current, resistance and dc circuits

current

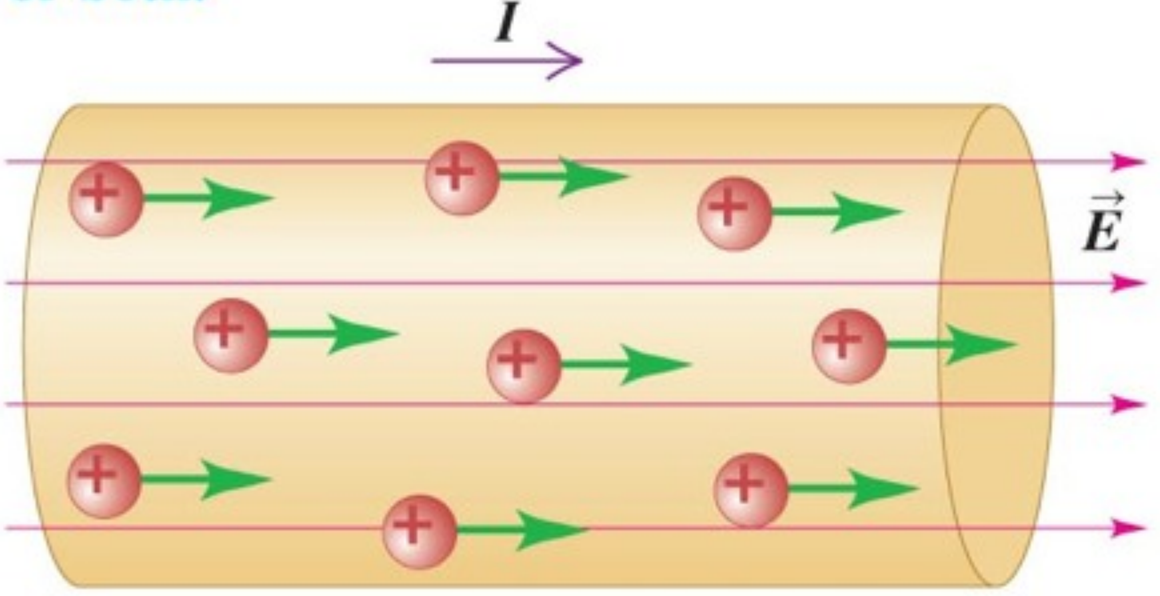
➜ previously we considered electrostatic situations in which no ***E***-field could exist inside a conductor

➜ now we move to the case where an **electric field is maintained within a conductor** by an external source and the conductor forms a complete circuit

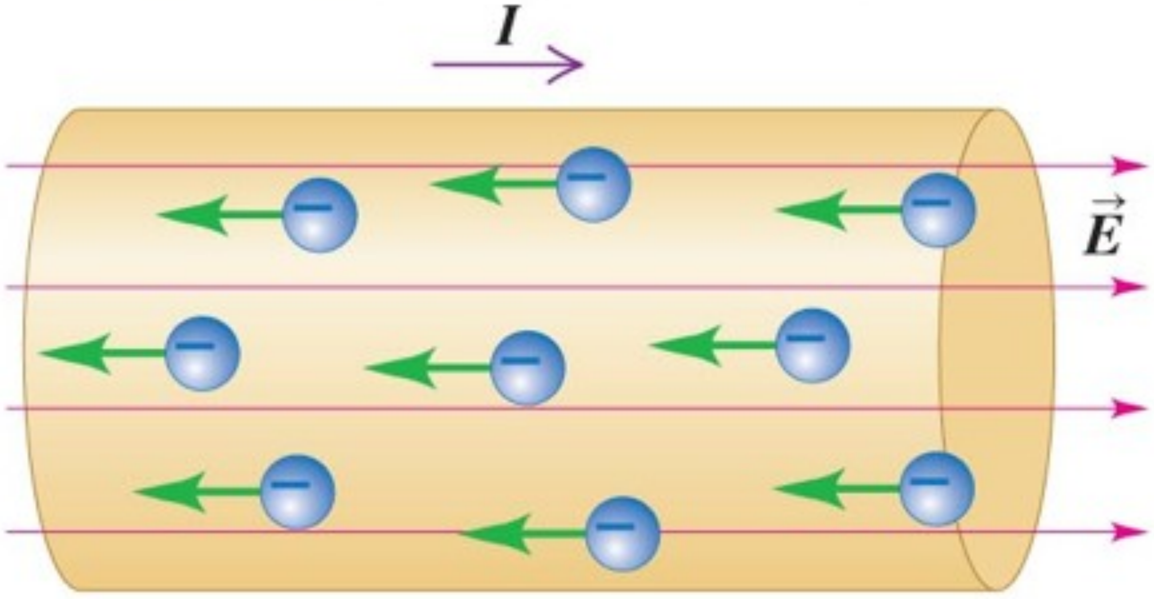
➜ this electric field applies a force to charges within the conductor

electrical current $$I = \frac{\Delta Q}{\Delta t}$$ *Coulombs/second = Amperes*

A conventional current is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.



