Comparison of Inductance Calculation Techniques

Tony Morcos
Magnequench Technology Center
Research Triangle Park, North Carolina

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

VCM Baseline: Geometry

Axially-magnetized MQ3-F 42 NdFeB disk

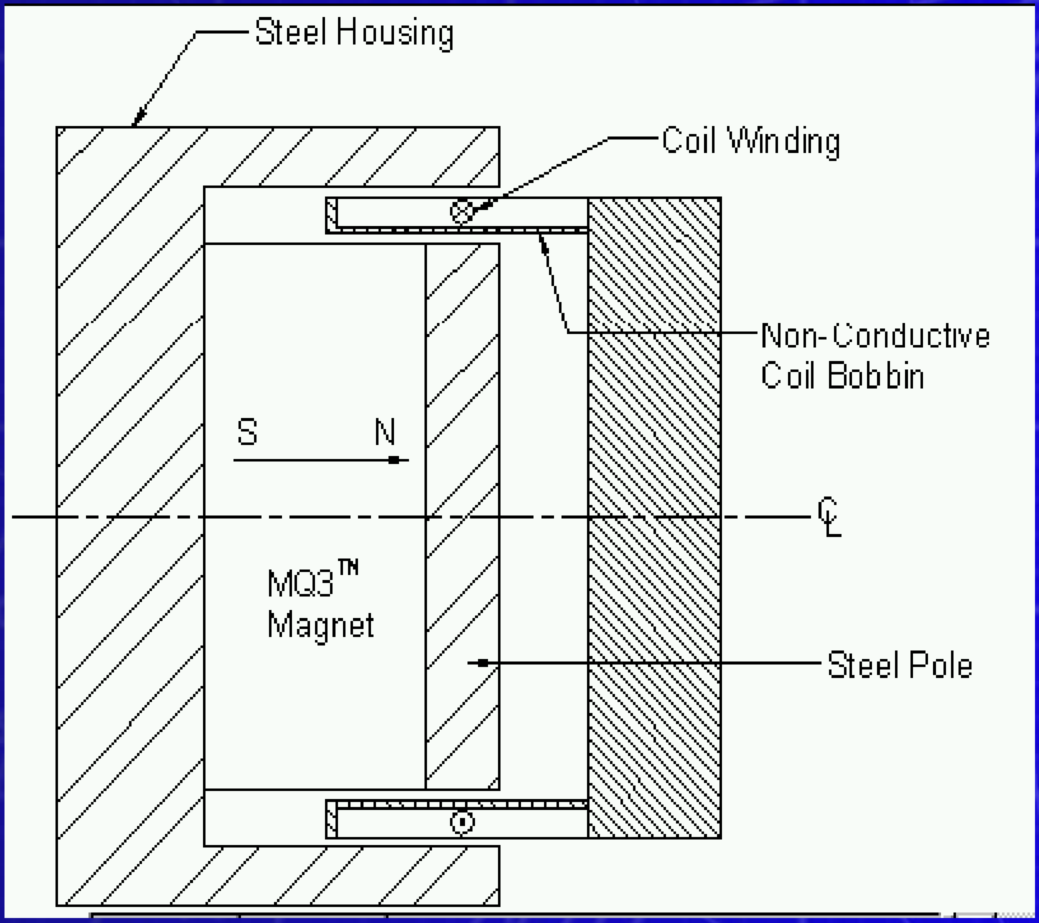
- Br = 13.1 kG
- Hc = 12.3 kOe

1018 CRS Pole and Housing

Layer-Wound Coil

- Round Cu wire, 0.2 mm diameter
- 320 turns, R = 17 Ohms

Non-conductive coil bobbin



Magnequench

All rights reserved. This presentation is Copyright © Magnequench, Inc. 2000

VCM Baseline: Static FEA

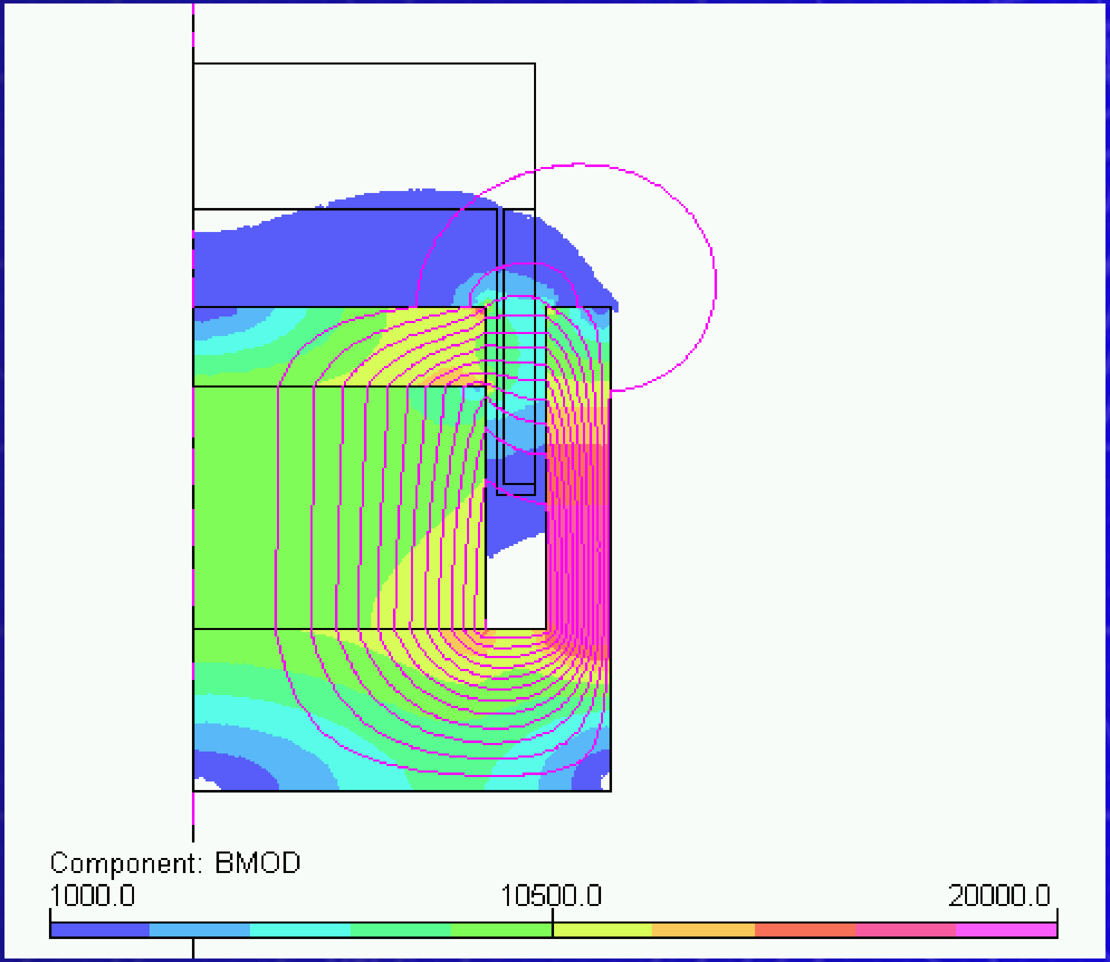
Axi-Symmetric Static DC FEA

