CURRENT, RESISTANCE AND ELECTROMOTIVE FORCE

Electric current

Up to now, we have considered charges at rest (ELECTROSTATICS)

But $\overline{\mathbf{E}}$ exerts a force ⇒ charges move if they are free to do so.

Definition: ELECTRIC CURRENT is the rate of flow of charge: $I = \frac{dQ}{dt}$

Units of current: 1 Ampere (A) ≡ 1 C s^{-1} (Coulombs/second)

Convention: Current flows in the direction of $\overline{\mathbf{E}}$ (i.e., it is taken to be carried by positive charges – this is usually not so)

Conductivity and Resistivity

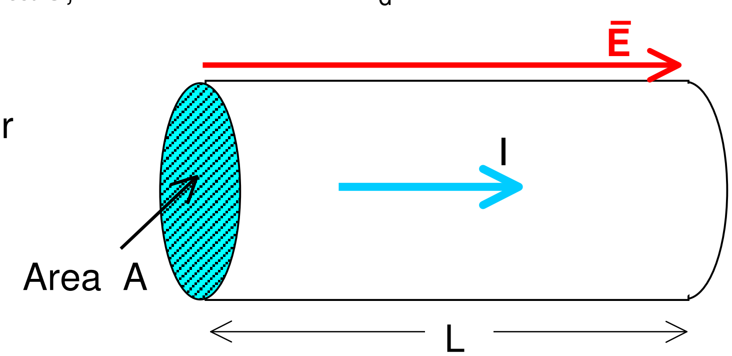
The effect of $\overline{\mathbf{E}}$ is to give the electrons in the conductor a small DRIFT VELOCITY, $\overline{\mathbf{v}}_{\mathbf{d}}$, superimposed on their random thermal motion. Typically, v_d is very small around 0.1 mm s^{-1}.

Clearly, the current is proportional to drift velocity $I \propto v_d$

For many materials, including metals, $v_d \propto E$

So $I \propto E$

Consider a section of a conductor with cross sectional area A and length L.



Let a potential difference ΔV be applied between the two ends, so

$$E = \frac{\Delta V}{L}$$

Current: $I \propto E$ and $I \propto A$ (the larger the area, the easier it is for current to flow)

So $I = (\text{Constant})(A)\left[\frac{\Delta V}{L}\right]$

The constant of proportionality is called the CONDUCTIVITY, σ

$$I = \sigma\left[\frac{A}{L}\right]\Delta V$$

RESISTIVITY, ρ, is defined by $\rho = \frac{1}{\sigma}$

Resistance and Ohm's Law

The resistivity is a property of the substance. For a particular piece of the substance, the RESISTANCE, R, is defined by

$R = \frac{\Delta V}{I}$ where ΔV is the potential difference across the material and I is the current flowing through it.

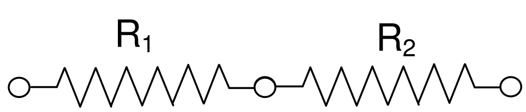
In the SI system, resistance is measured in Ohms (Ω): $1\ \Omega = \frac{1\ V}{1\ A}$

For a sample of uniform cross sectional area, A and length L, we have

$R = \rho\frac{L}{A}$ Units of ρ: Ω m Units of σ = $\Omega^{-1}\ m^{-1}$

Resistors in series and in parallel

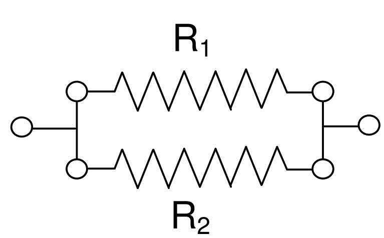
Series:



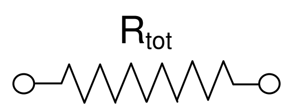
$\equiv$

$R_{tot} = R_1 + R_2$

Parallel:



$\equiv$



$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Electromotive force

If a circuit has any resistance to the flow of charge, then to make a current flow around a circuit, we need:

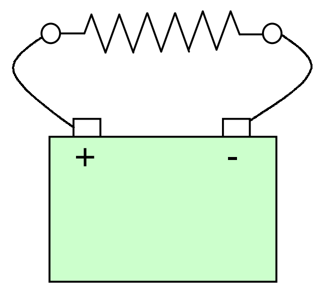
- A source of charge
- A sink for charge
- Some sort of “pump” to keep the charge moving

Example: An electric generator or battery

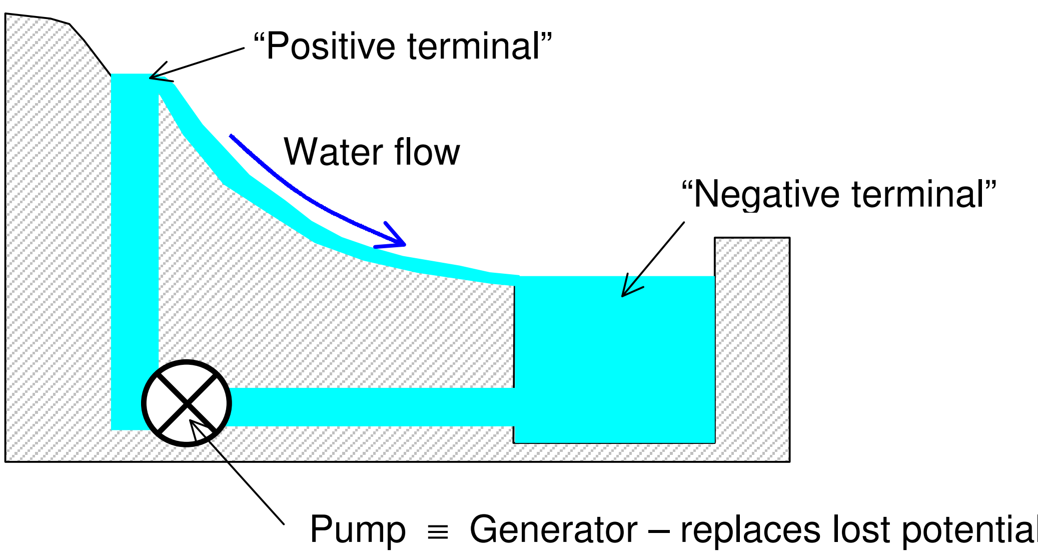
Positive terminal ≡ source

Negative terminal ≡ sink

Battery/generator ≡ pump
(Gives the charge electric PE)