In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

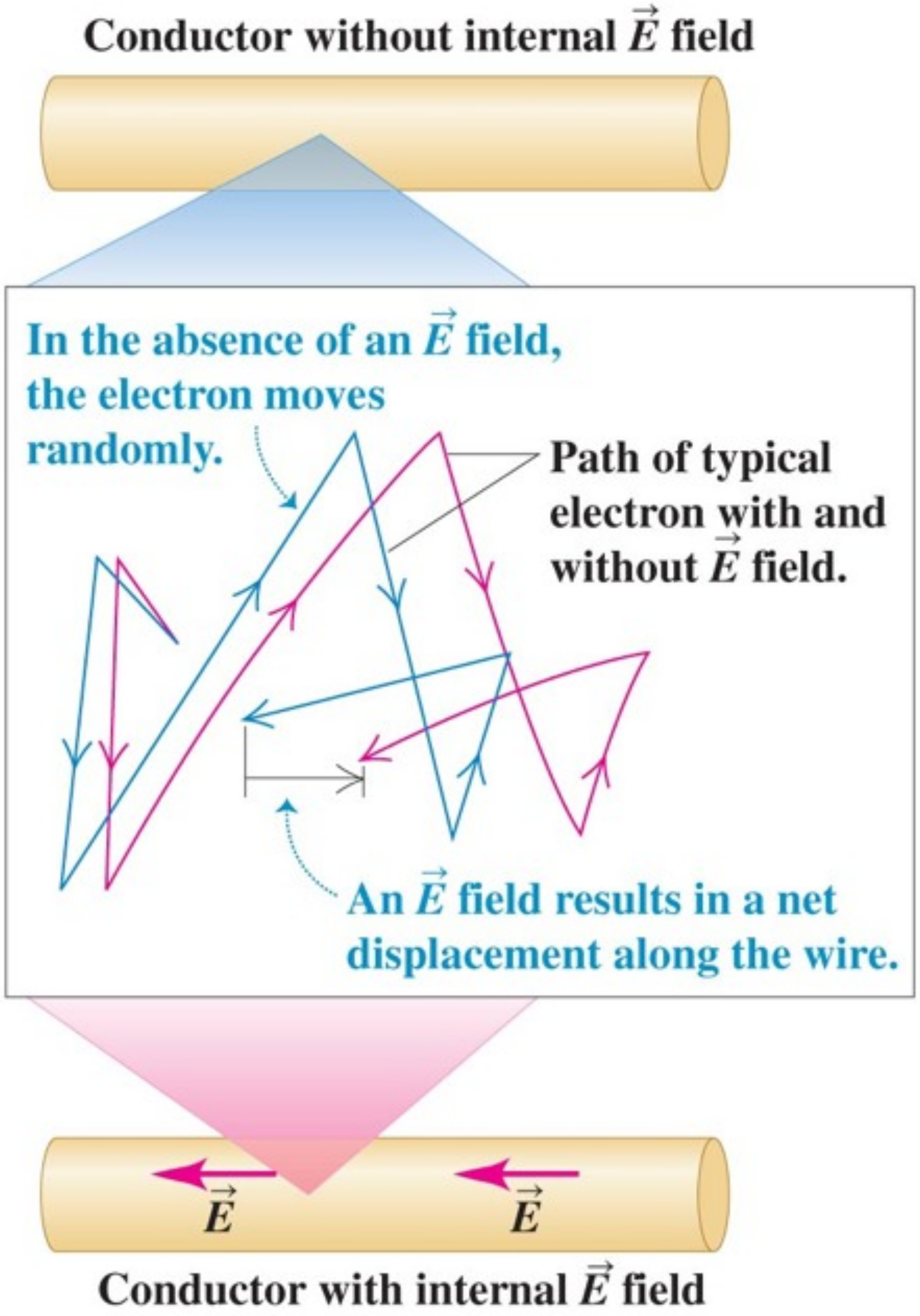


current

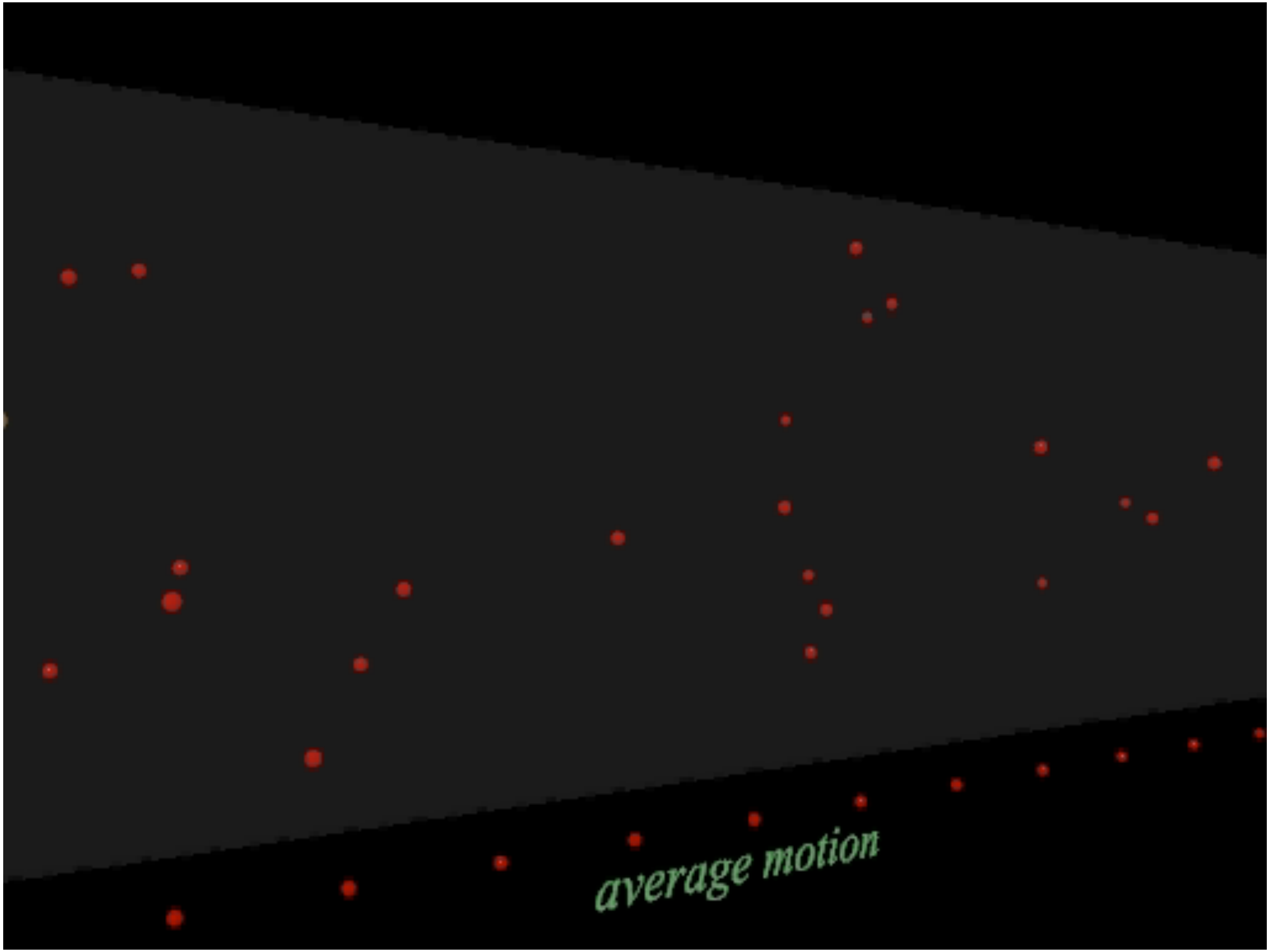
➜ rate of flow of charge is uniform throughout a conductor, else charge would accumulate in certain areas

➜ at non-zero temperature, electrons in a metal move a lot without any applied field

➜ although the drift velocity of electrons is rather slow ~ 10^{-4} m/s, the electric fields propagate through the metal at close to the speed of light, so we never notice a delay when 'switching on'



current

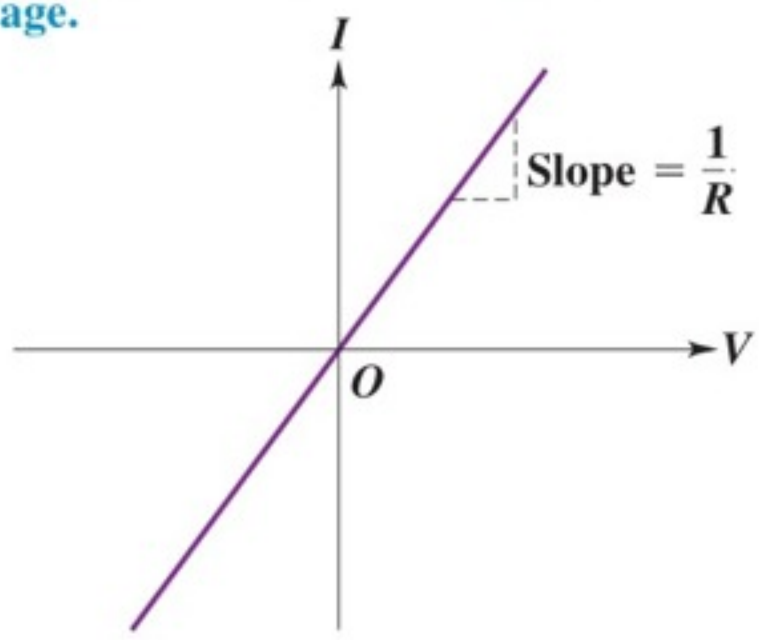


current

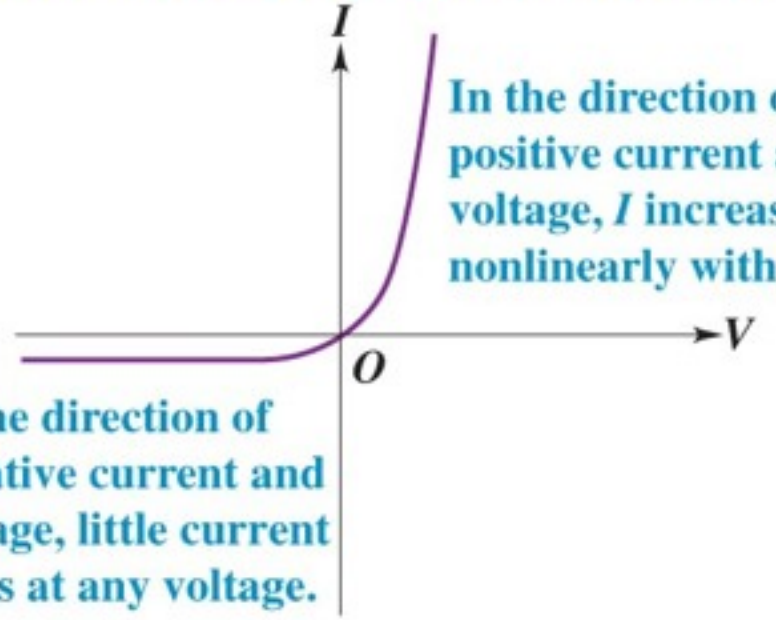
➜ what determines how much current we get for a given applied electric field or potential difference ?

➜ properties of the conductor determine the “resistance”

Ohmic resistor (e.g., typical metal wire): At a given temperature, current is proportional to voltage.



Semiconductor diode: a non-ohmic resistor



In the direction of positive current and voltage, *I* increases nonlinearly with *V*.

In the direction of negative current and voltage, little current flows at any voltage.

Ohm's law

➜ what determines how much current we get for a given applied electric field or potential difference ?

➜ properties of the conductor determine the "resistance"

➜ (conventional) current flows from a point of high potential to a point of lower potential along the direction of the ***E***-field

➜ the current is proportional to the drift velocity of the charges in the conductor

➜ in experiments, for many materials over a range of temperature, the drift velocity is approximately proportional to the electric field magnitude ***E*** and hence to the potential difference *V*

➜ from this follows the empirical Ohm's law

$$V = IR$$

with *R* a constant

resistance, *R*,
measured in
Ohms, Ω = V/A

if you like mysteries think about whether this agrees with F=ma

resistance of a wire

➜ we find that the resistance of a cylindrical ‘wire’

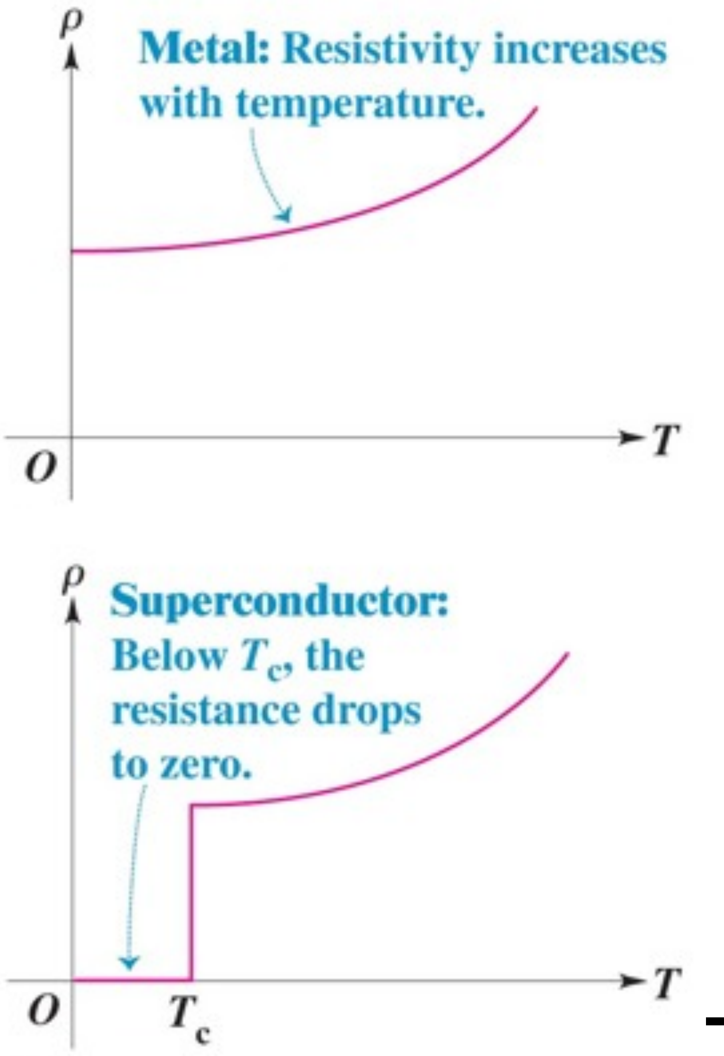
- ➜ increases with increasing length of wire
  (charge has further to “push through”)
- ➜ decreases with increasing cross-sectional area of wire
  (charge has more possible paths through the wire)
- ➜ depends upon the material the wire is made of

$$R = \rho \frac{L}{A}$$

ρ is the **resistivity** which is a property of the material at a given temperature