OD = 31.8 mm

L = 28.6 mm

K_f = 9.8 N/A

Moderate saturation of steel housing



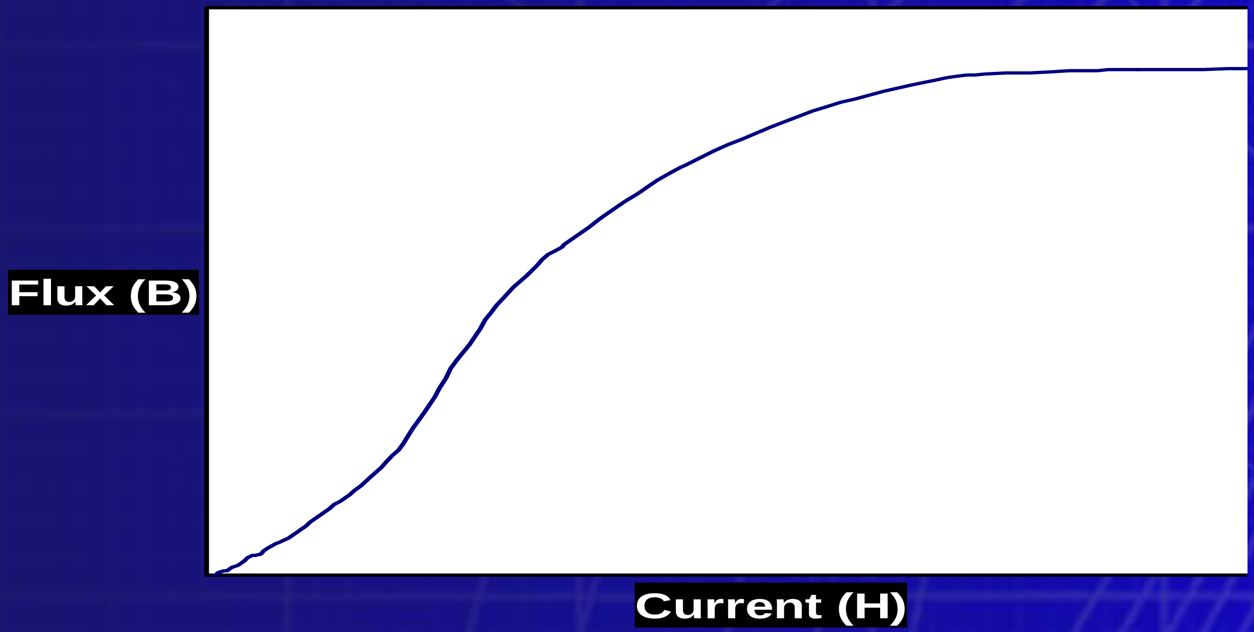
Magnequench

All rights reserved. This presentation is Copyright © Magnequench, Inc. 2000

Definitions of Inductance

Inductance (L) is the change in magnetic flux with respect to a change in coil current:

$$L = \frac{\Delta\phi}{\Delta i} = \frac{d\phi}{di}$$



All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

Definitions of Inductance: Units

The SI unit for inductance is the Henry:

$$L = \frac{N\phi}{I} = Henries$$

$$L = \frac{Weber \times turn}{Ampere}$$

$$L = \frac{Volt \times \sec ond \times turn}{Ampere}$$

Magnequench

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

Definitions of Inductance

L is proportional to the square of the number of coil turns and inversely proportional to the magnetic circuit reluctance:

$$L = \frac{N^2}{\Re}$$

L is also proportional to the energy in the magnetic field produced by the current:

$$L = \frac{2E}{I^2}$$

Magnequench

All rights reserved. This presentation is Copyright © Magnequench, Inc. 2000

Definitions of Inductance: General Concepts

Inductance is *electrical inertia*

In time domain, L limits current rise in coil winding when driven with step voltage

Voltage drop across an inductor:

$$V = L\frac{di}{dt}$$

In frequency domain, L limits bandwidth

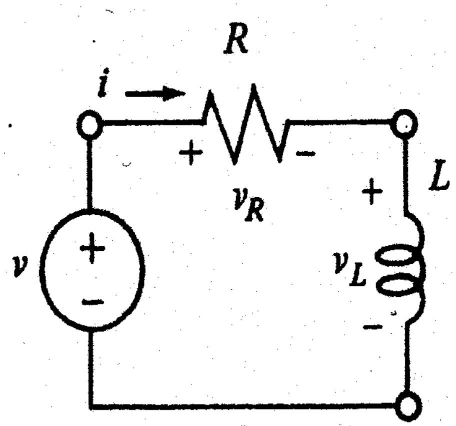
Magnequench

All rights reserved. This presentation is Copyright © Magnequench, Inc. 2000

VCM Equivalent Circuit

$$V = IR + L\frac{di}{dt} + K_B(velocity)$$

Assume stationary coil: velocity = 0



$$v(t) = u(t)$$

$$i(t) = \frac{1}{R}(1 - e^{-Rt/L})u(t)$$

$$v_R(t) = (1 - e^{-Rt/L})u(t)$$

$$v_L(t) = e^{-Rt/L}u(t)$$

$$v(t) = \delta(t)$$

$$i(t) = \frac{1}{L}e^{-Rt/L}u(t)$$

$$v_R(t) = \frac{R}{L}e^{-Rt/L}u(t)$$

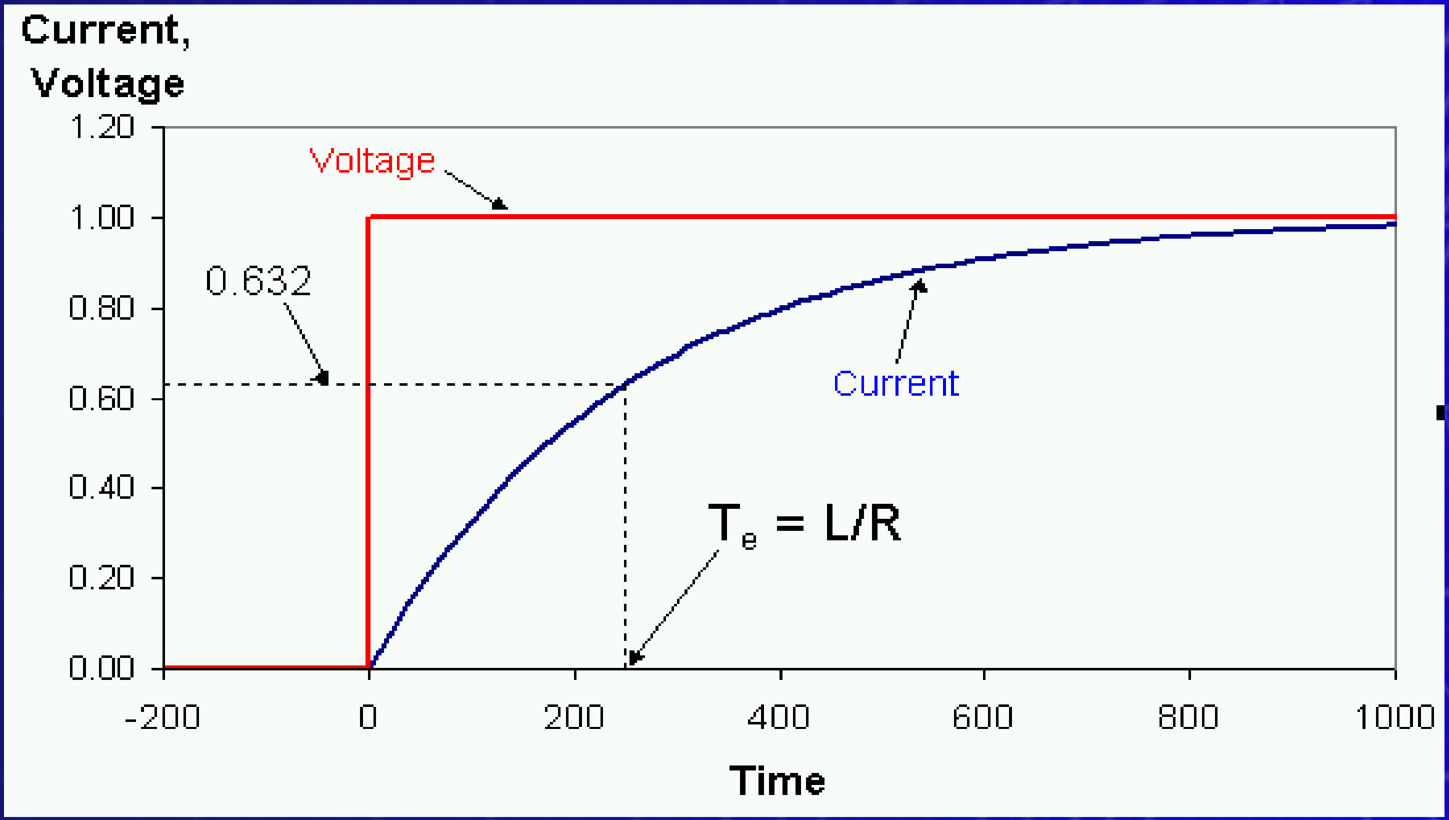
$$v_L(t) = \delta(t) - \frac{R}{L}e^{-Rt/L}u(t)$$

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

L Measurement: Step Response

Apply step voltage to coil, monitor current rise:

