Chapter 14

Inductance and Magnetic Fields

Objectives

When you have studied the material in this chapter you should be able to:

- **explain the meaning and significance of terms such as magnetic field strength, magnetic flux, permeability, reluctance and inductance;**
- **outline the basic principles of electromagnetism and apply these to simple calculations of magnetic circuits;**
- **describe the mechanisms of self-induction and mutual induction;**
- **estimate the inductance of simple inductors from a knowledge of their physical construction;**
- **describe the relationship between the current and voltage in an inductor for both DC and AC signals;**
- **calculate the energy stored in an inductor in terms of its inductance and its current;**
- **describe the operation and characteristics of transformers;**
- **explain the operation of a range of inductive sensors.**

14.1 Introduction

We noted in Chapter 13 that capacitors store energy by producing an electric field within a piece of dielectric material. Inductors also store energy, but in this case it is stored within a *magnetic* field. In order to understand the operation and characteristics of inductors, and related components such as transformers, first we need to look at *electromagnetism*.

14.2 Electromagnetism

A wire carrying an electrical current causes a **magnetomotive force** (**m.m.f.**), F, which produces a **magnetic field** about it, as shown in Figure 14.1(a). One can think of an m.m.f. as being similar in some ways to an e.m.f. in an electric circuit. The presence of an e.m.f. results in an electric field and in the production of an electric current. Similarly, in magnetic

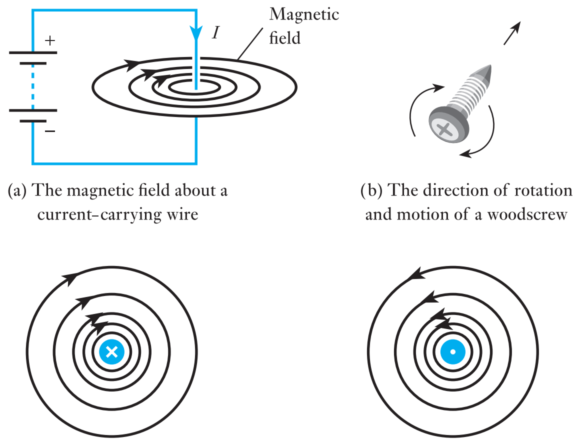


Figure 14.1 The magnetic effects of an electric current in a wire

(a) The magnetic field about a current-carrying wire

(b) The direction of rotation and motion of a woodscrew

(c) The magnetic field about a current flowing into the page

(d) The magnetic field about a current flowing out of the page

circuits, the presence of an m.m.f. results in a magnetic field and the production of magnetic flux. The m.m.f. has units of amperes and for a single wire F is simply equal to the current I.

The magnitude of the field is defined by the **magnetic field strength**, H, which in this arrangement is given by

$$H = \frac{I}{l} \tag{14.1}$$

where I is the current flowing in the wire and l is the length of the magnetic circuit. The units of H are amperes per metre. The length of the circuit increases as the circumference of the circles increases, and hence the field gets weaker as we move further from the wire. Since the circumference of a circle is linearly related to its radius (being equal to $2\pi r$), the field strength is directly proportional to the current I and inversely proportional to the distance from the wire.

Example 14.1 **A straight wire carries a current of 5 A. What is the magnetic field strength, H, at a distance of 100 mm from the wire?**

Since the field about a straight wire is symmetrical, the length of the magnetic path at a distance r from the wire is given by the circumference of a circle of this radius. When $r = 100$ mm, the circumference is equal to $2\pi r = 0.628$ m. Therefore, from Equation 14.1

$$\text{magnetic field strength, } H = \frac{I}{l} = \frac{5}{0.628} = 7.96 \text{ A/m}$$

The direction of the electric field is determined by the direction of the current in the wire. For a long straight wire the electric field is circular about its axis, and one way of remembering the direction of the magnetic field is to visualise a woodscrew lying along the axis of the wire. In this arrangement, the rotation of the screw bears the same relationship to the direction of motion of the screw as the direction of the magnetic field has to the flow of current in the wire. This is shown in Figure 14.1(b). If we imagine a wire running perpendicular through this page, then a current flowing into the page would produce a clockwise magnetic field, while one flowing out of the page would result in an anticlockwise field, as shown in Figures 14.1(c) and 14.1(d). The direction of current flow in these figures is indicated by a cross to show current into the page and a dot to show current coming out of the page. To remember this notation, you may find it useful to visualise the head or the point of the screw of Figure 14.1(b).

The magnetic field produces a **magnetic flux** that flows in the same direction as the field. Magnetic flux is given the symbol Φ, and the unit of flux is the **weber** (Wb).

The strength of the flux at a particular location is measured in terms of the **magnetic flux density**, B, which is the flux per unit area of cross-section. Therefore

$$B = \frac{\Phi}{A} \tag{14.2}$$

The unit of flux density is the tesla (T), which is equal to 1 Wb/m^2.

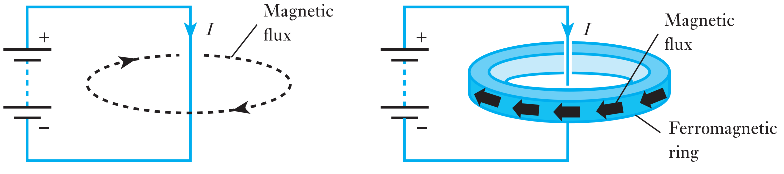
The flux density at a point depends on the strength of the field at that point, but it is also greatly affected by the material present. If a current-carrying wire is surrounded by air, this will result in a relatively small amount of magnetic flux as shown in Figure 14.2(a). However, if the wire is surrounded by a ferromagnetic ring, the flux within the ring will be orders of magnitude greater, as illustrated in Figure 14.2(b).