TABLE 19.1 **Resistivities at room temperature**

Substance	ρ ($\Omega \cdot$m)	Substance	ρ ($\Omega \cdot$m)
Conductors:		Mercury	95×10^{-8}
Silver	1.47×10^{-8}	Nichrome alloy	100×10^{-8}
Copper	1.72×10^{-8}	Insulators:	
Gold	2.44×10^{-8}	Glass	$10^{10} - 10^{14}$
Aluminum	2.63×10^{-8}	Lucite	$> 10^{13}$
Tungsten	5.51×10^{-8}	Quartz (fused)	75×10^{16}
Steel	20×10^{-8}	Teflon®	$> 10^{13}$
Lead	22×10^{-8}	Wood	$10^{8} - 10^{11}$

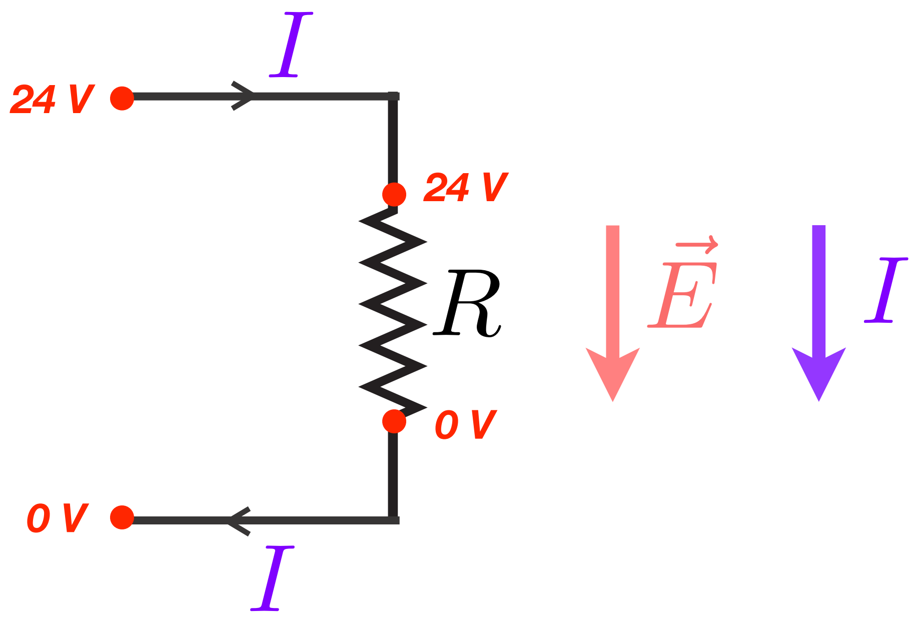


Metal: Resistivity increases with temperature.

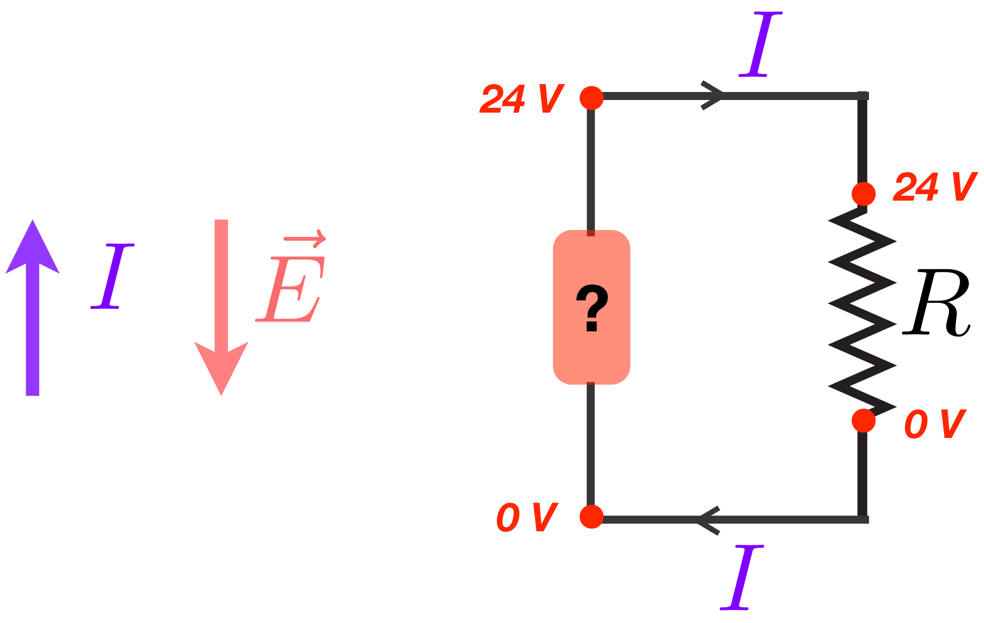
Superconductor: Below T_c, the resistance drops to zero.

electromotive force

current will always 'flow' from high potential to lower potential - along the direction of the electric field



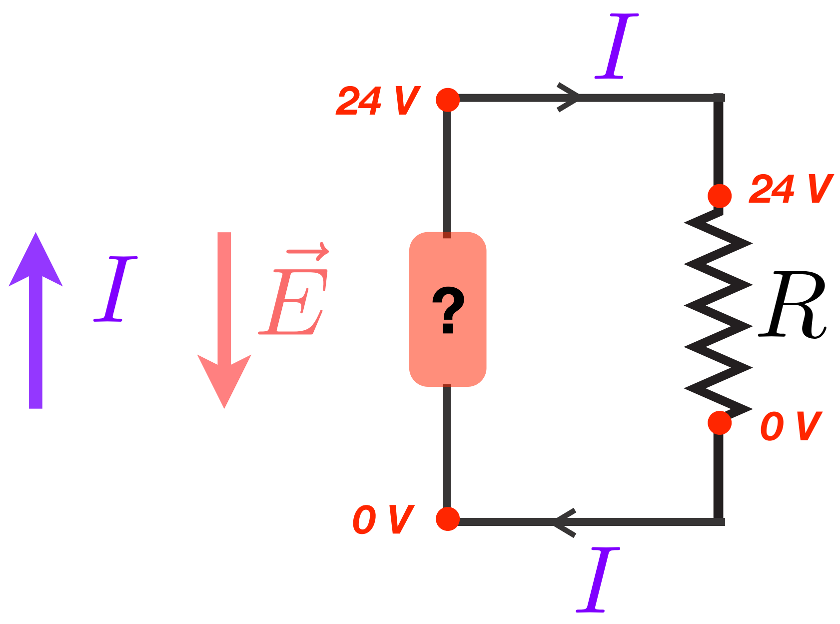
if we want a complete circuit with resistance we will need a device in which current can flow from lower to higher potential **against the direction of the electric field**



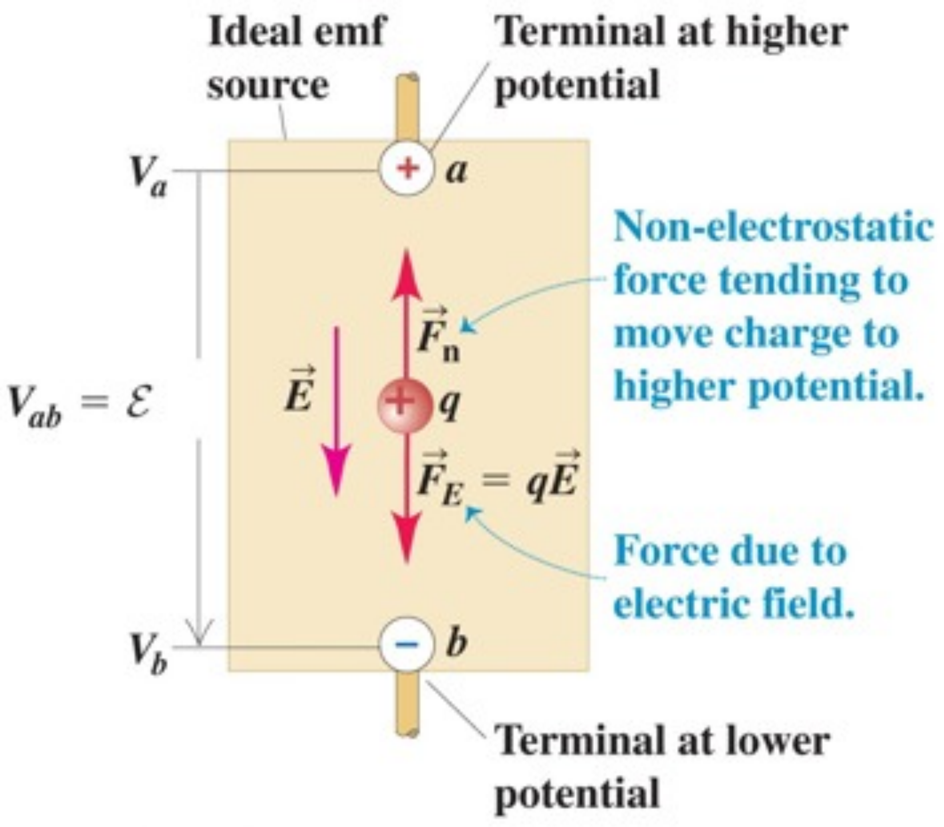
electromotive force

this is called an “electromotive force”

examples include batteries, electric generators, solar cells, thermocouples …



ideal emf sources maintain a constant potential difference between the terminals

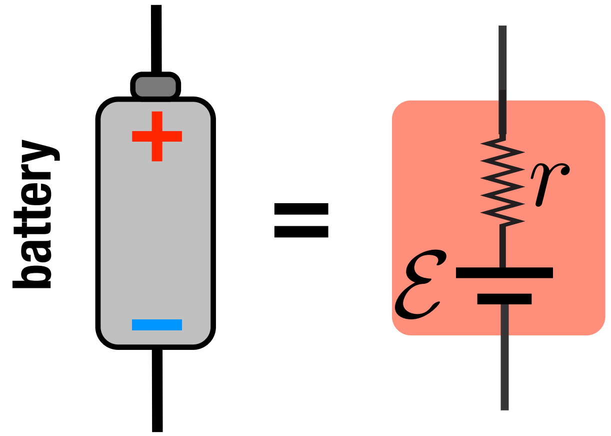


the device uses a force other than $\vec{F} = q\vec{E}$ to move charges from low to high potential

real emf sources

➜ real sources of emf have an internal resistance experienced by the current as it flows through the source.

