\frac{d}{l}\right].
$$

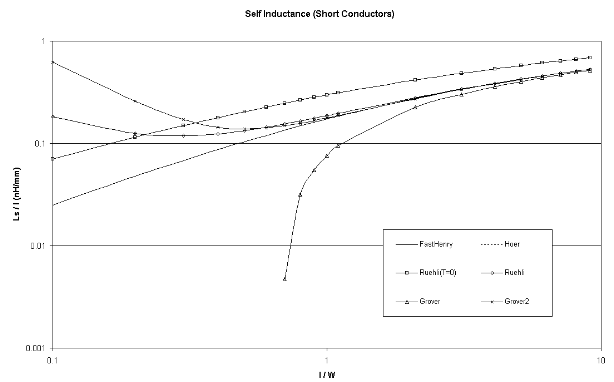

Fig. 4. (a) self inductance for normalized length 0.1-10

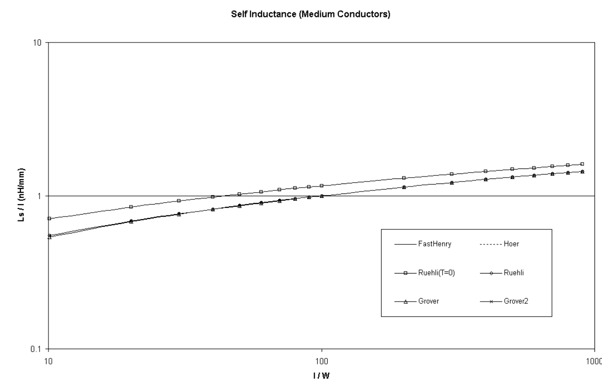

Fig. 4. (b) self inductance for normalized length 10-1000

An interesting point to mention is that although this formula seems different from the Ruehli's, they are mathematically equivalent.

3.3 Hoer's Formula

The difference between Grover/Ruehli's and Hoer's formula is the number of filaments are used. While Grover/Ruehli's formula assumes only one filament for each conductor while Hoer formula assumes multiple filaments for each conductor. As a result, Hoer's formula is more accurate and much more complicated than Grover/Ruehli's.

The Hoer's mutual inductance formula between two parallel conductors with width (W_1 *and* W_2), thickness (T_1 *and* T_2), and length (l_1 *and* l_2), respectively is as follows [3].

$$L_{ij} = \frac{0.001}{W_1 W_2 T_1 T_2} \Bigg[\Bigg[\Bigg[\left(\frac{y^2 z^2}{4} - \frac{y^4}{24} - \frac{z^4}{24}\right) \cdot x \cdot \ln\left(\frac{x+\sqrt{x^2+y^2+z^2}}{\sqrt{y^2+z^2}}\right)$$

$$+ \left(\frac{z^2 x^2}{4} - \frac{x^4}{24} - \frac{z^4}{24}\right) \cdot y \cdot \ln\left(\frac{y+\sqrt{x^2+y^2+z^2}}{\sqrt{z^2+x^2}}\right)$$

$$+ \left(\frac{x^2 y^2}{4} - \frac{x^4}{24} - \frac{y^4}{24}\right) \cdot z \cdot \ln\left(\frac{z+\sqrt{x^2+y^2+z^2}}{\sqrt{x^2+y^2}}\right)$$

$$+ \frac{1}{60}\left(x^4+y^4+z^4-3x^2y^2-3y^2z^2-3z^2x^2\right)\sqrt{x^2+y^2+z^2}$$

$$- \frac{xyz^3}{6}\tan^{-1}\left(\frac{xy}{z\sqrt{x^2+y^2+z^2}}\right) - \frac{xy^3 z}{6}\tan^{-1}\left(\frac{xz}{y\sqrt{x^2+y^2+z^2}}\right)$$

$$- \frac{x^3 yz}{6}\tan^{-1}\left(\frac{yz}{x\sqrt{x^2+y^2+z^2}}\right) \Bigg]_{D_y+W_2-W_1,\,D_y}^{D_y-W_1,\,D_y+W_2} (x) \Bigg]_{D_x+T_2-T_1,\,D_x}^{D_x-T_1,\,D_x+T_2} (y) \Bigg]_{D_z+l_2-l_1,\,D_z}^{D_z-l_1,\,D_z+l_2} (z)$$