Magnetic flux density is related to the field strength by the expression

$$B = \mu H \tag{14.3}$$

where μ is the **permeability** of the material through which the field passes. One can think of the permeability of a material as a measure of the ease with which a magnetic flux can pass through it. This expression is often rewritten as

$$B = \mu_0 \mu_r H \tag{14.4}$$

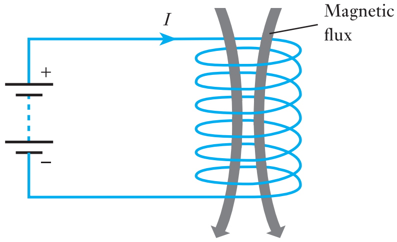


(a) The magnetic flux about a current-carrying wire in air

(b) The effect of adding a ferromagnetic ring

Figure 14.2 Magnetic flux associated with a current-carrying wire

Figure 14.3 The magnetic field in a coil



where μ_0 is the permeability of free space, and μ_r is the relative permeability of the material present. μ_0 is a constant with a value of $4\pi \times 10^{-7}$ H/m. μ_r is the ratio of the flux density produced in a material to that produced in a vacuum. For air and most non-magnetic materials, $\mu_r = 1$ and $B = \mu_0 H$. For ferromagnetic materials, μ_r may have a value of 1000 or more. Unfortunately, for ferromagnetic materials μ_r varies considerably with the magnetic field strength.

When a current-carrying wire is formed into a coil, as shown in Figure 14.3, the magnetic field is concentrated within the coil, and it increases as more and more turns are added. The m.m.f. is now given by the product of the current I and the number of turns of the coil N, so that

$$F = IN \tag{14.5}$$

For this reason, the m.m.f. is often expressed in *ampere-turns*, although formally its units are amperes, since the number of turns is dimensionless.

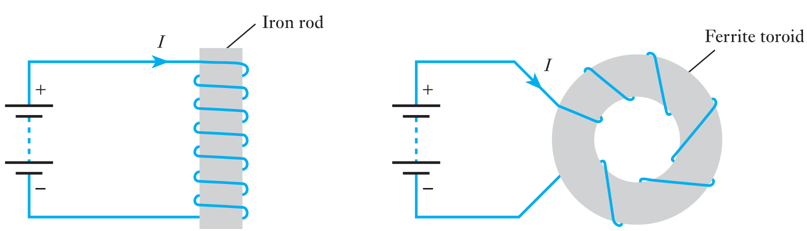
In a long coil with many turns, most of the magnetic flux passes through the centre of the coil. Therefore, it follows from Equations 14.1 and 14.5 that the magnetic field strength produced by such a coil is given by

$$H = \frac{IN}{l} \tag{14.6}$$

where l is the length of the flux path as before.

As discussed earlier, the flux density produced as a result of a magnetic field is determined by the permeability of the material present. Therefore, the introduction of a ferromagnetic material in a coil will dramatically increase the flux density. Figure 14.4 shows examples of arrangements that use such materials in coils. The first shows an iron bar placed within a

Figure 14.4 The use of ferromagnetic materials in coils



(a) A coil wound on an iron rod to increase the flux density

(b) A coil wound on a ferrite toroid

linear coil to increase its flux density. The second shows a coil wound on a ferrite toroid (a ring with a circular cross-section).

Example 14.2

A coil is formed by winding 500 turns of wire onto a non-magnetic toroid that has a mean circumference of 400 mm and a cross-sectional area of 300 mm^2. If the current in the coil is 6 A, calculate:

(a) the magnetomotive force;
(b) the magnetic field strength within the coil;
(c) the flux density in the coil;
(d) the total flux.

How would these quantities be affected if the toroid were replaced by one of similar dimensions but constructed of a magnetic material with a μ_r of 100?

(a) The magnetomotive force is given by the 'ampere-turns' of the coil and is therefore

$$\begin{aligned} F &= IN \\ &= 6 \times 500 \\ &= 3000 \text{ ampere-turns} \end{aligned}$$

(b) The magnetic field strength is given by the m.m.f. divided by the length of the magnetic path. In this case, the length of the magnetic path is the mean circumference of the coil, so

$$\begin{aligned} H &= \frac{IN}{l} \\ &= \frac{3000}{0.4} \\ &= 7500 \text{ A/m} \end{aligned}$$

(c) For a non-magnetic material $B = \mu_0 H$, so

$$\begin{aligned} B &= \mu_0 H \\ &= 4\pi \times 10^{-7} \times 7500 \\ &= 9.42 \text{ mT} \end{aligned}$$

(d) The total flux can be deduced from Equation 14.2, from which it is clear that $\Phi = BA$. Hence

$$\begin{aligned} \Phi &= BA \\ &= 9.42 \times 10^{-3} \times 300 \times 10^{-6} \\ &= 2.83 \ \mu\text{Wb} \end{aligned}$$

If the toroid were replaced by a material with a μ_r of 100, this would have no effect on (a) and (b) but would increase (c) and (d) by a factor of 100.

14.3 Reluctance

As we know, in electric circuits, when an electromotive force is applied across a resistive component a current is produced. The ratio of the voltage to the resultant current is termed the *resistance* of the component and is a measure of how the component opposes the flow of electricity.

A directly equivalent concept exists in magnetic circuits. Here a magnetomotive force produces a magnetic flux, and the ratio of one to the other is termed the **reluctance**, S, of the magnetic circuit. In this case, the reluctance is a measure of how the circuit opposes the flow of *magnetic flux*. Just as resistance is equal to V/I, so the reluctance is given by the m.m.f. (F) divided by the flux (Φ) and hence

$$S = \frac{F}{\Phi} \tag{14.7}$$

The units of reluctance are amperes per weber (A/Wb).

14.4 Inductance

A changing magnetic flux induces an electrical voltage (an e.m.f.) in any conductor within the field. The magnitude of the effect is given by **Faraday's law**, which states that:

> The magnitude of the e.m.f. induced in a circuit is proportional to the rate of change of the magnetic flux linking the circuit.

Also of importance is Lenz's law, which states that:

> The direction of the e.m.f. is such that it tends to produce a current that opposes the change of flux responsible for inducing that e.m.f.

