Lecture 10. Resistor Circuits, Batteries and EMF

Outline:

- Connection of Resistors: In Parallel and In Series.
- Batteries.
- Non-ideal batteries: internal resistance.
- Potential distribution around a complete circuit

Lecture 9:

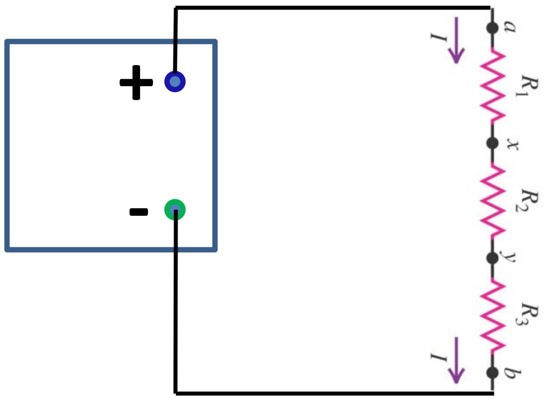
DC current: flow of charge carriers, requires $E \neq 0$ in a conductor. To keep current running, we need to maintain a non-zero potential difference across a conductor.

Microscopic picture: electron "mosquito cloud" slowly drifting in the field.

Linear regime: Ohm's Law = drift velocity $\propto E$

Resistance: the coefficient of proportionality between V and I, depends on materials parameters.

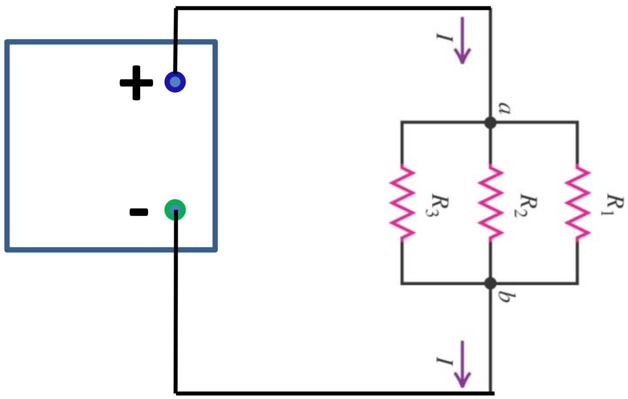
Connections: In Series vs. In Parallel



In Series:

current is the same through all elements

voltage across them can be different



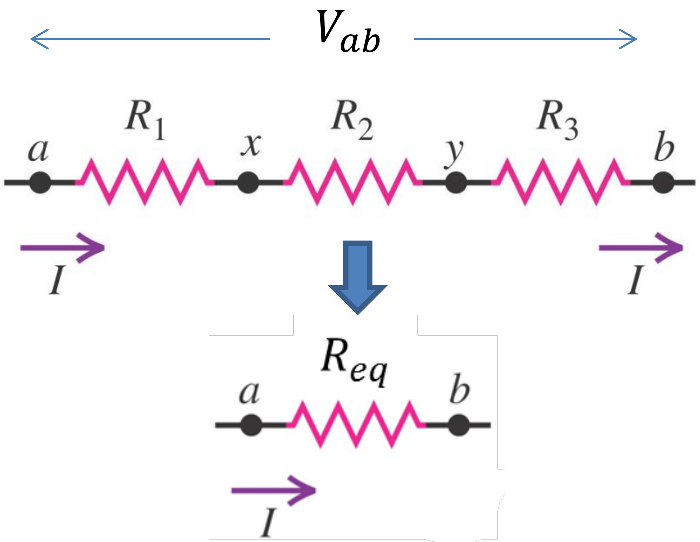
In Parallel:

voltage is the same across all elements

current through them can be different

(compare with capacitors: replace current with charge)