, where

$$\left[\left[\left[f(x,y,z) \right]_{q_2,q_4}^{q_1,q_3}(x)\right]_{r_2,r_4}^{r_1,r_3}(y)\right]_{s_2,s_4}^{s_1,s_3}(z) \equiv \sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{4} (-1)^{i+j+k+1} \cdot f(q_i, r_j, s_k)$$

Note that the above definition needs to add 64 terms since it invokes three levels of the summation. The runtime and complexity can be much larger than other formulae.

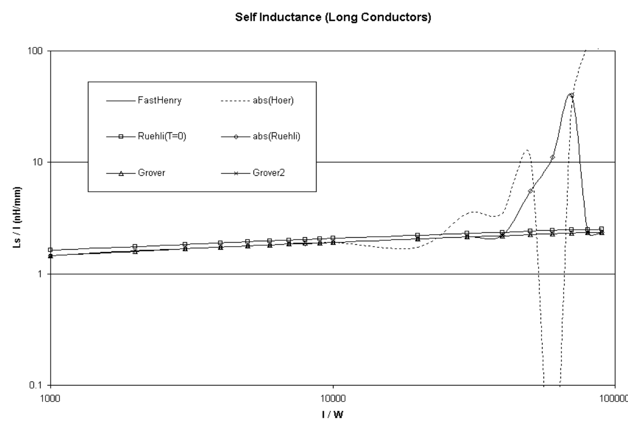

Fig. 4. (c) self inductance for normalized length 1000-100000

4. Computational Results

In this section, we present the evaluation results for the all the above-mentioned inductance formulae in the wide range of VLSI applications.

The computational results of self inductance with width equals to 0.25 μ m and thickness equals to 0.1 μ m are shown in Fig. 4. Fig. 4 (a), (b), and (c) show the normalized lengths of conductors (*l/W*) in 0.1-10, 10-1000, and 1000-100000, respectively.

Fig. 4 (a) shows that in the range when the normalized length is short (<10), many formulae present either irregular behavior or huge errors. For example, the inductance value of Grover 2 (without table) is increasing as the length goes down and the value of Grover's first formula decreases too much. Furthermore, Grover's first formula can even generate negative inductance values when the normalized length is shorter than 0.5. The Ruehli's first formula becomes flat. Fig. 4 (a) also shows that Hoer's and FastHenry's formulae are the most consistent and reliable when the normalized length is short. Although the values Ruehli's second formula(with zero thickness) shows consistency for this application range, it is overestimated by 20% comparing to Hoer's and FastHenry's result. It is quite often that the normalized length is less than 10. For example, when the conductor width is 100

Table 2. Comparisons of self inductance formulae

Formula	Short length (l/W < 10)	Medium length (10 ~ 1000)	Large length (l/W > 10000)
Hoer	O	O	X
FastHenry	O	O	O
Ruehli	X	O	X
Ruehli(T=0)	O (T/W <0.01) (30% larger)	O (T/W <0.01) (30% larger)	O (T/W <0.01) (30% larger)
Grover	X	O	O
Grover 2	X	O	O

micron, the conductor length has to be larger than 1000 micron to make normalized length larger than 10.

Fig. 4 (b) shows that when the normalized lengths are within 10-1000, all formulae present sound behavior and all the values are almost identical.

Fig. 4 (c) shows that although Hoer's formula is accurate for short normalized length, it has large oscillations when the normalized length is larger than 10000. Similar behavior also occurred for the Ruehli's formula.