When a circuit forms a single loop, the e.m.f. induced by changes in the magnetic flux associated with that circuit is simply given by the rate of change of the flux. When a circuit contains many loops, then the resulting e.m.f. is the sum of the e.m.f.s produced by each loop. Therefore, if a coil of N turns experiences a change in magnetic flux, then the induced voltage V is given by

$$V = N\frac{\mathrm{d}\Phi}{\mathrm{d}t} \tag{14.8}$$

where $\mathrm{d}\Phi/\mathrm{d}t$ is the rate of change of flux in Wb/s.

This property, whereby an e.m.f. is induced into a wire as a result of a changes in magnetic flux, is referred to as **inductance**.

14.5 Self-inductance

We have seen that a current flowing in a coil (or in a single wire) produces a magnetic flux about it, and that changes in the current will cause changes in the magnetic flux. We have also seen that when the magnetic flux associated with a circuit changes, this induces an e.m.f. in that circuit which opposes the changing flux. It follows, therefore, that when the current in a coil changes, an e.m.f. is induced in that coil which tends to oppose the change in the current. This process is known as **self-inductance**.

The voltage produced across the inductor as a result of changes in the current is given by the expression

$$V = L\frac{\mathrm{d}I}{\mathrm{d}t} \tag{14.9}$$

where L is the inductance of the coil. The unit of inductance is the **henry** (symbol H), which can be defined as the inductance of a circuit when an e.m.f. of 1 V is induced by a change in the current of 1 A/s.

14.5.1 Notation

It should be noted that some textbooks assign a negative polarity to the voltages of Equations 14.8 and 14.9 to reflect the fact that the induced voltage *opposes* the change in flux or current. This notation reflects the implications of Lenz's law. However, either polarity can be used provided that the calculated quantity is applied appropriately, and in this text we will use the *positive* notation since this is consistent with the treatment of voltages across resistors and capacitors.

Example 14.3 **The current in a 10 mH inductor changes at a constant rate of 3 A/s. What voltage is induced across this coil?**

From Equation 14.9

$$V = L\frac{\mathrm{d}I}{\mathrm{d}t}$$

$$= 10 \times 10^{-3} \times 3$$

$$= 30 \text{ mV}$$

14.6 Inductors

Circuit elements that are designed to provide inductance are called **inductors**. Typical components for use in electronic circuits will have an inductance of the order of microhenries or millihenries, although large components may have an inductance of the order of henries.

Small-value inductors can be produced using air-filled coils, but larger devices normally use ferromagnetic materials. As we noted earlier, the presence of a ferromagnetic material dramatically increases the flux density in a coil and consequently also increases the rate of change of flux. Therefore, adding a ferromagnetic core to a coil greatly increases its inductance. Inductor cores may take many forms, including rods, as in Figure 14.4(a), or rings, as in Figure 14.4(b). Small inductor cores are often made from iron oxides called **ferrites**, which have very high permeability. Larger components are often based on laminated steel cores.

Unfortunately, the permeability of ferromagnetic materials decreases with increasing magnetic field strength, making inductors non-linear. Air does not suffer from this problem, so air-filled inductors are linear. For this reason, air-filled devices may be used in certain applications even though they may be physically larger than components using ferromagnetic cores.

14.6.1 Calculating the inductance of a coil

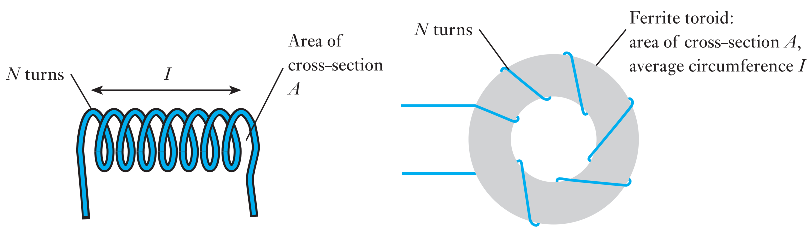
The inductance of a coil is determined by its dimensions and by the material around which it is formed. Although it is fairly straightforward to calculate the inductance of simple forms from first principles, designers often use standard formulae. Here we will look at a couple of examples, as shown in Figure 14.5.

Figure 14.5(a) shows a simple, helical, air-filled coil of length l and cross-sectional area A. The characteristics of this arrangement vary with the dimensions, but provided that the length is much greater than the diameter, the inductance of this coil is given by the expression

$$L = \frac{\mu_0 A N^2}{l} \tag{14.10}$$

Figure 14.5(b) shows a coil wound around a toroid that has a mean circumference of l and a cross-sectional area of A. The inductance of this arrangement is given by

$$L = \frac{\mu_0 \mu_r A N^2}{l} \tag{14.11}$$



(a) An air-filled coil

(b) A coil wound on a ferrite toroid

Figure 14.5 Examples of standard inductor formats where μ_r is the relative permeability of the material used for the toroid. If this is a non-magnetic material then μ_r will be equal to 1, and the inductance becomes

$$L = \frac{\mu_0 A N^2}{l} \tag{14.12}$$

which is the same as for the long air-filled coil described earlier (although the meaning of l is slightly different). Although these two examples have very similar equations, other coil arrangements will have difference characteristics.

In these two examples, and in many other inductors, the inductance increases as the square of the number of turns.

Example 14.4 **Calculate the inductance of a helical, air-filled coil 200 mm in length, with a cross-sectional area of 30 mm² and having 400 turns.**

From Equation 14.10

$$L = \frac{\mu_0 A N^2}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 30 \times 10^{-6} \times 400^2}{200 \times 10^{-3}}$$

$$= 30\ \mu\text{H}$$

14.6.2 Equivalent circuit of an inductor

So far we have considered inductors as idealised components. In practice, all inductors are made from wires (or other conductors) and therefore all real components will have resistance. We can model a real component as an **ideal inductor** (that is, one that has inductance but no resistance) in series with a resistor that represents its internal resistance. This is shown in Figure 14.6.