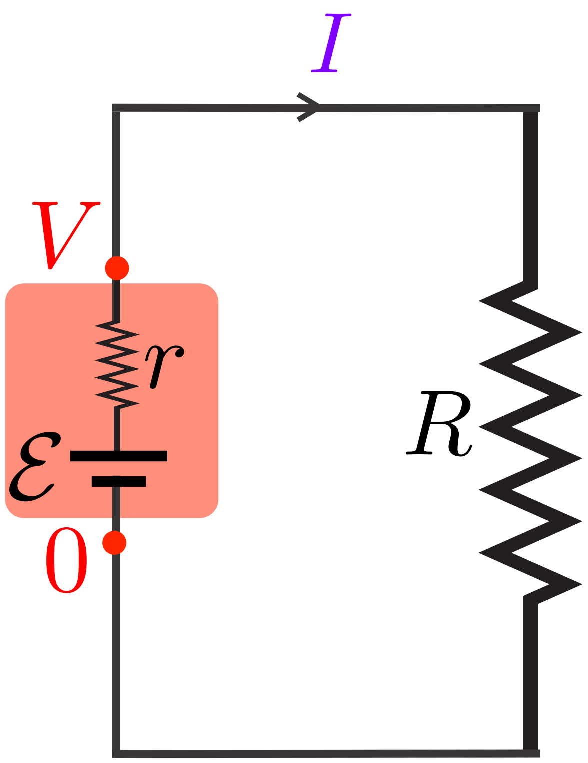
we denote this by a lower case **r**



➜ hence the potential difference between the terminals is reduced and depends upon the current flowing

$$V = \mathcal{E} - Ir$$

➜ so connected in a circuit with a resistor we have

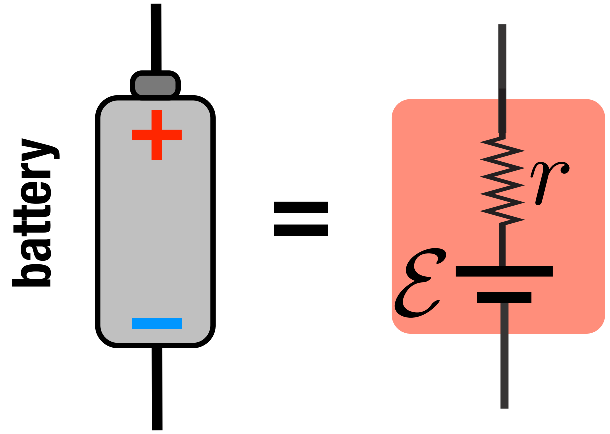


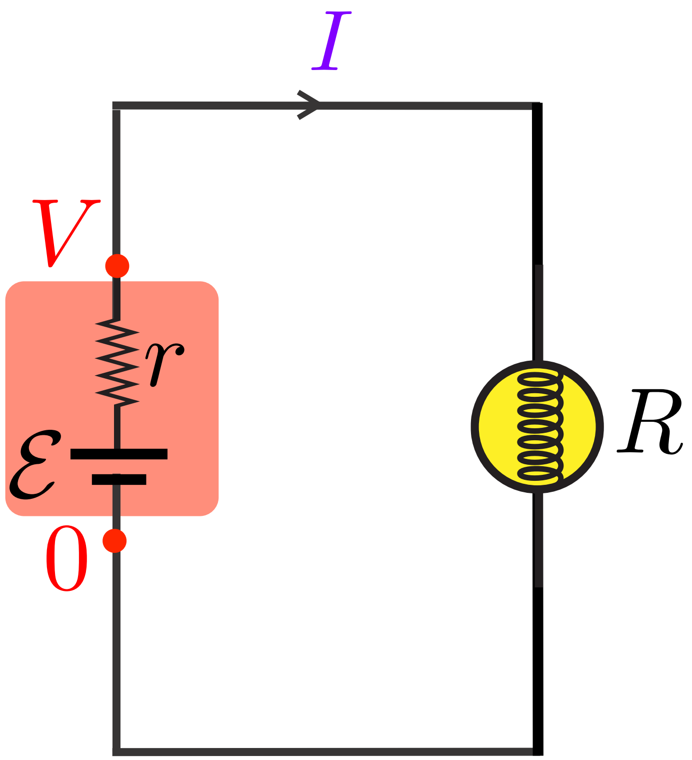
$$V = IR$$

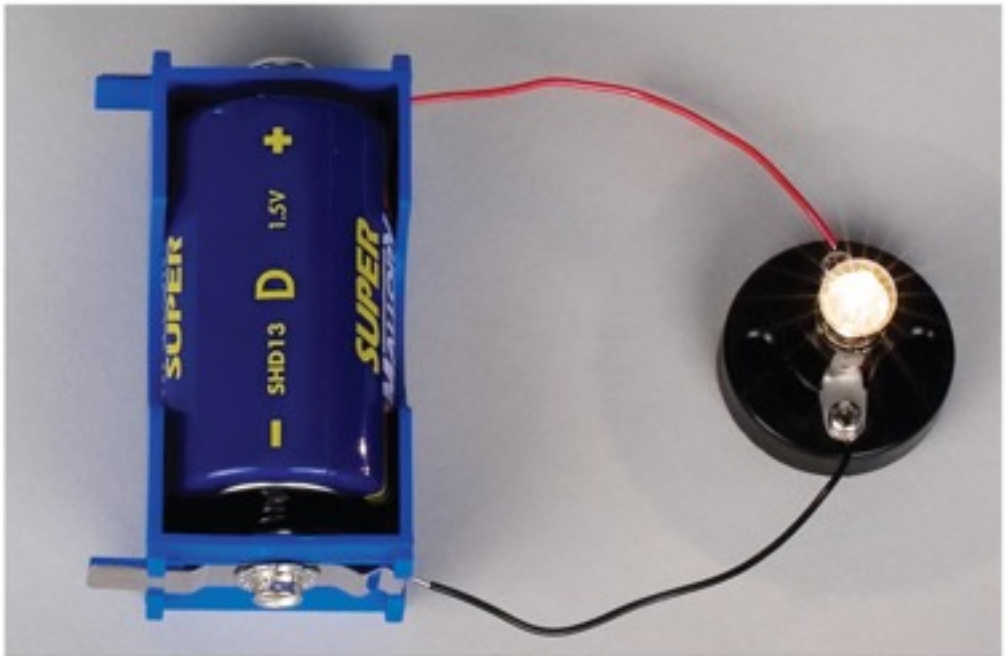
$$I = \frac{\mathcal{E}}{r + R}$$

dimming bulb

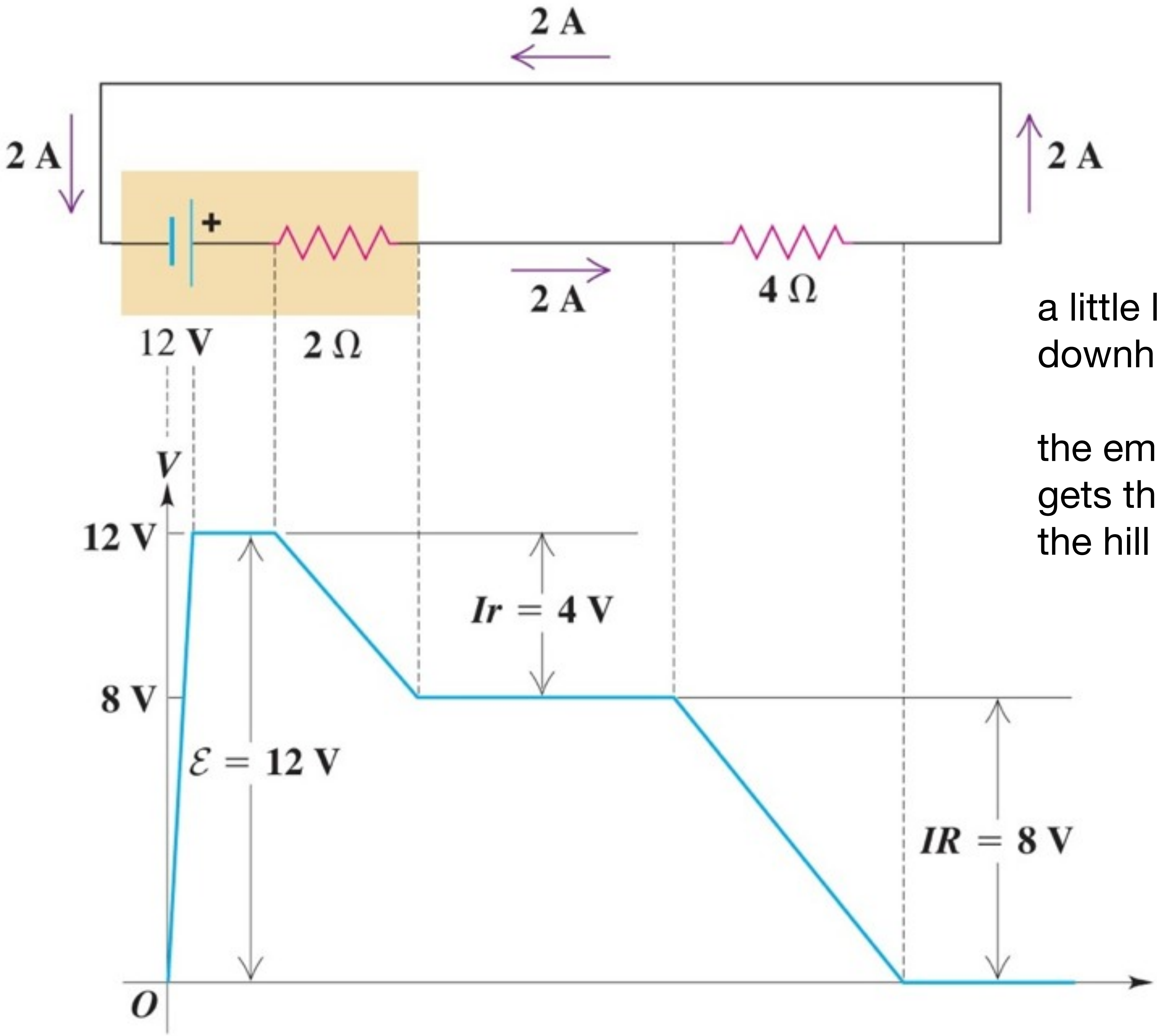
As a flashlight battery ages its emf stays approximately constant, but its internal resistance increases. A fresh battery has an emf of 2.5 V and negligible internal resistance. When the battery needs replacing its emf is still 2.5 V but its internal resistance has increased to 1000 Ω. If this old battery is supplying 0.5 mA of current, what is its terminal voltage?