Resistors: Connection in Series and in Parallel



In Series

Common quantity: current

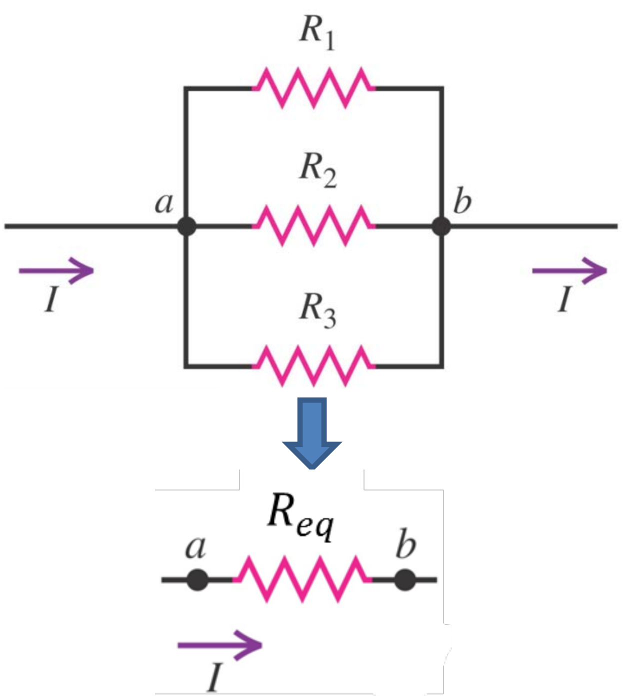
$$\boldsymbol{R_{eq} \equiv \frac{V_{ab}}{I}}$$

V_{ab} - voltage difference across R_1 , R_2, R_3

I - ***common current***

$$V_{ab} = V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3)$$

$$R_{eq} = \sum_i R_i$$



In Parallel

Common quantity: voltage difference

$$\boldsymbol{R_{eq} \equiv \frac{V_{ab}}{I}}$$

V_{ab} - ***common voltage difference***

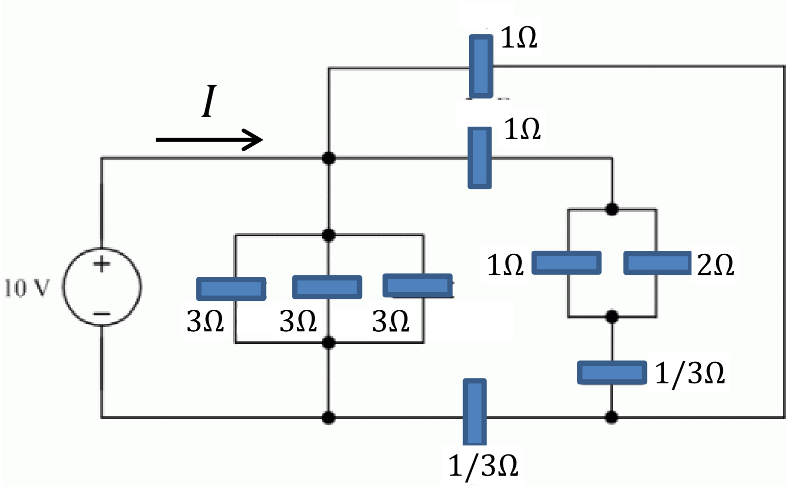
I - total current *I*

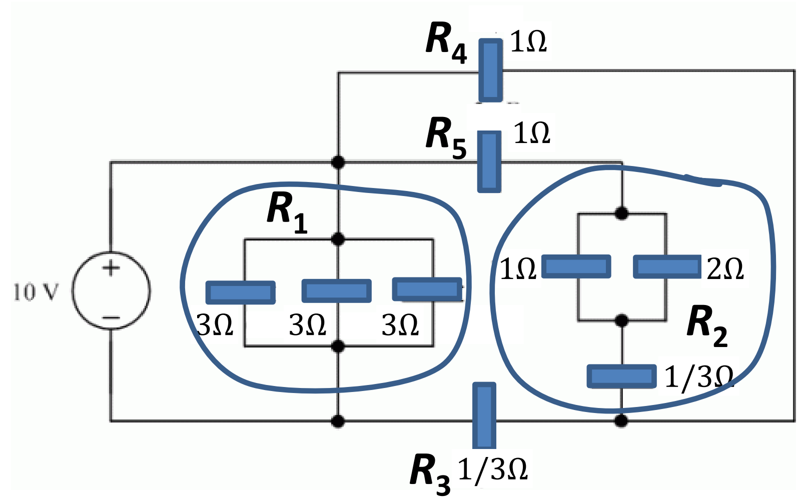
$$I = I_1 + I_2 + I_3 = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\sum_i \frac{1}{R_i}\right)^{-1}$$