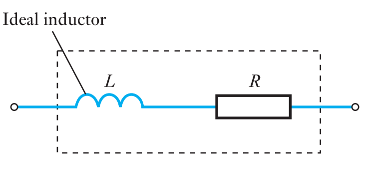


Figure 14.6 An equivalent circuit of a real inductor

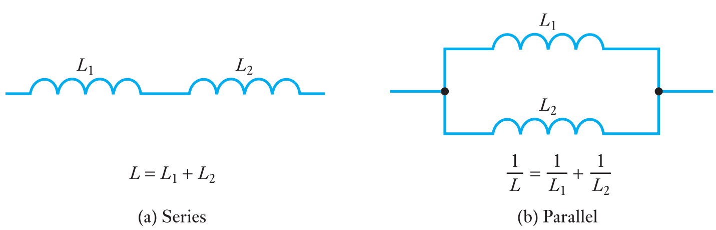
14.6.3 Stray inductance

While circuit designers will often use inductors to introduce inductance into circuits, the various conductors in *all* circuits introduce **stray inductance** that is often unwanted. We have seen that even a straight wire exhibits inductance, and though this is usually small (perhaps 1 nH per mm length of wire) the combined effects of these small amounts of inductance can dramatically affect circuit operation – particularly in high-speed circuits. In such cases, great care must be taken to reduce both stray inductance and stray capacitance (as discussed in Chapter 13).

14.7 Inductors in series and parallel

When several inductors are connected together, their effective inductance is computed in the same way as when resistors are combined, *provided that they are not linked magnetically*. Therefore, when inductors are connected in series their inductances add. Similarly, when inductors are connected in parallel their combined inductance is given by the reciprocal of the sum of the reciprocals of the individual inductances. This is shown in Figure14.7.

Figure 14.7 Inductors in series and parallel



Example 14.5

Calculate the inductance of:

(a) a 10 H and a 20 H inductor in series;
(b) a 10 H and a 20 H inductor in parallel.

(a) Inductances in series add

$$L = L_1 + L_2$$
$$= 10\text{ H} + 20\text{ H}$$
$$= 30\text{ H}$$

(b) Inductances in parallel sum as their reciprocals

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$
$$= \frac{1}{10} + \frac{1}{20}$$
$$= \frac{30}{200}$$
$$L = 6.67\text{ H}$$

14.8 Relationship between voltage and current in an inductor

From Equation 14.9, we know that the relationship between the voltage across an inductor and the current through it is given by

$$V = L\frac{dI}{dt}$$

This implies that when a constant current is passed through an inductor ($dI/dt = 0$) the voltage across it is zero. However, when the current changes a voltage is produced that tends to oppose this change in current. Another implication of the equation is that the current through an inductor cannot change instantaneously, since this would correspond to $dI/dt = \infty$ and would produce an infinite induced voltage opposing the change in current. Thus inductors tend to stabilise the *current* flowing through them. (You may recall that in capacitors the voltage cannot change instantaneously, so capacitors tend to stabilise the *voltage* across them).

The relationship between the voltage and the current in an inductor is illustrated in Figure 14.8. In the circuit of Figure 14.8(a), the switch is initially open and no current flows in the circuit. If now the switch is closed (at $t = 0$), then the current through the inductor cannot change instantly, so initially $I = 0$, and consequently $V_R = 0$. By applying Kirchhoff's voltage law around the circuit, it is clear that $V = V_R + V_L$, and if initially $V_R = 0$, then the entire supply voltage V will appear across the inductor, and $V_L = V$.

The voltage across the inductor dictates the initial rate of change of the current (since $V_L = L\, dI/dt$) and hence the current steadily increases. As I grows the voltage across the resistor grows and V_L falls, reducing dI/dt.

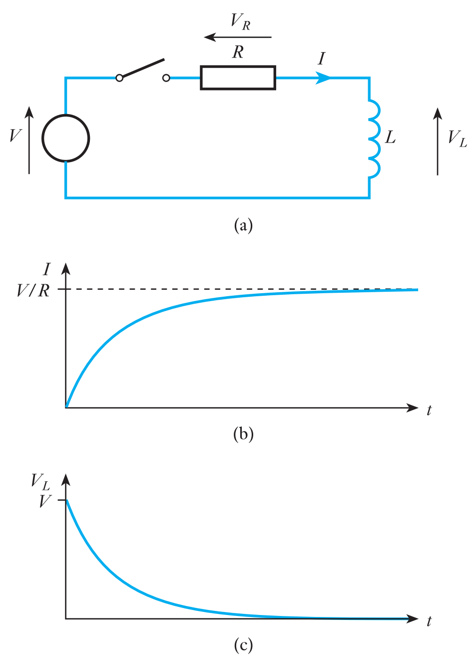


Figure 14.8 Relationship between voltage and current in an inductor

Therefore, the rate of increase of the current decreases with time. Gradually, the voltage across the inductor tends to zero and all the applied voltage appears across the resistor. This produces a steady-state current of V/R. The result is that the current is initially zero but increases exponentially with time, and the voltage across the inductor is initially V but falls exponentially with time. This behaviour is shown in Figures 14.8(b) and 14.8(c). You might like to compare these curves with the corresponding results produced for a capacitor in Figure 13.7.

Computer Simulation Exercise 14.1

Simulate the circuit of Figure 14.8(a) with $V = 1$ V, $R = 1\ \Omega$ and $L = 1$ H. Include in your circuit a switch that closes at $t = 0$. Use transient simulation to investigate the behaviour of the circuit during the first 5 s after the switch changes. Plot V_L and I against time on separate graphs and confirm that the circuit behaves as expected. Experiment with different values of the circuit components and note the effects on the voltage and current graphs.

In Chapter 13, we noted that the time taken for a capacitor to charge increases with both the capacitance C and the series resistance R, and we defined a termed called the time constant, equal to the product CR, which determines the charging time. In the inductor circuit discussed above, the rate at which the circuit approaches its steady-state condition increases with the inductance L but *decreases* with the value of R. The reason for this effect will become clear in Chapter 18, but for the moment we will simply note that circuits of this type have a time constant (T) equal to L/R.