T_e = Electrical time constant = L/R

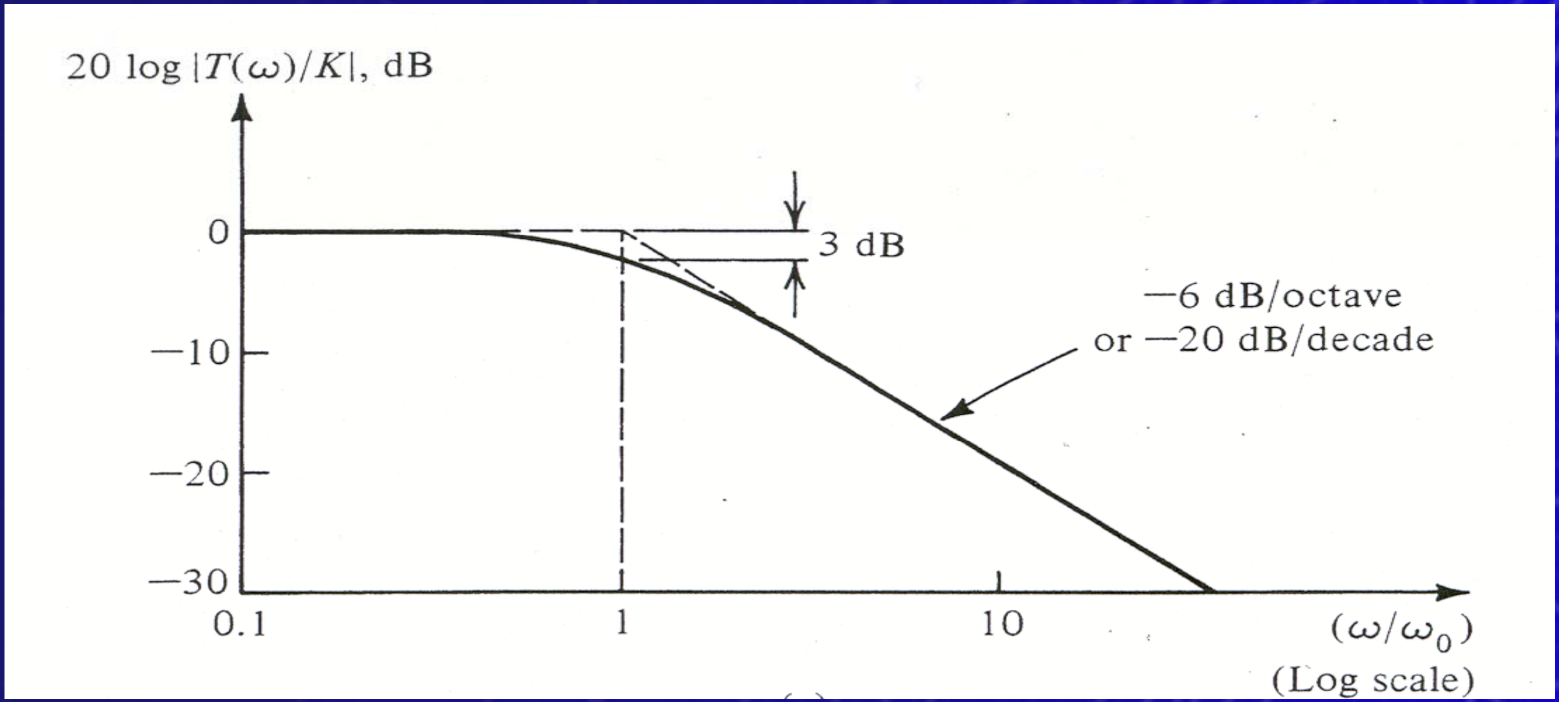


All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

L Measurement: Bode Plot

Sweep frequency, monitor Gain = V_{out} / V_{in}

f_o = 3dB cutoff frequency = $1/T_e$

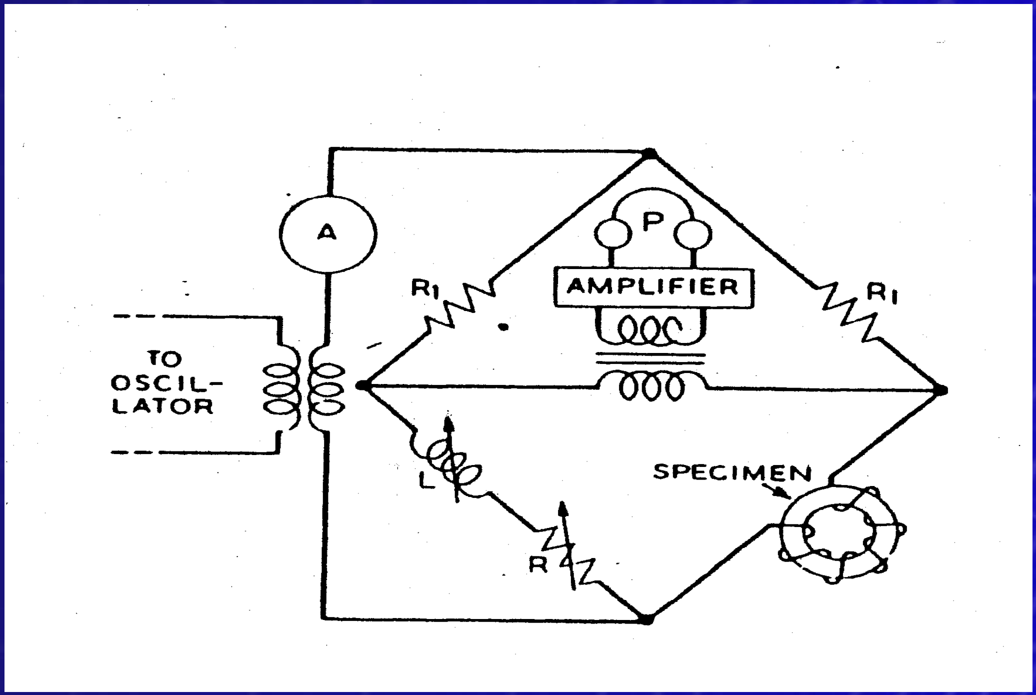


Magnequench
All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

L Measurement: Bridge Circuit

Bridge circuit suitable for small signal measurements

Allows for L measurement at various frequencies to account for eddy currents ("polluted" or "apparent" L)

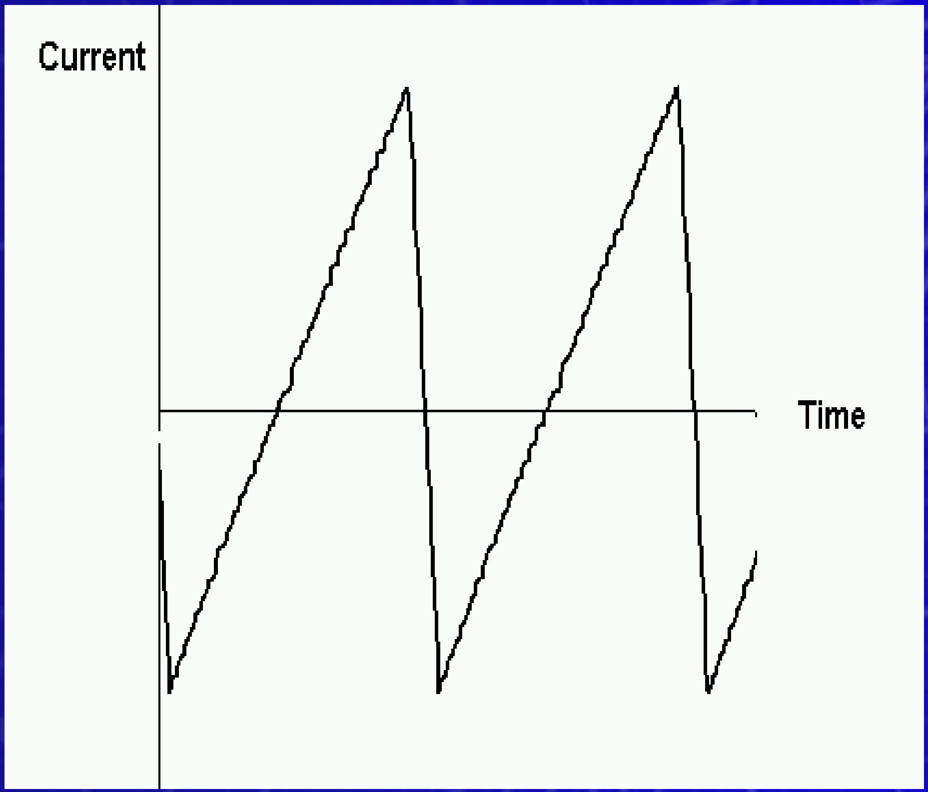


All rights reserved. This presentation is Copyright © Magnequench, Inc. 2000

L Measurement: Ramped Current

$$V = L\frac{di}{dt}$$

$$L = V/(\frac{di}{dt})$$



Sencore is only meter manufacturer to employ this method

Small signal measurement

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

L Calculation: Grover's Formulas

"Inductance Calculations: Working Formulas and Tables" by Frederick W. Grover

Written in 1946 -- still valid

ISBN: 0-87664-513-9

Open air inductance only: no magnetic fields or ferromagnetic coupling

Valid only at low frequencies: eddy currents not accounted for

$$L = 2.66mH$$

Magnequench

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

L Calculation: Flux Linkages

Axisymmetric analysis, solution type = Modified R*A :