$$R_1 \parallel R_2: \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

More Complex Circuits





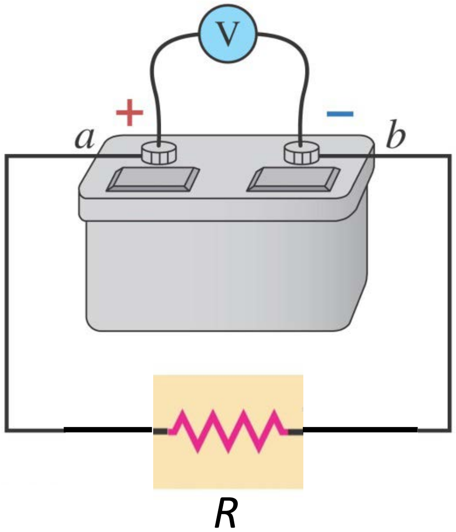
$$R_1 = \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right)^{-1} = 1\Omega \qquad R_2 = \frac{1 \cdot 2}{1+2} + \frac{1}{3} = 1\Omega \qquad R_2 \ldots R_5 = 1 + 1 = 2\Omega$$

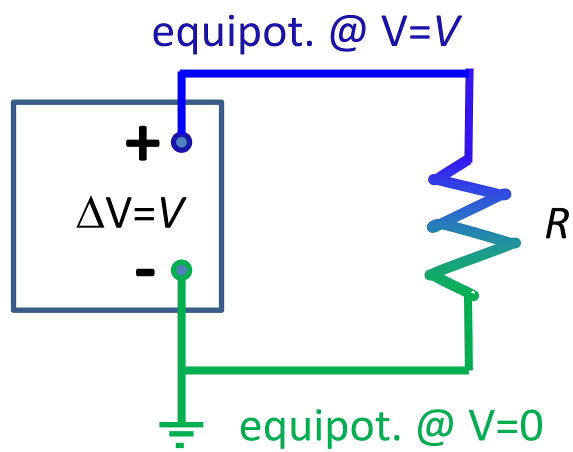
$$R_4 \parallel (R_2 \ldots R_5) = \frac{1 \cdot 2}{1+2} = \frac{2}{3}\Omega \qquad R_{3\ldots}[R_4 \parallel (R_2 \ldots R_5)] = \frac{2}{3} + \frac{1}{3} = 1\Omega$$

$$R_{eq} = R_1 \parallel \{R_{3\ldots}[R_4 \parallel (R_2 \ldots R_5)]\ \} = \frac{1 \cdot 1}{1+1} = 0.5\Omega$$

$$I = \frac{10V}{0.5\Omega} = 20A \qquad P = IV = 200W$$

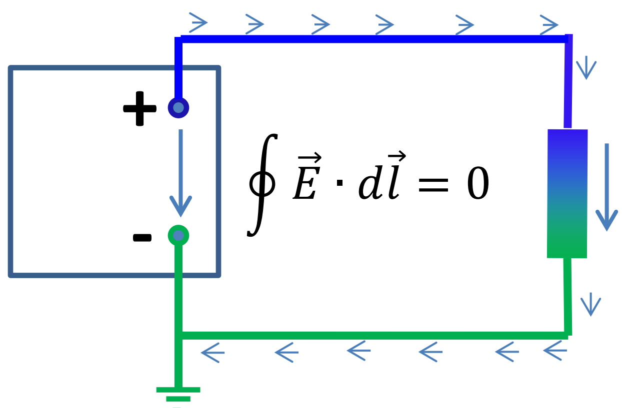
Ideal Batteries (no energy dissipation inside battery)

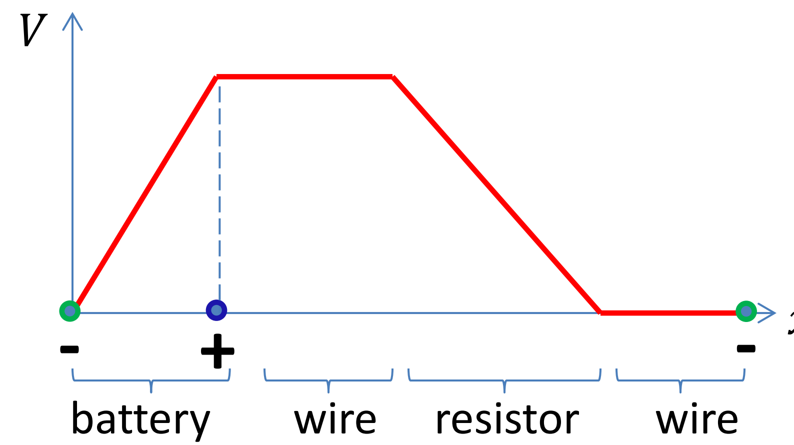




We assume that the wires have resistance much smaller than the load *R* (all voltage difference provided by the battery is applied to the load *R*).