Computer Simulation Exercise 14.2

Repeat Computer Simulation Exercise 14.1 noting the effect of different component values. Begin with the same values as in the previous simulation exercise and then change the values of L and R while keeping the ratio L/R constant. Again plot V_L and I against time on separate graphs and confirm that the characteristics are unchanged. Hence confirm that the characteristics are determined by the time constant L/R rather than the actual values of L and R.

It is interesting to consider what happens in the circuit of Figure 14.8(a) if the switch is opened some time after being closed. From Figure 14.8(b), we know that the current stabilises at a value of V/R. If the switch is now opened, this would suggest that the current would instantly go to zero. This would imply that dI/dt would be infinite and that an infinite voltage would be produced across the coil. In practice, the very high induced voltage appears across the switch and causes ‘arcing’ at the switch contacts. This maintains the current for a short time after the switch is operated and reduces the rate of change of current. This phenomenon is used to advantage in some situations such as in automotive ignition coils. However, arcing across switches can cause severe damage to the contacts and also generates electrical interference. For this reason, when it is necessary to switch inductive loads, we normally add circuitry to reduce the rate of change of the current. This circuitry may be as simple as a capacitor placed across the switch.

So far in this section we have assumed the use of an ideal inductor and have ignored the effects of any internal resistance. In Section 14.6, we noted that an inductor with resistance can be modelled as an ideal inductor in series with a resistor. In Chapter 15, we will look at the characteristics of circuits containing elements of various types (resistive, inductive and capacitive), so we will leave the effects of internal resistance until that time.

14.9 Sinusoidal voltages and currents

Having looked at the relationship between voltage and current in a DC circuit containing an inductor, it is now time to turn our attention to circuits using sinusoidal quantities.

Consider the arrangement of Figure 14.9(a), where an alternating current is passed through an inductor. Figure 14.9(c) shows the sinusoidal current

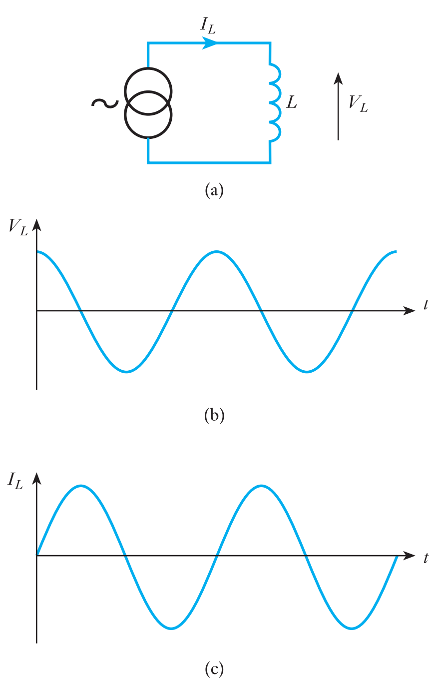


Figure 14.9 Inductors and alternating quantities

waveform in the inductor, which in turn dictates the voltage across the inductor. From Equation 14.9 we know that the voltage across an inductor is given by $L\,dI/dt$, so the voltage is directly proportional to the *time differential* of the current. Since the differential of a sine wave is a cosine wave, we obtain a voltage waveform as shown in Figure 14.9(b). The current waveform is phase-shifted with respect to the voltage waveform by 90° (or π/2 radians). It is also clear that the current waveform *lags* the voltage waveform. You might like to compare this result with that shown in Figure 13.8 for a capacitor. You will note that in a capacitor the current *leads* the voltage, while in an inductor the current *lags* the voltage. We will return to the analysis of sinusoidal waveforms in Chapter 15.

Computer Simulation Exercise 14.3

Simulate the circuit of Figure 14.9(a) using any value of inductor. Use a sinusoidal current source to produce a current of 1 A peak at 1 Hz and use transient analysis to display the voltage across the inductor, and the current through it, over a period of several seconds. Note the phase relationship between the two waveforms and hence confirm that the current lags the voltages by 90° (or π/2 radians). Note the effect of varying the inductor value, and the frequency used.

14.10 Energy storage in an inductor

Inductors store energy within a magnetic field. The amount of energy stored in this way can be determined by considering an initially unenergised inductor of inductance L, in which a current is gradually increased from zero to I amperes. If the rate of change of the current at a given time is di/dt, then the instantaneous voltage across the inductor (v) will be given by

$$v = L\frac{di}{dt}$$

In a small amount of time dt, the amount of energy added to the magnetic field is equal to the product of the instantaneous voltage (v), the instantaneous current (i) and the time interval (dt).

$$\begin{aligned}\text{Energy added} &= vi\,dt \\ &= L\frac{di}{dt}i\,dt \\ &= Li\,di\end{aligned}$$

Therefore, the energy added to the magnetic field as the current increases from zero to I is given by

$$\text{Stored energy} = L\int_0^I i\mathrm{d}t$$

$$\text{Stored energy} = \frac{1}{2}LI^2 \qquad (14.13)$$

Example 14.6 **What energy is stored in an inductor of 10 mH when a current of 5 A is passing through it?**

From Equation 14.3

$$\text{Stored energy} = \frac{1}{2}LI^2$$

$$= \frac{1}{2} \times 10^{-2} \times 5^2$$

$$= 125 \text{ mJ}$$

14.11 Mutual inductance

If two conductors are linked magnetically, then a changing current in one of these will produce a changing magnetic flux associated with the other and will result in an induced voltage in this second conductor. This is the principle of **mutual inductance**.