potential in a circuit - an analogy



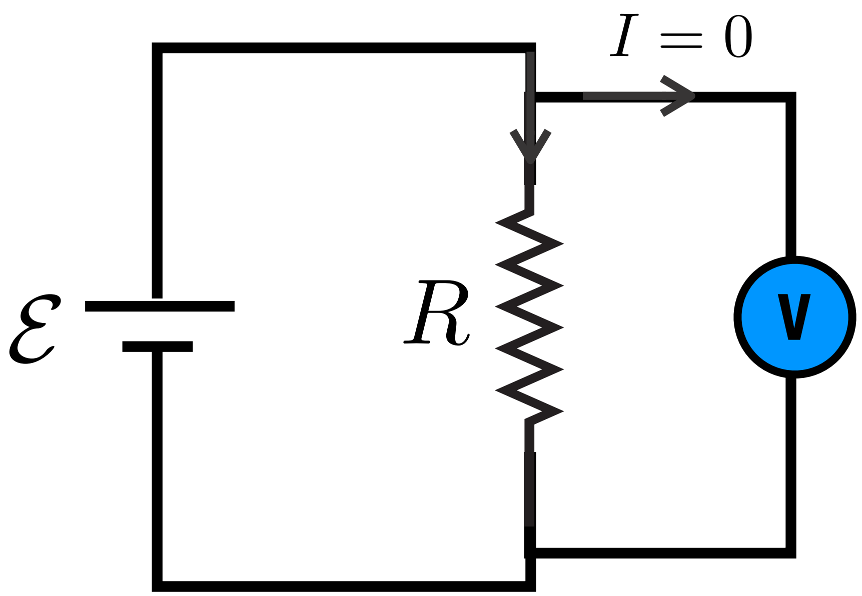
a little like water flowing downhill ?

the emf is like the pump that gets the water to the top of the hill

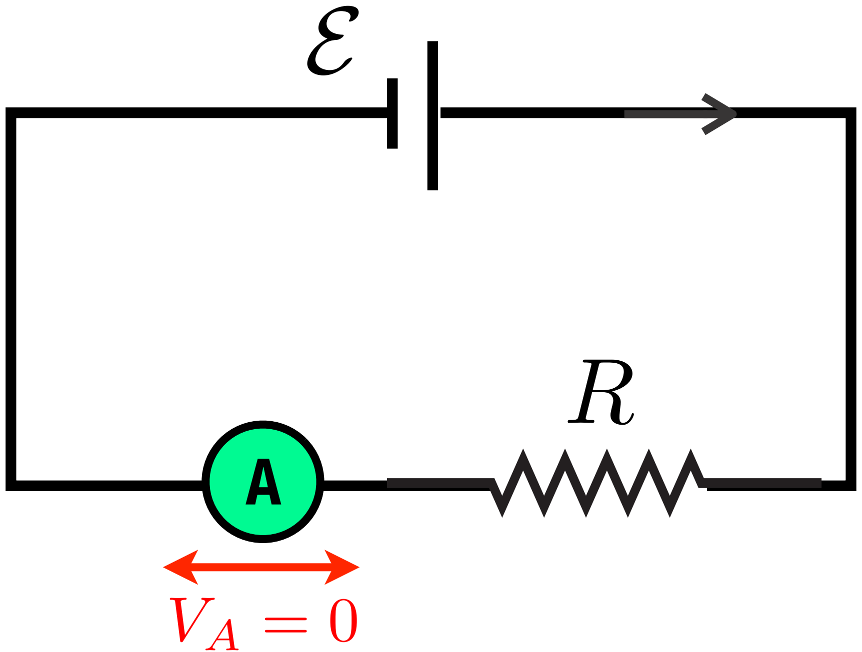
measurement, voltmeters and ammeters

it’s handy to define idealised measurement tools

➜ ideal voltmeter draws no current (infinite resistance)



➜ ideal ammeter causes no change in potential (zero resistance)



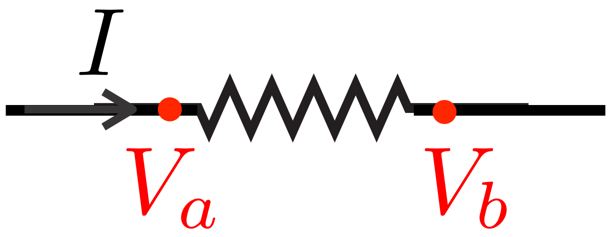
power in electrical circuits

see the textbook for a derivation that shows the power transferred **into** a circuit element is

$$P = IV$$

$$V = V_a - V_b$$

suppose the circuit element is a resistor



then Ohm's law holds

$$V = IR$$

so then

$$P = I^2R = \frac{V^2}{R}$$

is the power lost from the circuit (as heat)

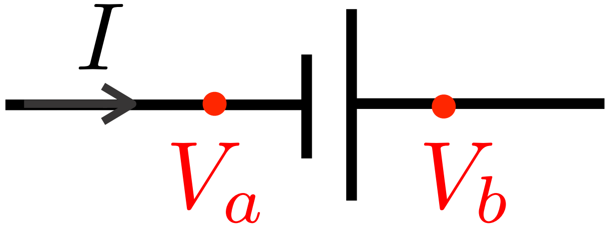
power in electrical circuits

see the textbook for a derivation that shows the power transferred into a circuit element is

$$P = IV$$

$$V = V_a - V_b$$

suppose the circuit element is an ideal emf source



$$V_b = V_a + \mathcal{E}$$

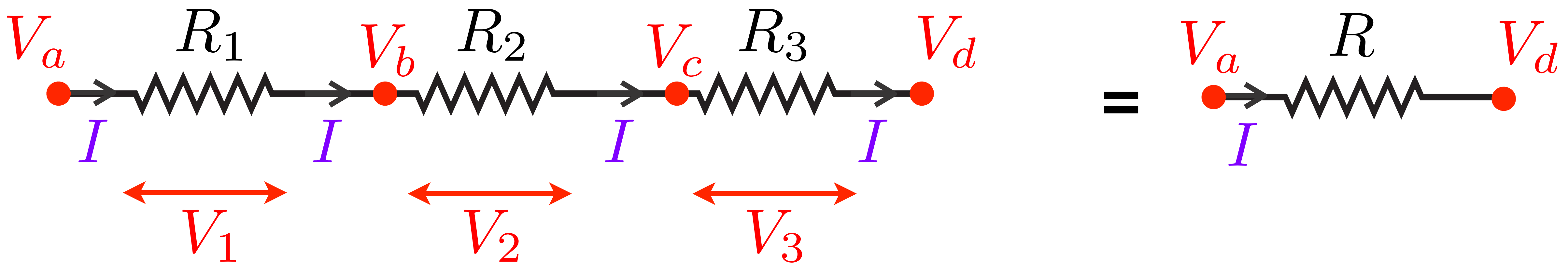
$$V = -\mathcal{E}$$

$$P = -\mathcal{E}I$$

negative indicates that power is transferred **out** of the circuit element

resistors in series

Consider three resistors connected in series (part of a larger circuit)



$$V_a - V_d = IR$$

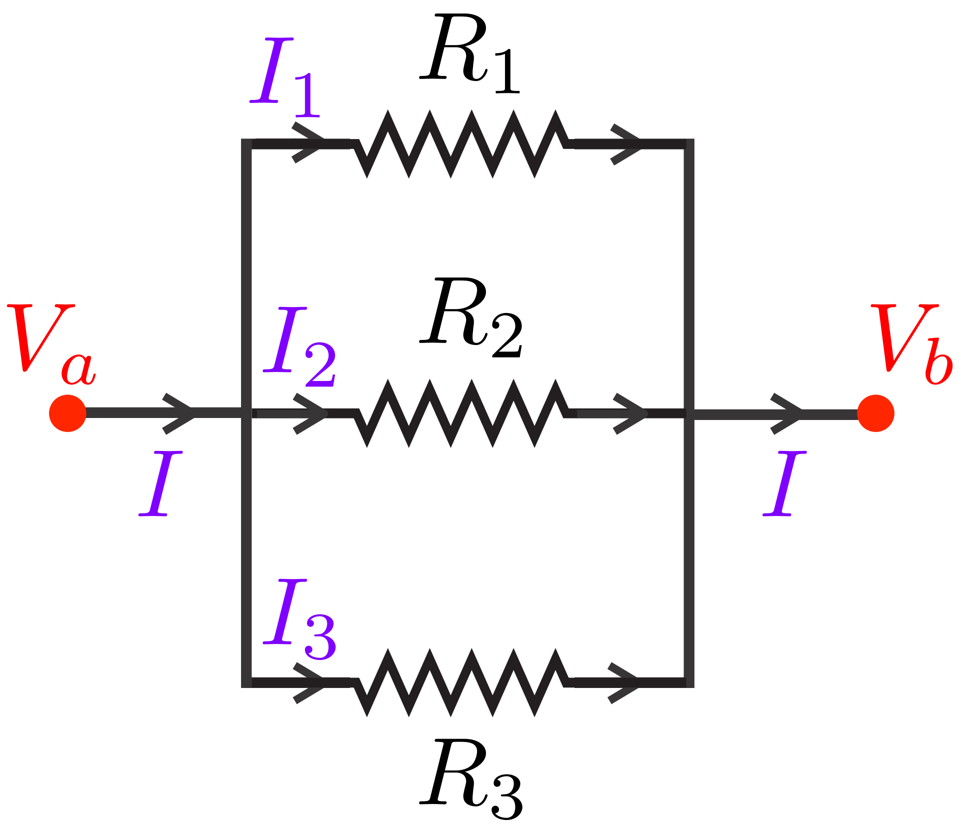
$$\left.\begin{aligned} V_1 &= IR_1 \\ V_2 &= IR_2 \\ V_3 &= IR_3 \end{aligned}\right\} \quad \begin{aligned} V_a - V_d &= V_1 + V_2 + V_3 \\ &= IR_1 + IR_2 + IR_3 \end{aligned}$$

$$IR = IR_1 + IR_2 + IR_3$$

$$\boxed{R = R_1 + R_2 + R_3}$$

resistors in parallel

Consider three resistors connected in parallel (part of a larger circuit)