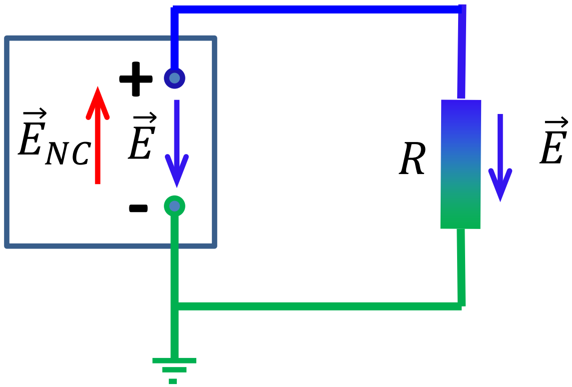
Sketch of a (***conservative***) electric field

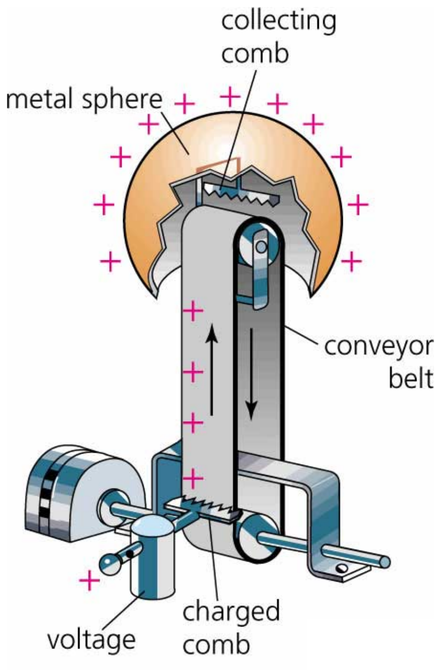




Conclusion: an "external agent" inside the battery (***not the electrostatic E !!***) forces the charge carriers to climb "the potential hill".

External Forces (not Electrostatic Field) separate Charges



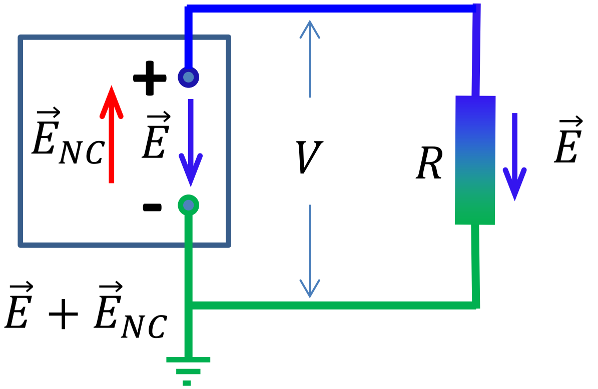


Mechanical analogy: van de Graaff generator

Demonstrations: "lemon" battery, dynamo

Fictitious field $\vec{E}_{NC}$ inside the battery: a source of an additional force on charge carriers. This field is ***non-conservative***: $\oint \vec{E}_{NC} \cdot d\vec{l} \neq 0$ ($\vec{E}_{NC}$ is zero outside the battery). Inside the battery, electrons are driven by the total electric field $\vec{E}_{net} = \vec{E} + \vec{E}_{NC}$.