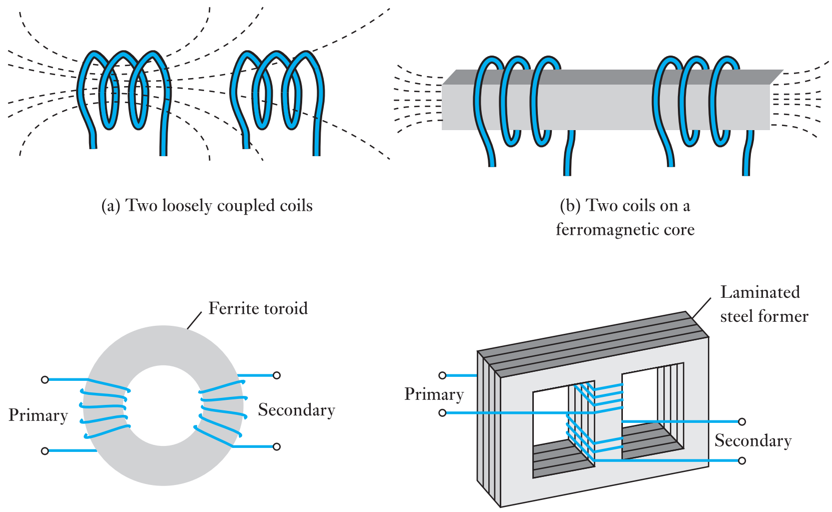
Mutual inductance is quantified in a similar way to self-inductance, such that if a current I_1 flows in one circuit, the voltage induced in a second circuit V_2 is given by

$$V_2 = M\frac{\mathrm{d}I_1}{\mathrm{d}t} \qquad (14.14)$$

where M is the mutual inductance between the two circuits. The unit of mutual inductance is the henry, as for self-inductance. Here, a henry would be defined as the mutual inductance between two circuits when an e.m.f. of 1 V is induced in one by a change in the current of 1 A/s in the other. The mutual inductance between two circuits is determined by their individual inductances and the magnetic linkage between them.

Often our interest is in the interaction of coils, as in a **transformer**. Here a changing current in one coil (the primary) is used to induce a changing current in a second coil (the secondary). Figure 14.10 shows arrangements of two coils that are linked magnetically. In Figure 14.10(a), the two coils are loosely coupled with a relatively small part of the flux of the first coil linking with the second. Such an arrangement would have a relatively low mutual inductance. The degree of coupling between circuits is described by their **coupling coefficient**, which defines the fraction of the flux of one coil that links with the other. A value of 1 represents total flux linkage, while a

Figure 14.10 Mutual inductance between two coils



(a) Two loosely coupled coils

(b) Two coils on a ferromagnetic core

(c) Coils on a ferrite toroid

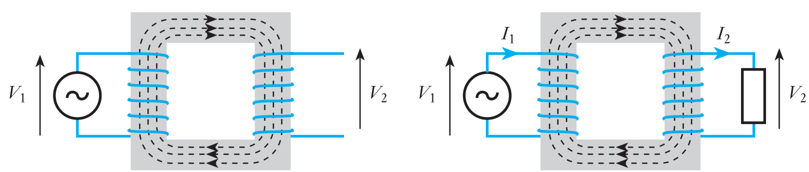
(d) Coils wound on a laminated steel core

value of 0 represents no linkage. The coupling between the two coils can be increased in a number of ways, such as by moving the coils closer together, by wrapping one coil around the other, or by adding a **ferromagnetic core** as in Figure 14.10(b). Excellent coupling is achieved by wrapping coils around a continuous ferromagnetic loop as in Figures 14.10(c) and 14.10(d). In these examples, the cores increase the inductance of the coils and increase the flux linkage between them.

14.12 Transformers

The basic form of a transformer is illustrated in Figure 14.11(a). Two coils, a primary and a secondary, are wound onto a ferromagnetic core or former in an attempt to get a coupling coefficient as close as possible to unity. In practice, many transformers are very efficient and for the benefit of this discussion we will assume that all the flux from the primary coil links with the secondary. That is, we will assume an ideal transformer with a coupling coefficient of 1.

Figure 14.11 A transformer



(a) An unloaded transformer

(b) A transformer with a resistive load

If an alternating voltage V_1 is applied to the primary, this will produce an alternating current I_1, which in turn will produce an alternating magnetic field. Since the variation in the magnetic flux associated with the primary coil is the same as that associated with the secondary, the voltage induced *in each turn* of the primary and the secondary will be the same. Let us call this V_T. Now, if the number of turns in the primary is N_1, then the voltage induced across the primary will be N_1V_T. Similarly, if the number of turns in the secondary is N_2, then the voltage across the secondary will be N_2V_T. Therefore, the ratio of the output voltage V_2 to the input voltage V_1 is given by

$$\frac{V_2}{V_1} = \frac{N_2V_T}{N_1V_T}$$

and thus

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad (14.15)$$

Thus the transformer works as a voltage amplifier with a gain determined by the ratio of the number of turns in the secondary to that in the primary. N_2/N_1 is often called the **turns ratio** of the transformer.

However, there are, several points to note about this arrangement. The first is that this voltage amplification clearly applies only to alternating voltages – a constant voltage applied to the primary will not produce a changing magnetic flux and consequently no output voltage will be induced. Second, it must be remembered that this 'amplifier' has no energy source other than the input signal (that is, it is a passive amplifier) and consequently the power delivered at the output cannot be greater than that absorbed at the input. This second point is illustrated in Figure 14.11(b), where a resistive load has been added to our transformer. The addition of a load means that a current will now flow in the secondary circuit. This current will itself produce magnetic flux, and the nature of induction means that this flux will oppose that generated by the primary circuit. Consequently, the current flowing in the secondary coil tends to reduce the voltage in that coil. The overall effect of this mechanism is that when the secondary is open-circuit, or when the output current is very small, the output voltage is as predicted by Equation 14.15, but as the output current increases the output voltage falls.

The efficiency of modern transformers is very high and therefore the power delivered at the output is almost the same as that absorbed at the input. For an ideal transformer

$$V_1I_1 = V_2I_2 \qquad (14.16)$$

If the secondary of a transformer has many more turns than the primary we have a **step-up transformer**, which provides an output voltage that is much higher than the input voltage, but it can deliver a smaller output current. If the secondary has fewer turns than the primary we have a **step-down transformer**, which provides a smaller output voltage but can supply a greater current. Step-down transformers are often used in power supplies for low-voltage electronic equipment, where they produce an output voltage of a few volts from the supply voltage. An additional advantage of this arrangement is that the transformer provides **electrical isolation** from the supply lines, since there is no electrical connection between the primary and the secondary circuits.