To understand the irrational behavior for the exact formulae provided by Hoer, we carefully exam the numerical result of each term of Hoer's formula. We conclude that the major source of error of Hoer's formula comes from the numerical dispersion since the high order term such as z^4 terms create huge numbers while the values of other terms are small. As a result, carefulness must be deployed when we implement Hoer's formula. For example, we should avoid directly summation or subtraction operations between large and small numbers. We suggest to switch over to Grover's or FastHenry's formulae when the normalized length is larger than 10000.

As a result, we conclude that only FastHenry's formula is valid and numerically stable for all range of VLSI applications and all other formulae have their applicable ranges. Table 2 summarizes our conclusion.

We carefully implemented all the four mutual formulae and present the experimental results in Fig. 5 and Fig. 6. Since the mutual inductance for two conductors with different lengths and positions can be obtained from the special case when two conductors are parallel and with equal-length. We only present the result for this special case. The thickness of both conductors is 0.1 μm and width is 0.25 μm for our experiments. Since Ruehli's formula is equivalent to Grover's and FastHenry uses Grover's formula as well, we use Grover/Ruehli to represent the values for Grove, Ruehli, and FastHenry.

The relationships of the distance to the mutual inductance between two conductors are shown in Fig. 5. The relationships of the length to the mutual inductance between two conductors are shown in Fig. 6. Comparing Fig. 5 and Fig. 6, we know that mutual inductance is a weaker function in the distance of two conductors rather than the lengths.

Unlike self inductance formulae, all the mutual inductance formulae are consistent and the values are matched. The maximum error is within 0.1%. We conclude that all the mutual

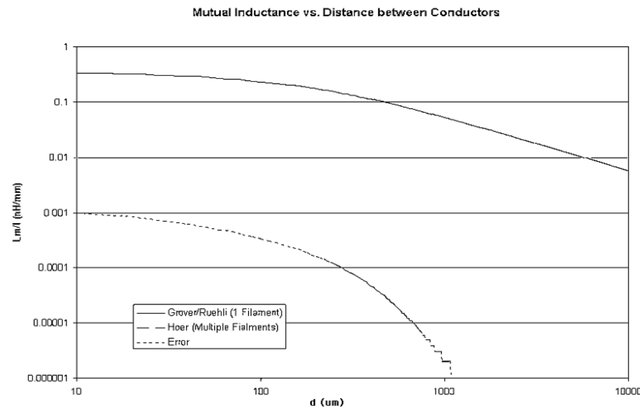

Fig. 5. Mutual Inductance vs. Distance between conductors

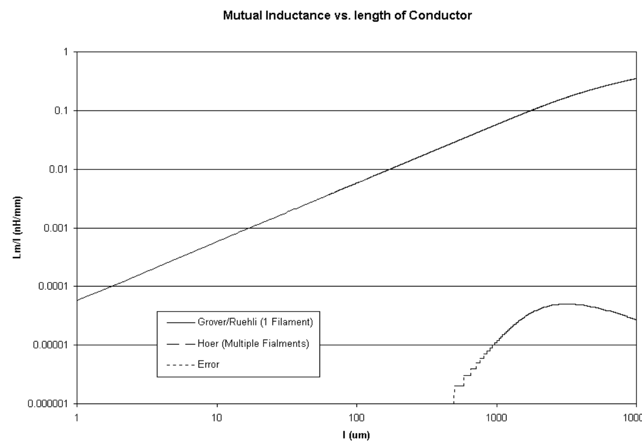

Fig. 6. Mutual Inductance vs. length of conductor

inductance formulae are reliable in the VLSI application.

5. Conclusions

In this paper, we extensively tested several widely used partial self and mutual inductance formulae. The experimental results show that only FastHenry's formula is numerically sound for all range of VLSI applications for self inductance while all mutual inductance formulae are showing excellent matches with each other. As a result, we must pay attention while applying these formulae.

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