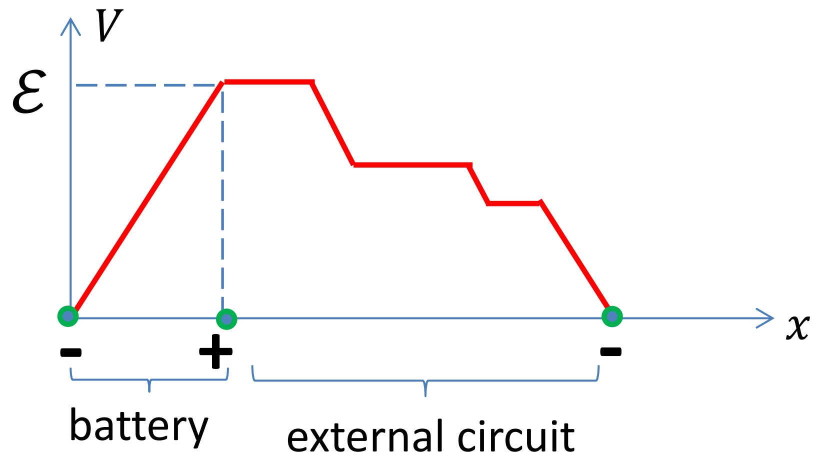
Electromotive Force (EMF)



$$\vec{E}_{net} = \vec{E} + \vec{E}_{NC}$$

$$\mathcal{E} \equiv \int_{-}^{+} \vec{E}_{NC} \cdot d\vec{l}$$ - the electromotive force

Units: volts



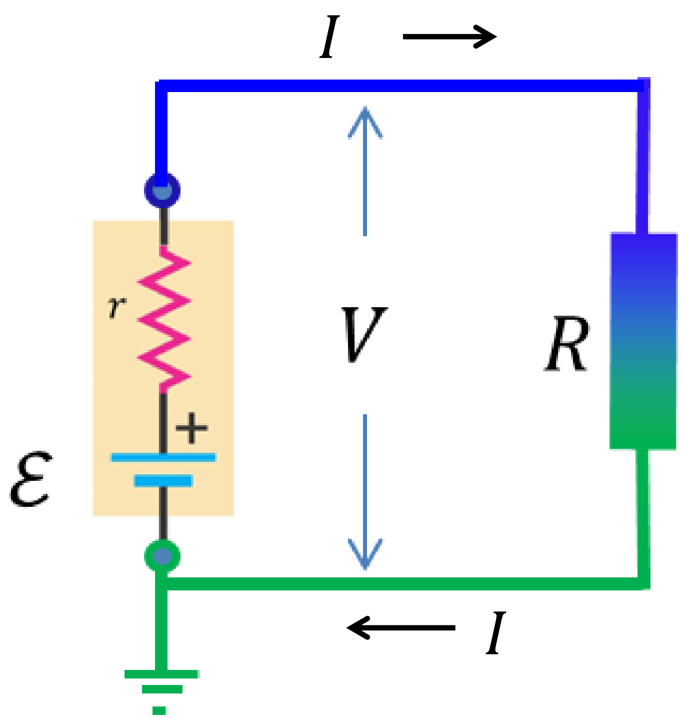
Ideal battery:

$$\mathcal{E} = V = IR$$

V – voltage difference between the battery's electrodes

R – net resistance of the external circuit

Non-ideal Battery



$\boldsymbol{r}$ – the ***internal resistance*** of the battery

What is the output voltage?

$$R_{eq} = R + r$$

$$I = \frac{\mathcal{E}}{r + R}$$

$$V = \mathcal{E} - Ir = \mathcal{E}\frac{R}{r + R}$$

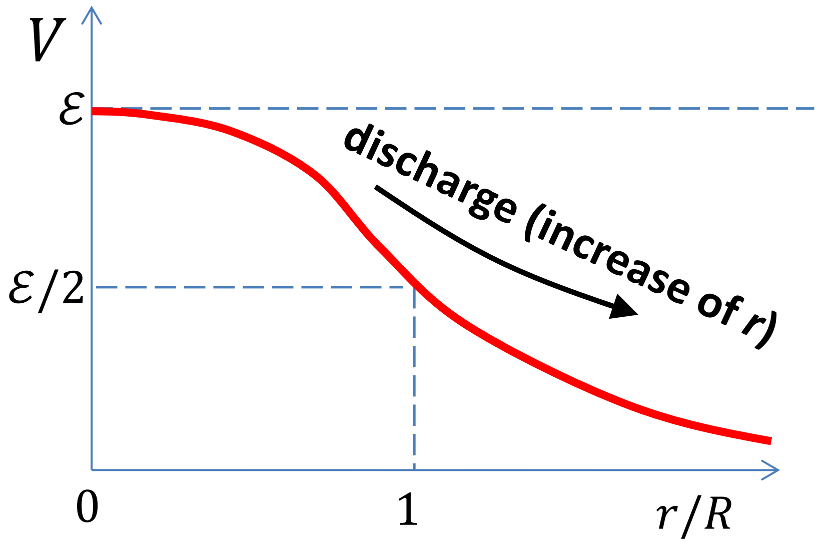
$$V = \mathcal{E}\frac{R}{r + R}$$

Demonstration of the internal resistance.

