

Mutual Inductance and Transformers

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This lecture is concerned with introducing a new type of electric effect that can occur in circuits, called **mutual inductance**. Mutual inductance refers to electrical coupling between multiple wire coils (inductors). We will discuss how circuits with coupled inductors can be analyzed. We will also introduce **transformers**—mutual inductance-based devices that are used in communications and power systems to establish required current and voltage levels.

1 Inductor Physics

Unfortunately, we will not have much time to discuss the physics of wire coils (inductors). For now, the discussion will be limited to a conceptual description of mutual inductance. If time permits, we will consider inductor physics in more detail at the end of the course.

Current in a coiled wire generates a magnetic field in and around the coil. Changes in this magnetic field (caused by changes in current) generate a force on charges in nearby conducting materials, and hence generate a voltage in these conductors. One conductor in which a voltage is generated by the changing current is the coiled wire itself: this is what we call **inductance**, or more properly **self-inductance**.

It is sometimes useful to build circuits in which two coiled wires are nearby (often because they are wound around the same **core**, or material in which the magnetic field is generated). In this case, changing currents in each wire generate voltages across the other wire, in addition to generate a voltage across itself. This effect is called mutual inductance. If the core material is non-magnetic, then circuits with coupled inductors of this sort satisfy linear differential equations, and the coupling is the "same" in both directions (please see your Nilsson and Riedel for more details). We will consider circuits of this type.

One detail is worth keeping in mind. The polarities of the induced voltages depend on the orientation of the wire coils. To analyze circuits, we will need a convention for keeping track of coil winding orientation.

2 Mutual Inductance in Circuits

Consider two coupled coils. In a circuit schematic, the two coils are represented as follows:

Notice the following features of the circuit diagram:

- Coils 1 and 2 have self-inductances L_1 and L_2 , respectively.
- There is a mutual inductance M that describes the strength of the interactions between the two coils. It turns out that $M = k\sqrt{L_1L_2}$, where the **coupling coefficient** k is between 0 and 1. If the current going through inductor 1 is i_1 , the voltage induced on inductor 2 by inductor 1 has magnitude $M\frac{di_1}{dt}$. Similarly, the voltage induced on inductor 1 by inductor 2 is $M\frac{di_2}{dt}$.
- There is a **dot** by one terminal of each inductor. These dots allow us to determine the polarity of the voltage induced by the coupling of the inductors. The **dot convention** is as follows: if the reference current enters the inductor through the dot, then the reference polarity of the voltage it induces at the other terminal is positive at the dotted end (i.e., the difference between the voltages at the dotted and undotted ends is $M\frac{di}{dt}$, rather than $-M\frac{di}{dt}$). If the reference current leaves the inductor through the dot, then the reference polarity of the voltage on the other inductor is positive at the undotted end.

In our circuits, the voltages due to self-inductance and mutual inductance add. Thus, in the example given during class, the equations governing the two inductors are

$$\begin{aligned}v_1 &= L_1\frac{di_1}{dt} - M\frac{di_2}{dt} \\v_2 &= -L_2\frac{di_2}{dt} + M\frac{di_1}{dt}.\end{aligned}$$

Do you see how to come up with the proper plus and minus signs?

Now let's study an example circuit with these coils. As we'll see in this example, it's a good idea to use mesh currents rather than node voltages when analyzing circuits with mutual inductances.

Example:

3 Linear Transformer

A **transformer** is a special circuit with mutually inductive coils, that can be used to **match** (cancel, shape) impedances and establish voltage and current levels. First, we'll talk about the **linear transformer**, and then discuss a special case of the linear transformer called the **ideal transformer**.

We have already developed all the circuit theory needed to study the linear transformer, so it's just a matter of analyzing the circuit below. For a change, let's do the analysis in general, in the same way as in your textbook.

4 Ideal Transformer

An ideal transformer is a linear transformer with the following three special properties:

- The coupling coefficient is unity.
- The self-inductance of each coil is infinite.
- The coil losses due to parasitic resistances are zero.

It turns out that two coils wrapped tightly on the same ferromagnetic core approximate an ideal transformer.

Because of the properties of an ideal transformer, it turns out that a special set of equations that govern its behavior can be derived. We will not go through this derivation here, but it is in

Nillson and Riedel if you are interested. The result of the analysis is that voltages and currents on either side of the transformer are simply related, by the ratio of the number of turns of the coil on either side. We discuss this relation in the next paragraph, but first we draw the circuit diagram for an ideal transformer.

For an ideal transformer, the following two relations hold:

- $\frac{v_1}{N_1} = \pm \frac{v_2}{N_2}$, where N_1 and N_2 are the number of turns on coil 1 and 2, respectively.
- $i_1 N_1 = \pm i_2 N_2$

The sign of the relation is determined by the dot markings. Here's the convention:

Let's study an example circuit with an ideal transformer. This circuit gives an indication of how a transformer can be used to shape impedances.