14.13 Circuit symbols

We have looked at several forms of inductor and transformer, and some of these may be indicated through the use of different circuit symbols. Figure 14.12 shows various symbols and identifies their distinguishing characteristics. Figure 14.12(f) shows a transformer with two secondary coils. This figure also illustrates what is termed the **dot notation** for indicating the polarity of coil windings. Current flowing *into* each winding at the connection indicated by the dot will produce magnetomotive forces in the same direction within the core. Reversing the connections to a coil will invert the corresponding voltage waveform. The dot notation allows the required connections to be indicated on the circuit diagram.

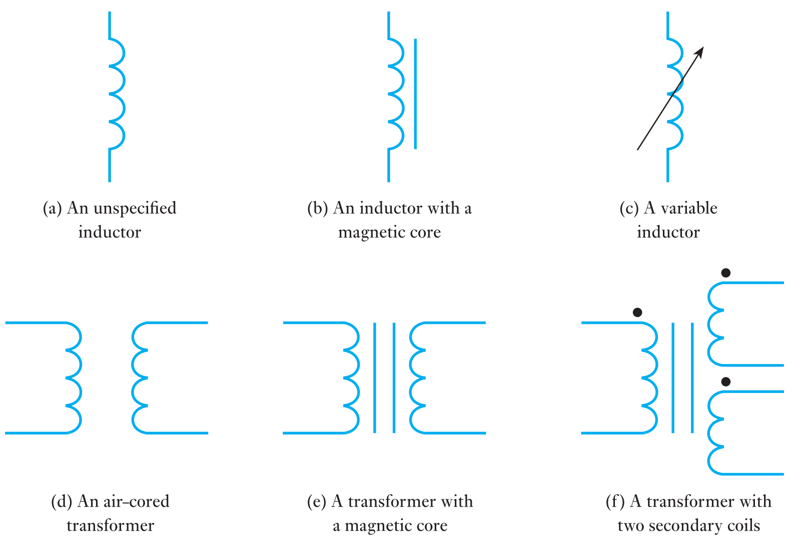


Figure 14.12 Circuit symbols for inductors and transformers

14.14 The use of inductance in sensors

Inductors and transformers are used in a wide range of electrical and electronic systems, and we shall be meeting several such applications in later chapters. However, at this point, it might be useful to look at a couple of situations where inductance is used as a means of measuring physical quantities. The first of these we have already encountered in Chapter 3.

14.14.1 Inductive proximity sensors

We looked briefly at inductive proximity sensors in Section 3.6 and looked at some real devices in Figure 3.7. The essential elements of such a sensor are shown in Figure 14.13. The device is basically a coil wrapped around a ferromagnetic rod. The arrangement is used as a sensor by combining it with a ferromagnetic plate (attached to the object to be sensed) and a circuit to measure the self-inductance of the coil. When the plate is close to the coil it increases its self-inductance, allowing its presence to be detected. The sensor can be used to measure the separation between the coil and the plate but is more often used in a binary mode to sense its presence or absence.

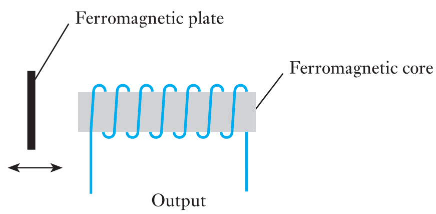


Figure 14.13 An inductive proximity sensor

14.14.2 Linear variable differential transformers (LVDTs)

An LVDT consists of three coils wound around a hollow, non-magnetic tube, as shown in Figure 14.14. The centre coil forms the primary of the transformer and is exited by an alternating voltage. The remaining coils form identical secondaries, positioned symmetrically either side of the primary. The two secondary coils are connected in series in such a way that their output voltages are out of phase (note the position of the dots in Figure 14.14) and therefore cancel. If a sinusoidal signal is applied to the primary coil, the symmetry of the arrangement means that the two secondary coils produce identical signals that cancel each other, and the output is zero. This assembly is turned into a useful sensor by the addition of a movable 'slug' of ferromagnetic material inside the tube. The material increases the mutual inductance between the primary and the secondary coils and thus increases

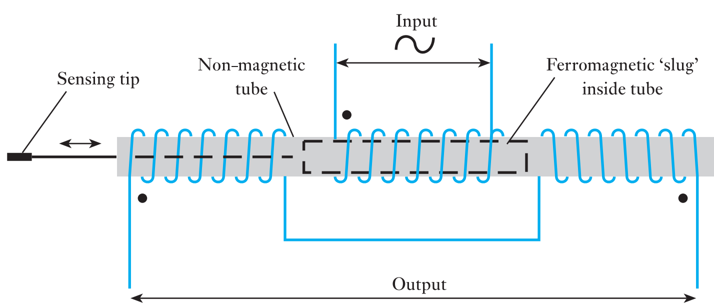


Figure 14.14 A linear variable differential transformer (LDVT)

the magnitude of the voltages induced in the secondary coils. If the slug is positioned centrally with respect to the coils, it will affect both coils equally and the output voltages will still cancel. However, if the slug is moved slightly to one side or the other, it will increase the coupling to one and decrease the coupling to the other. The arrangement will now be out of balance, and an output voltage will be produced. The greater the displacement of the slug from its central position the greater the resulting output signal. The output is in the form of an alternating voltage where the magnitude represents the offset from the central position and the phase represents the direction in which the slug is displaced. A simple circuit can be used to convert this alternating signal into a more convenient DC signal if required.

LVDTs can be constructed with ranges from a few metres down to a fraction of a millimetre. They typically have a resolution of about 0.1 percent of their full range and have good linearity. Unlike resistive potentiometers, they do not require a frictional contact and so can have a very low operating force and long life.