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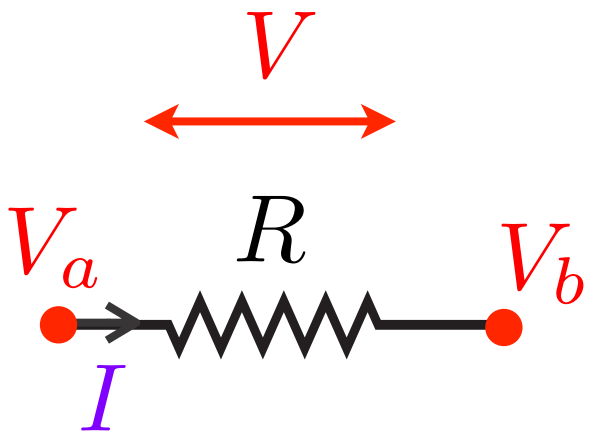


$$V = IR$$

$$V_a - V_b = I_1 R_1$$

$$V_a - V_b = I_2 R_2$$

$$V_a - V_b = I_3 R_3$$

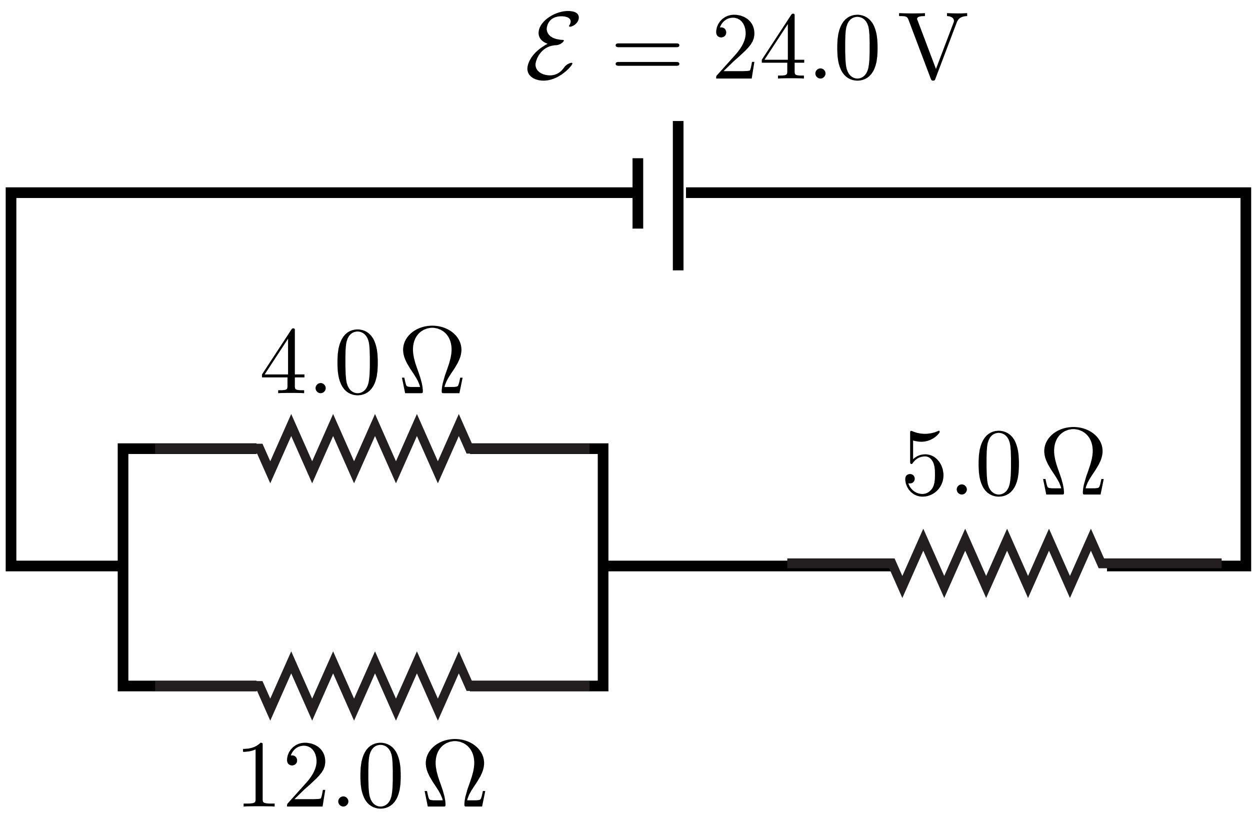
$$I = I_1 + I_2 + I_3$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

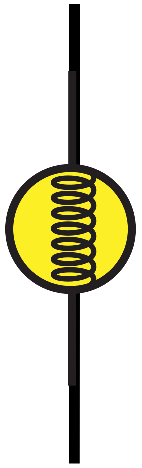
$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

an example

find the current through each resistor



bulbs are resistors



➜ bulbs act like resistors - power loss is through heat and light

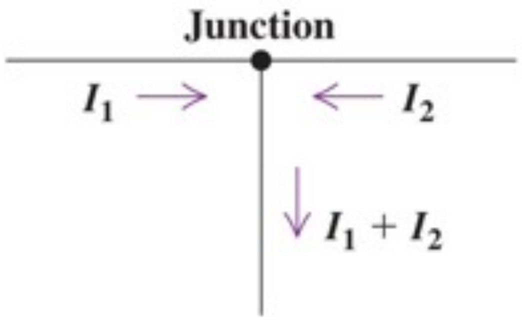
➜ their brightness is proportional to the power loss in them, $I^2R = V^2/R$

➜ so the larger the current, the brighter the bulb
(or, the larger the potential drop, the brighter the bulb)

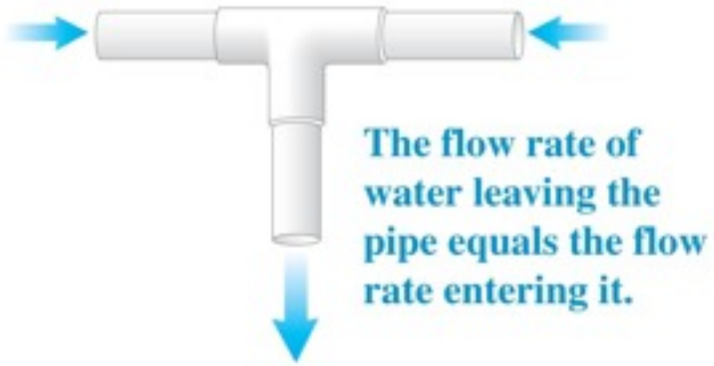
Kirchhoff’s rules

- ➜ some circuits can’t be expressed in terms of parallel and series set-ups
  - ➜ Kirchhoff’s rules let us deal with these
  - ➜ just things we already know

➜ **junction rule**



(a) Kirchhoff’s junction rule


The flow rate of water leaving the pipe equals the flow rate entering it.

(b) Water-pipe analogy for Kirchhoff’s junction rule

“current into a junction = current out of a junction”

“no current is lost in a junction”