Battery Discharge

The internal resistance of a battery increases in the process of work: it is small (***in comparison with a typical load resistor***) for a "fresh" battery, and becomes large for an "old" (discharged) one.

$$V = \mathcal{E}\frac{R}{r+R} = \mathcal{E}\frac{1}{r/R+1}$$



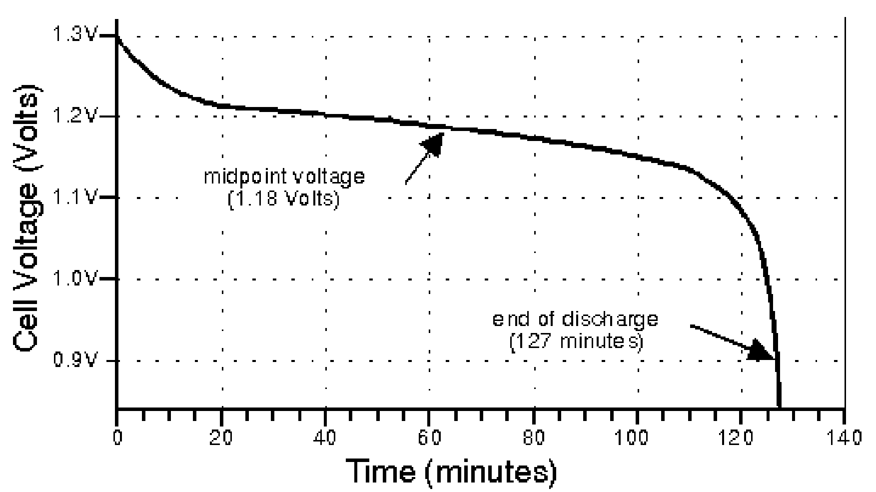
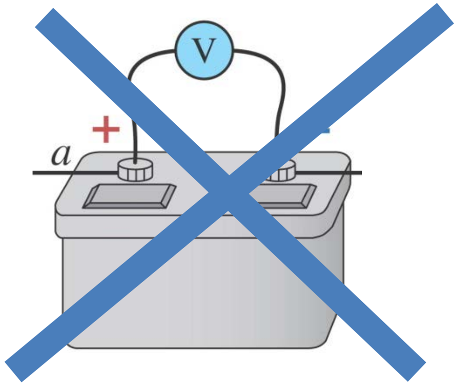
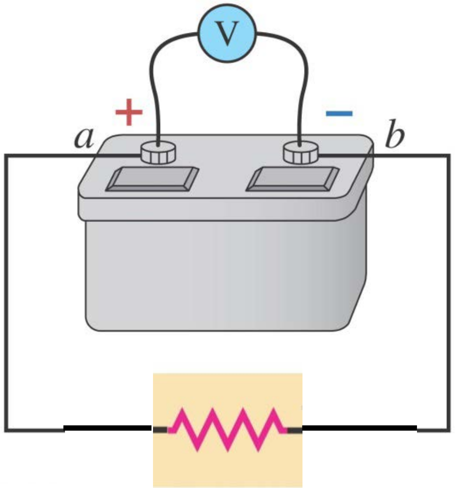


Figure 2. Single Cell 300mA Discharge Curve.

Battery Tester

Can we use a voltmeter (very high *r*) to test the "freshness" of a battery?