Analogy: A water pump provides gravitational PE to the water:

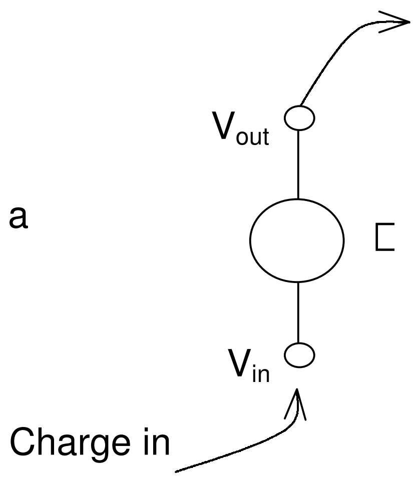


Definition: The ELECTROMOTIVE FORCE (emf), □, of a battery or generator is the potential energy gained by 1 C of charge as it passes from the negative terminal to the positive terminal.

Units of emf: $\square \equiv \dfrac{\text{Potential energy}}{\text{Charge}} \equiv \text{Volts}$

So, despite the name, emf is **NOT** a force

General symbol for a source of emf:



The increase in potential is

$$\square = \Delta V = V_{out} - V_{in}$$

The emf of a source is often called the “voltage”.

Example:

A 1.5-Volt battery provided 1 mA of current for 100 hours before running down. How much energy has it delivered?

By definition of emf, the battery does 1.5 J of work on every 1 C that passes through it.

$I = 1$ mA for 100 hrs $\Rightarrow Q_{tot} = (10^{-3})(100)(3600) = 360$ C

$\Rightarrow U_{tot} = (360)(1.5) = 540$ J

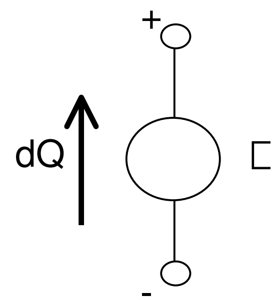
Typical sources of emf: batteries, electric generators, fuel cells, solar cells

Electric Power

Recall: To increase the potential energy of charge, a source of emf must DO WORK on the charge.

The rate of doing work is POWER. (Watts ≡ Joules/second)

Consider a source of emf, □:



Let a charge dQ pass through in time dt

Energy gained: $dU = \square dQ$

$$\text{Power } P = P = \frac{dU}{dt} = \frac{dQ}{dt} = \square I$$

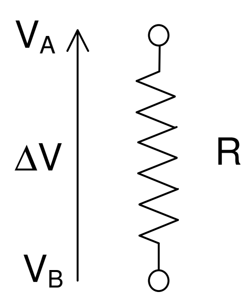
So $P = \square I$ Power =(Voltage)(Current)

The energy gained by the charge could be

- converted to mechanical energy (e.g., powering an electric motor)
- converted to chemical energy (e.g., charging a battery)
- dissipated as heat

Power dissipation in a resistor

$\Delta V = V_A - V_B = IR$



Let dQ pass through the resistor

From the definition of potential difference,

PE lost by dQ is $dU = (\Delta V)(dQ)$

$$\text{Rate of loss of PE} = \frac{dU}{dt} = \Delta V \frac{dQ}{dt} = (\Delta V)I$$

So Power dissipated is $P = (\Delta V)I$ - the same as the formula derived above for the power delivered by a source of emf.

We can also write $P = (\Delta V)I = I^2R = (\Delta V)^2/R$

Kirchhoff's Rules

1. Kirchhoff's voltage rule (loop rule)

For any closed loop in an electric circuit, the sum of all emfs and potential drops is zero.

i.e., the change in PE of a charge on going around the circuit is zero.

This holds because $\overline{\mathbf{E}}$ is a conservative field.

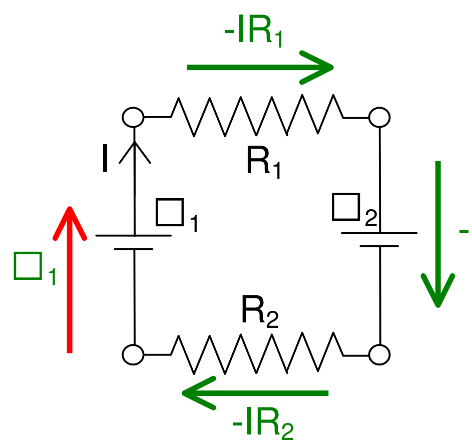
Example:

Start at any point, e.g., A

Go around the circuit adding up all the changes in potential:

$E_1 - IR_1 - E_2 - IR_2 = 0$

$$I = \frac{E_1 - E_2}{R_1 + R_2}$$



2. Kirchhoff's current rule (junction rule)

For any branch point (node) in an electric circuit, the sum of all currents entering it is equal to the sum of all currents leaving it.

i.e., charge does not build up at a node (a node has no capacitance)

e.g.: $I_1 + I_2 + I_3 = I_4 + I_5 + I_6$