the algebraic sum of the currents into a junction is zero $\sum I = 0$

Kirchhoff's rules

➜ some circuits can't be expressed in terms of parallel and series set-ups

   ➜ Kirchhoff's rules let us deal with these

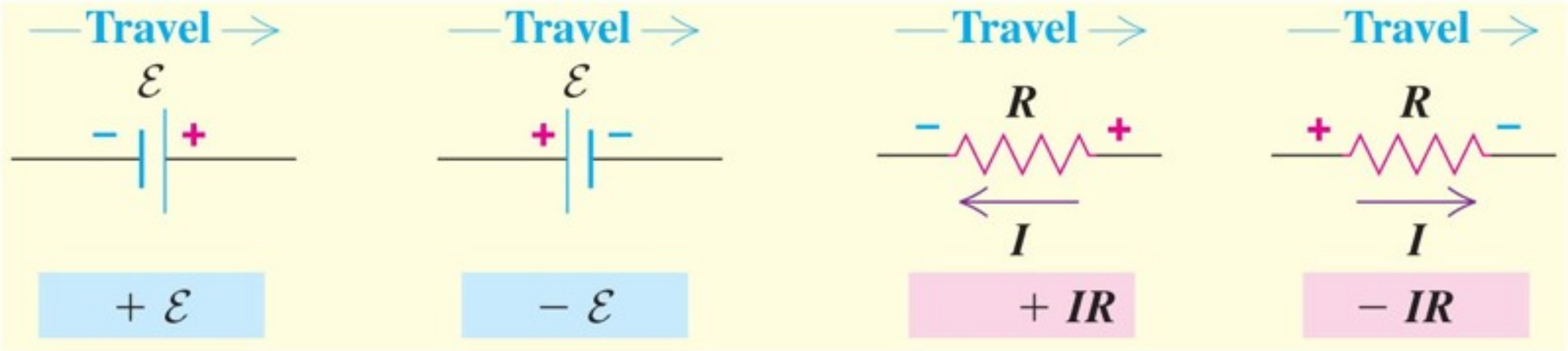
   ➜ just things we already know

➜ **loop rule**

the algebraic sum of the potential difference in any loop must equal zero

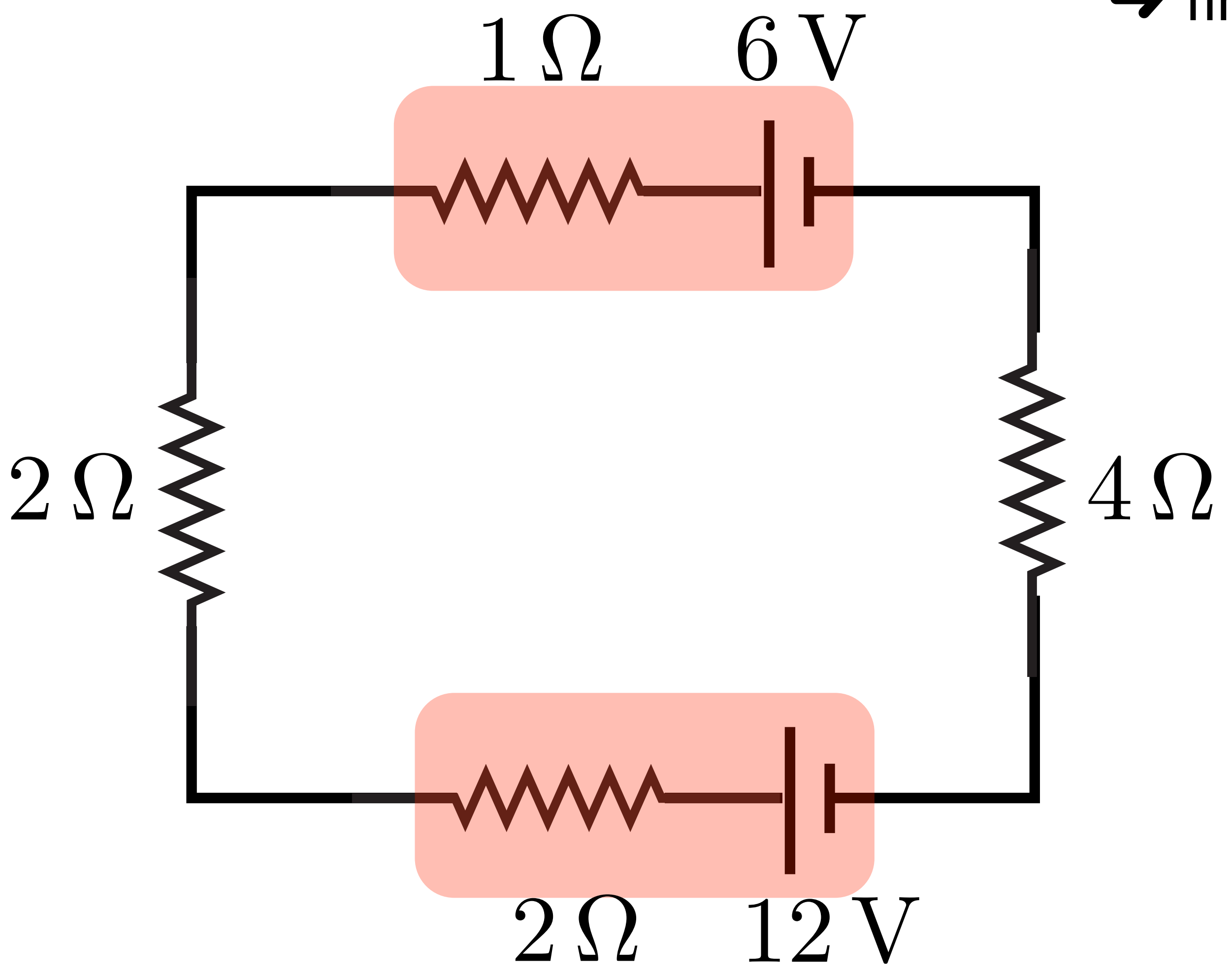
$$\sum_{\text{around loop}} V = 0$$

sign conventions



Kirchhoff's rules - example

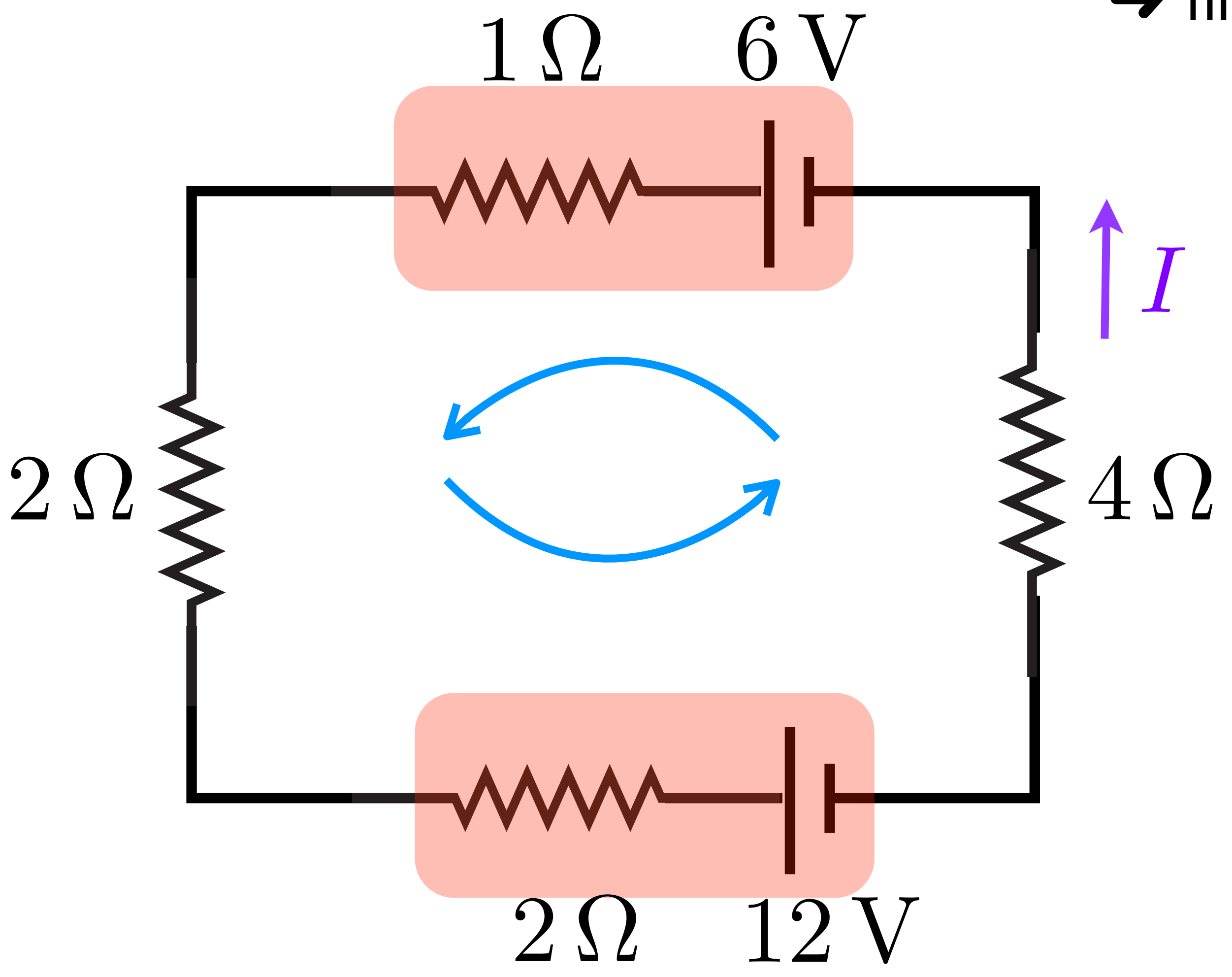
➜ just do a simple example to see how it works