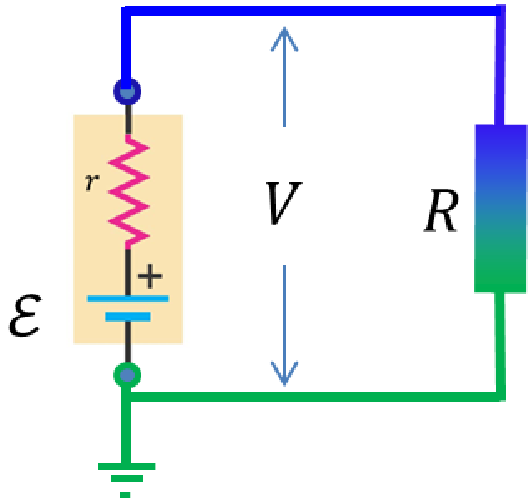




typical R_{load}

The voltmeter will measure $V \approx \mathcal{E}$ provided $r \gg R_{in}$. But R_s is usually as high as $10^6 - 10^7\ \Omega$, and even if $R_{in} \sim 10^3 - 10^4\ \Omega$, we won't notice the battery aging.

Voltage Source



$$V = \mathcal{E}\frac{R}{r+R}$$

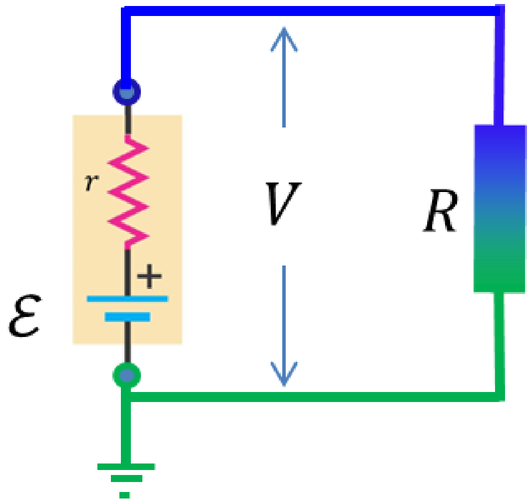
The goal*: *to provide output voltage that is independent of the load resistance.

An ideal voltage source:

A. has “zero” internal resistance (“zero” means that r << all possible values of load R).

B. has “infinite” internal resistance (“infinite” means that r >> all possible values of load R).

Voltage Source



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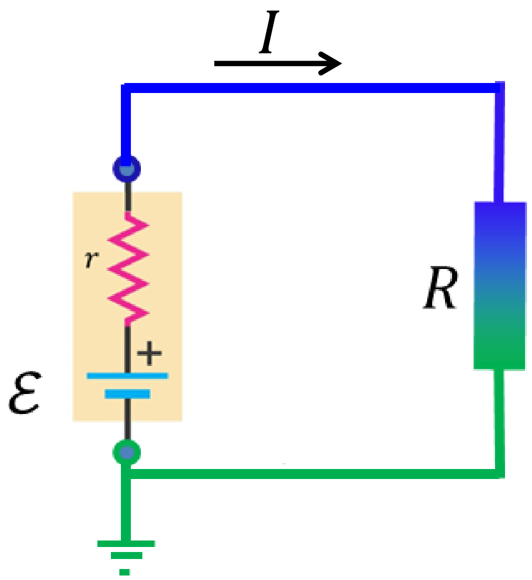
Current Source



$$I = \frac{\mathcal{E}}{r + R}$$

The goal: to provide output current that is independent of the load resistance.

An ideal current source:

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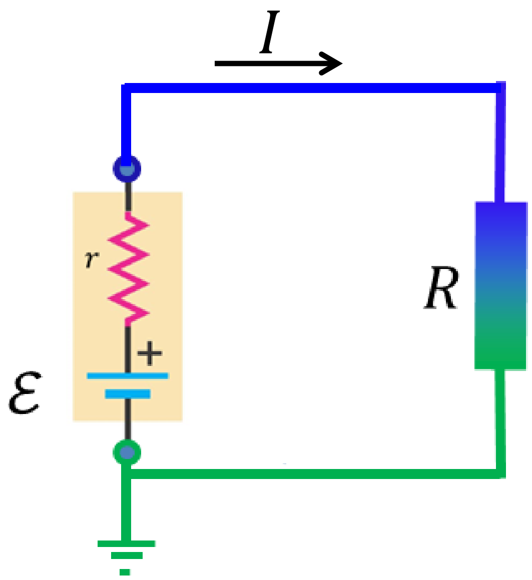
Current Source



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Conclusion

Batteries: the potential energy of charge carriers is increased by non-electrostatic (non-conservative) forces.

Non-ideal batteries: internal resistance.