$$\phi = \frac{2\pi}{CoilArea} \int_{coilarea} (rA)\,drdz$$

For linear problems with no permanent magnets:

$$L = \frac{\phi}{i}$$

To account for permanent magnet sources and non-linear materials, use “perturbation method”:

$$L = \frac{\Delta\phi}{\Delta i} = \frac{(\phi_2 - \phi_1)}{(i_2 - i_1)}$$

Magnequench

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

L Calculation: Flux Linkages

Series of static dc FEA runs:

$$I = 0.80 - 1.20A$$

$$\Delta I = 0.05A$$

Eddy currents neglected in static analysis

L is independent of current magnitude

$$L = 3.95mH$$

Magnequench

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

L Calculation: Stored Energy

For linear problems with no permanent magnets:

$$Energy = E = \frac{1}{2} L \times i^2$$

$$L = \frac{2E}{i^2}$$

To account for permanent magnet sources and non-linear materials, use “perturbation method”:

$$L = \frac{2\Delta E}{\Delta i^2} = \frac{2(E_2 - E_1)}{(i_2 - i_1)^2}$$

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

L Calculation: Stored Energy

Series of static dc FEA runs:

$$I = 0.80 - 1.20A$$

$$\Delta I = 0.05A$$

Eddy currents neglected in static analysis

L is dependent upon magnitude of current

$$L = 3.59 - 3.77mH$$

Magnequench

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

L Calculation: Transient FEA Step Response

Transient solver, axisymmetric 2-D, no motion of coil

- Adaptive time stepping
- Multiple output-time cases
- Drive function = Step input

Apply electrical conductivity values to copper, steel, magnet, and coil bobbin (if conductive)

Monitor rise of coil force vs. time

- Coil current directly proportional to coil force (Maxwell stress)
- Choose appropriate time steps for resolution of steep current rise

Calculate electrical time constant (T_e = L/R)

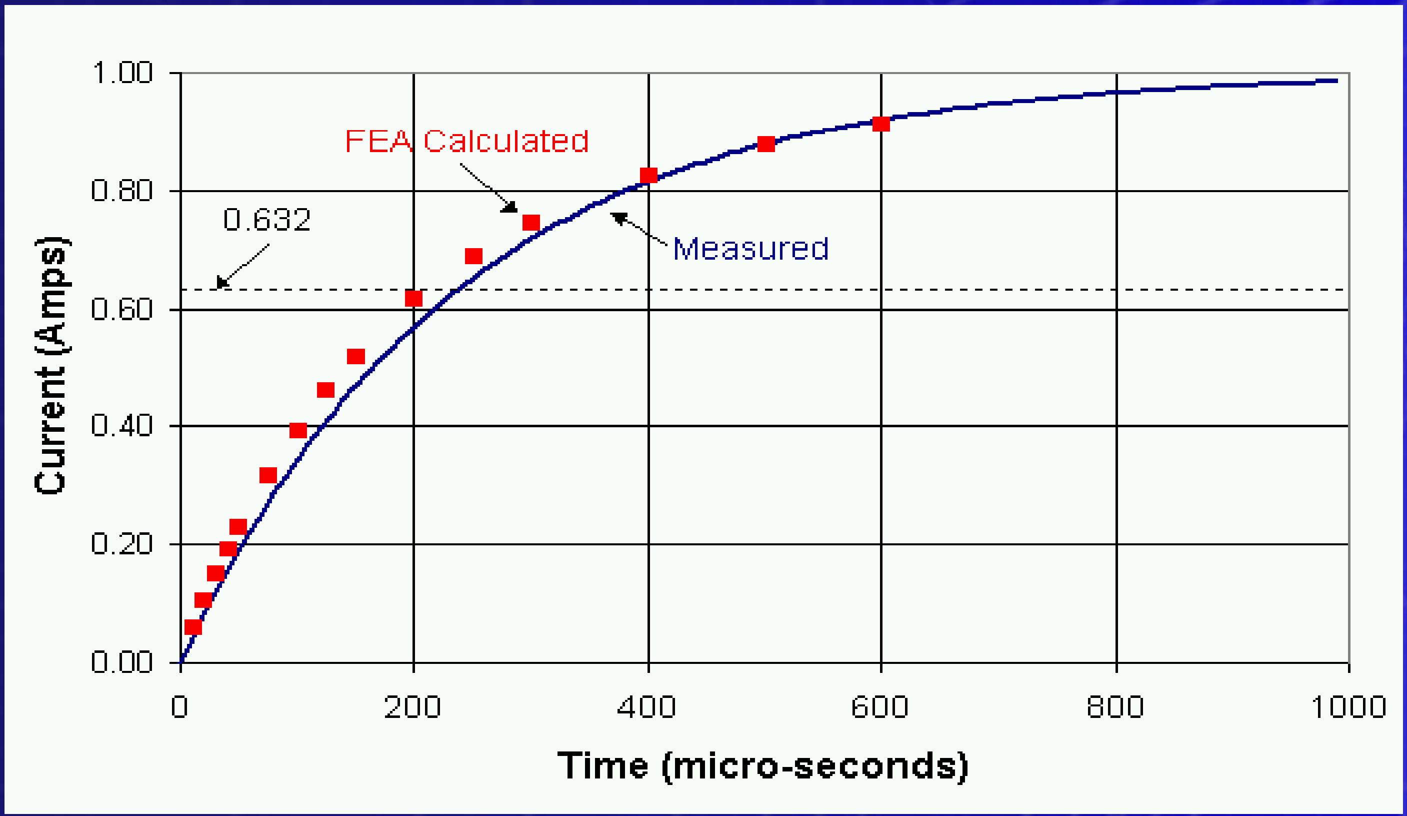
Eddy currents and saturation fully accounted for