➜ find the current in the circuit

Kirchhoff’s rules - example

➜ just do a simple example to see how it works



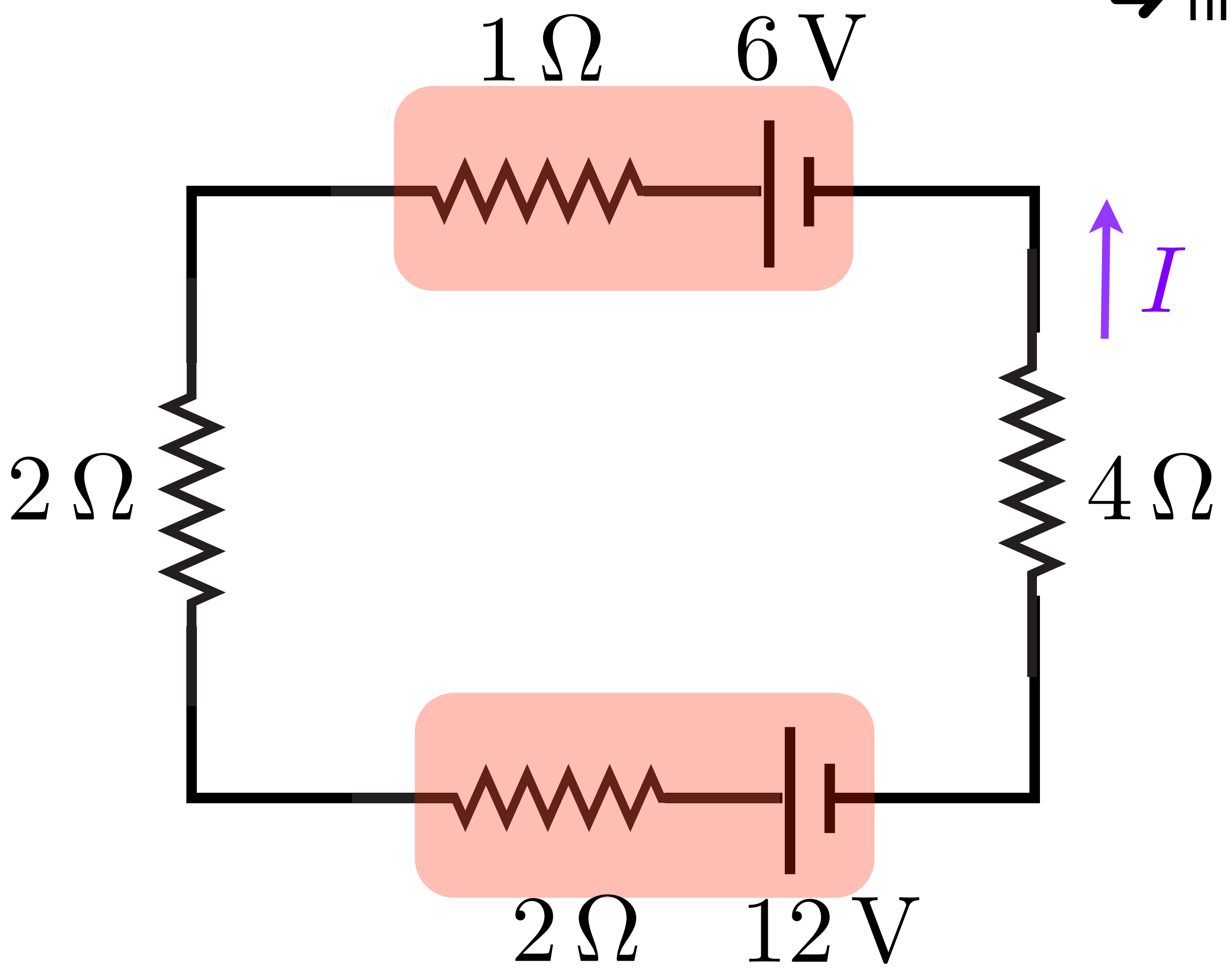
➜ find the current in the circuit

➜ choose a direction for the current - doesn’t matter if we’re wrong, we’ll just get a negative value for *I*

➜ choose whether to go around the loop clockwise or counterclockwise - shouldn’t change the result

Kirchhoff’s rules - example

➜ just do a simple example to see how it works



➜ find the current in the circuit

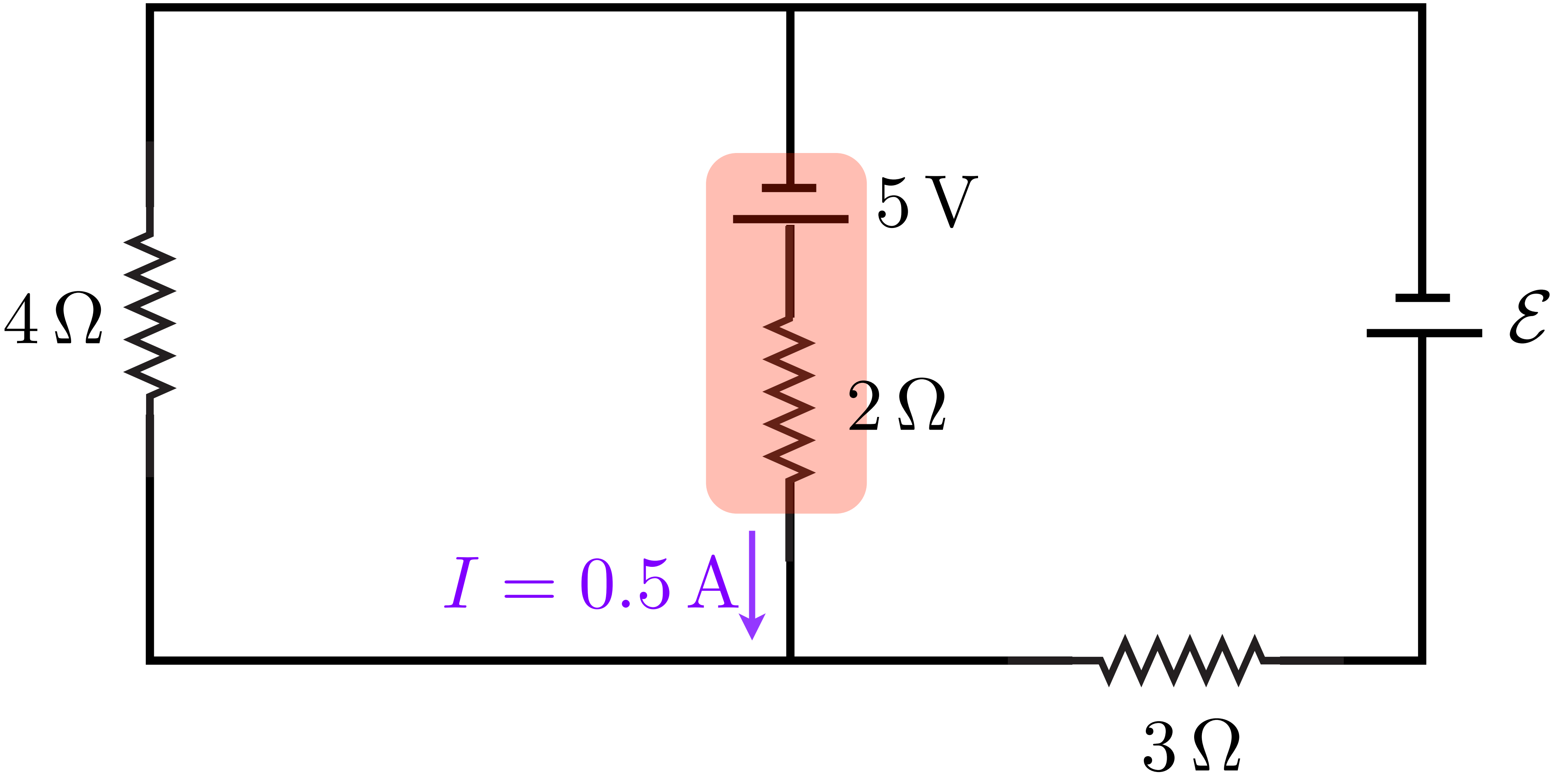
$$I = -\tfrac{2}{3}\,\text{A}$$

so we guessed the
current direction wrongly

Kirchhoff's rules - example

➜ a more challenging example that uses both rules

- ➜ find the unknown emf, $\mathcal{E}$, and the current in the leftmost resistor



Kirchhoff’s rules - example

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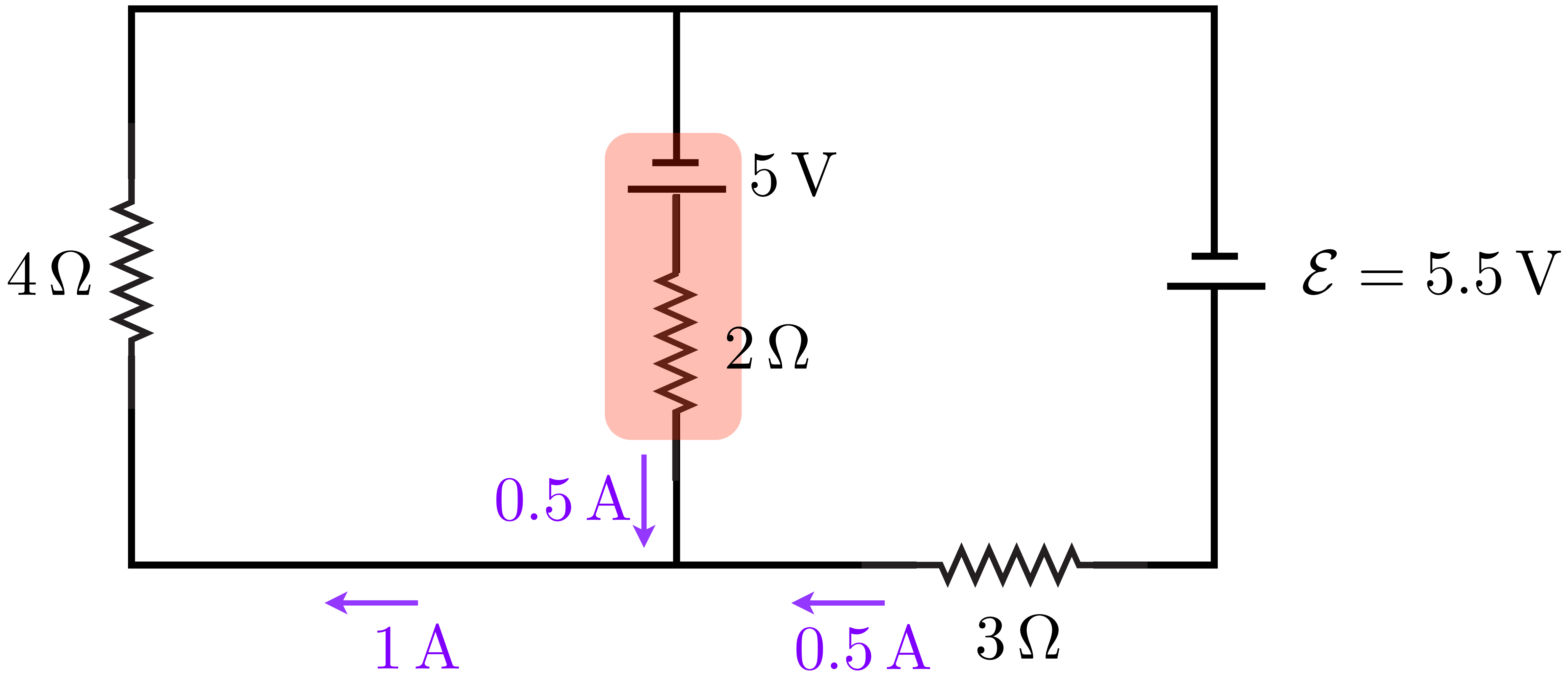

SI units summary

charge in Coulombs, C

current in Amperes, A = C/s   * actually the ‘base’ unit (along with m, kg, s)

potential in Volts, V

energy in Joules, J = C V

power in Watts, W = J/s

resistance in Ohms, Ω = V/A