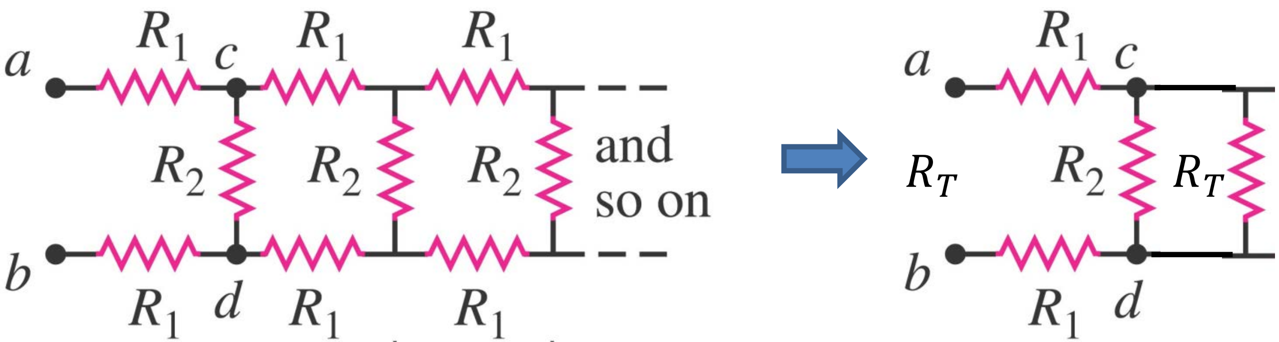
Potential distribution around a complete circuit.

Energy and power in electric circuits.

Next time: Lecture 11: Connection of Resistors, Kirchhoff's Rules

§§ 26.1 - 26.3

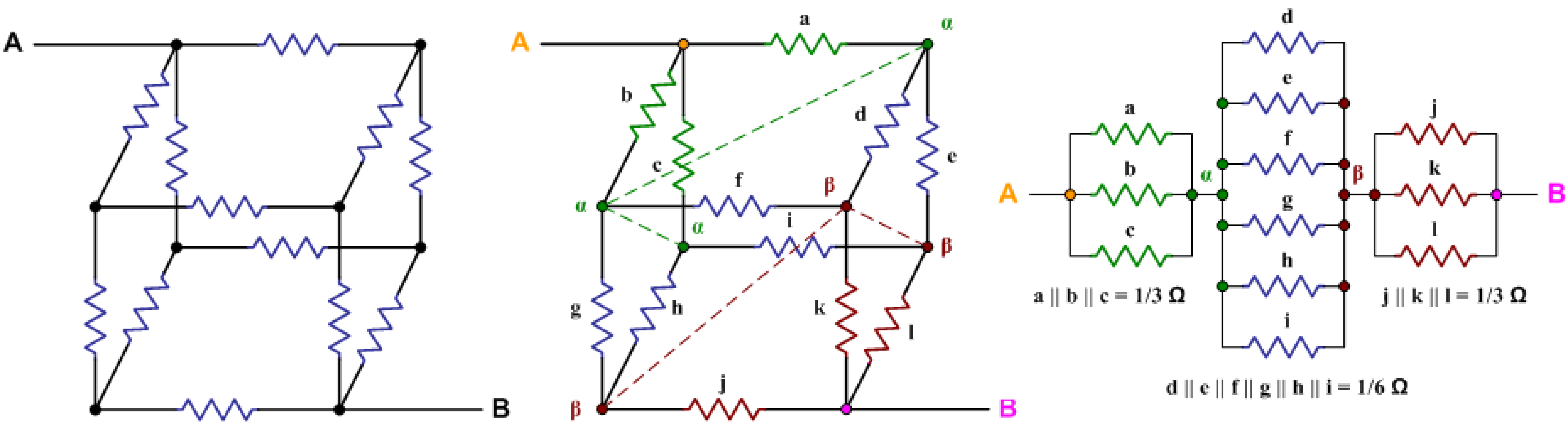
Appendix 1: More Complicated Resistor Circuits



$$R_T = 2R_1 + \frac{R_2 R_T}{R_2 + R_T} \qquad (R_T)^2 - 2R_1 R_T - 2R_1 R_2 = 0$$

All resistors are 1 Ω

All resistors are 1 Ω



Symmetry helps! Equipotential points can be connected (all *a*'s and *b*'s).