Key points

- Inductors store energy within a magnetic field.
- A wire carrying an electrical current causes a magnetomotive force (m.m.f.), which produces a magnetic field about it.
- The magnetic field strength, H, is proportional to the current and inversely proportional to the length of the magnetic circuit.
- The magnetic field produces a magnetic flux, Φ, which flows in the same direction.
- The flux density is determined by the field strength and the permeability of the material present.
- When a current-carrying wire is formed into a coil, the magnetic field is concentrated. The m.m.f. increases with the number of turns of the coil.
- A changing magnetic flux induces an electrical voltage in any conductors within the field.
- The direction of the induced e.m.f. is such that it opposes the change of flux.
- When the current in a coil changes, an e.m.f. is induced in that coil which tends to oppose the change in the current. This is self-inductance.
- The induced voltage is proportional to the rate of change of the current in the coil.
- Inductors can be made by coiling wire in air, but much greater inductance is produced if the coil is wound around a ferromagnetic core.

- All real inductors have some resistance.
- When inductors are connected in series their inductances add. When inductors are connected in parallel the resultant inductance is the reciprocal of the sum of the reciprocals of the individual inductances.
- The current in an inductor cannot change instantly.
- When using sinusoidal signals the current lags the voltage by 90° (or π/2 radians).
- The energy stored in an inductor is equal to $\frac{1}{2}LI^2$.
- When two conductors are linked magnetically, a changing current in one will induce a voltage in the other. This is mutual induction.
- When a transformer is used with alternating signals, the voltage gain is determined by the turns ratio.
- Several forms of sensor make use of variations in inductance.

Exercises

14.1 Explain what is meant by a magnetomotive force (m.m.f.).

14.2 Describe the field produced by a current flowing in a straight wire.

14.3 A straight wire carries a current of 3 A. What is the magnetic field at a distance of 1 m from the wire? What is the direction of this field?

14.4 What factors determine the flux density at a particular point in space adjacent to a current-carrying wire?

14.5 Explain what is meant by the permeability of free space? What are its value and units?

14.6 Give an expression for the magnetomotive force produced by a coil of N turns that is passing a current of I amperes?

14.7 A coil is formed by wrapping wire around a wooden toroid. The cross-sectional area of the coil is 400 mm^2, the number of turns is 600, and the mean circumference of the toroid is 900 mm. If the current in the coil is 5 A, calculate the magnetomotive force, the magnetic field strength in the coil, the flux density in the coil and the total flux.

14.8 If the toroid in Exercise 14.7 were to be replaced by a magnetic toroid with a relative permeability of 500, what effect would this have on the values calculated?

14.9 If a m.m.f. of 15 ampere-turns produces a total flux of 5 mWb, what is the reluctance of the magnetic circuit?

14.10 State Faraday's law and Lenz's law.

14.11 Explain what is meant by inductance.

14.12 Explain what is meant by self-inductance.

14.13 How is the voltage induced in a conductor related to the rate of change of the current within it?

14.14 Define the henry as it applies to the measurement of self-inductance.

14.15 The current in an inductor changes at a constant rate of 50 mA/s, and there is a voltage across it of 150 μV. What is its inductance?

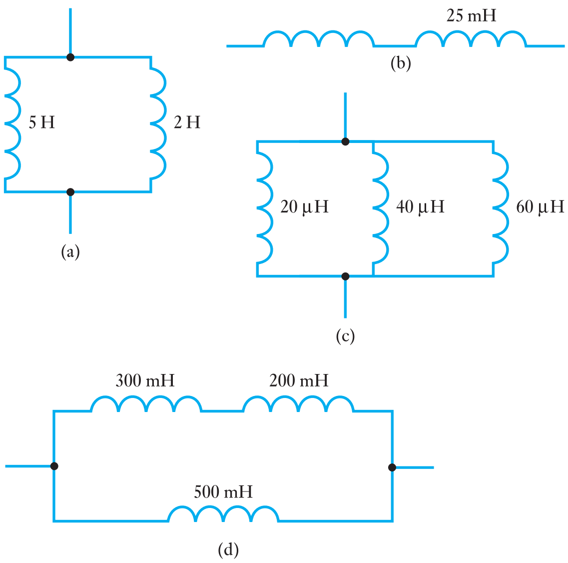
Exercises continued

14.16 Why does the presence of a ferromagnetic core increase the inductance of an inductor?

14.17 Calculate the inductance of a helical, air-filled coil 500 mm in length, with a cross-sectional area of 40 mm^2 and having 600 turns.

14.18 Calculate the inductance of a coil wound on a magnetic toroid of 300 mm mean circumference and 100 mm^2 cross-sectional area, if there are 250 turns on the coil and the relative permeability of the toroid is 800.

14.19 Calculate the effective inductance of the following arrangements.



14.20 Describe the relationship between voltage and current in an inductor.

14.21 Why is it not possible for the current in an inductor to change instantaneously?

14.22 Repeat Computer Simulation Exercise 14.1 with $V = 15$ V, $R = 5\ \Omega$ and $L = 10$ H. Plot the current through the inductor as a function of time and hence estimate the time taken for the inductor current to reach 2 A.

14.23 Explain what is meant by a time constant. What is the time constant of the circuit in Exercise 14.22?

14.24 The circuit of Exercise 14.22 is modified by changing R to 10 Ω. What value should be chosen for L so that the time taken for the inductor current to reach 2 A is unchanged?

14.25 Confirm your answer to Exercise 14.24 using computer simulation.

14.26 Discuss the implications of induced voltages when switching inductive circuits.

14.27 How do real inductors differ from ideal inductors?

14.28 What is the relationship between the sinusoidal current in an inductor and the voltage across it?

14.29 What is the energy stored in an inductor of 2 mH when a current of 7 A is passing through it?

14.30 Explain what is meant by mutual inductance.

14.31 Define the henry as it applies to the measurement of mutual inductance.

14.32 What is meant by a coupling coefficient?

14.33 What is meant by the turns ratio of a transformer?

14.34 A transformer has a turns ratio of 10. A sinusoidal voltage of 5 V peak is applied to the primary coil, with the secondary coil open-circuit. What voltage would you expect to appear across the secondary coil?

14.35 Explain the dot notation used when representing transformers in circuit diagrams.

14.36 Describe the operation of an inductive proximity sensor.

14.37 Describe the construction and operation of an LVDT.