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

L Calculation: Transient FEA Step Response

Measured: T_e = 255 microseconds -- L = 4.03 mH

Calculated: T_e = 210 microseconds -- L = 3.57 mH



All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

Measurements vs. Calculations

Summary of measured and calculated results:

In Field/ Open Air	Frequency (Hz)	Measurement/ Calculated	Method	Inductance (mH)
Open Air	100	Calculated	Grover's Formulas	2.66
Open Air	100	Measured	Bridge	2.66
Open Air	1000	Measured	Bridge	2.36
Open Air	10,000	Measured	Bridge	1.27
Open Air	5,000	Measured	Sencore	2.05
Open Air	N/A	Measured	Step Response	2.75
Open Air	N/A	Calculated	Flux Linkages	2.63
Open Air	N/A	Calculated	Stored Energy	2.63
Open Air	N/A	Calculated	Transient FEA	2.52
In Field	100	Measured	Bridge	4.35
In Field	1,000	Measured	Bridge	3.24
In Field	10,000	Measured	Bridge	0.95
In Field	5,000	Measured	Sencore	2.80
In Field	N/A	Measured	Step Response	4.03
In Field	N/A	Calculated	Flux Linkages	3.95
In Field	N/A	Calculated	Stored Energy	3.59 - 3.77
In Field	N/A	Calculated	Transient FEA	3.57

Magnequench

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

Measurements vs. Calculations

Less variation between open-air measurements and calculations: linear materials, no permanent magnets

Inductance is frequency-dependent: eddy currents retard changes in flux

Static FEA cannot account for eddy currents or for frequency dependence of inductance

Transient FEA accounts for frequency spectrum of step-response waveform

Magnequench

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

Error Sources

Measurement errors:

1. Is coil stationary with respect to magnet?
2. Nonlinear materials, saturation

Calculation errors:

1. Eddy currents (static analyses)
2. Floating point accuracy (perturbation)
3. Material properties - especially conductivity

Magnequench

All rights reserved. This presentation is
Copyright © Magnequench, Inc. 2000

Conclusions

There are many different methods of defining, measuring and calculating inductance, all of which yield different values

- Correlation of various methods within +/- 20% is reasonable

Servo system designers generally want "apparent" inductance

- Eddy currents and saturation effects should be accounted for

For FEA inductance calculations, only the transient step response method accounts for eddy currents

- Flux linkage and stored energy techniques are acceptable for low frequency situations, where eddy currents are not a significant factor

Magnequench

All rights reserved. This presentation is Copyright © Magnequench, Inc. 2000