Lecture 23
Mutual Induction and Transformers

Nov. 28, 2011

Material from Textbook by Alexander & Sadiku and Electrical Engineering: Principles & Applications, A. R. Hambley is used in lecture slides.

Magnetically Coupled Circuit
Chapter 13 in A & S

- Mutual Inductance
- Energy in a Coupled Circuit
- What is a transformer?
- Linear Transformers
- Ideal Transformers
- Applications

Inductance and Mutual Inductance

Definition of inductance L:

$$L = \frac{Flux\,linkages}{current} = \frac{\lambda}{i}$$

Substitute for the flux linkages using $\lambda = N\phi$

$$L = \frac{N\phi}{i}$$

Inductance and Mutual Inductance

Substituting $\phi = \dfrac{Ni}{\mathcal{R}}$ $L = \dfrac{N^2}{\mathcal{R}}$

Faraday's Law $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d}{dt}\oiint \vec{B}\cdot d\vec{S}$

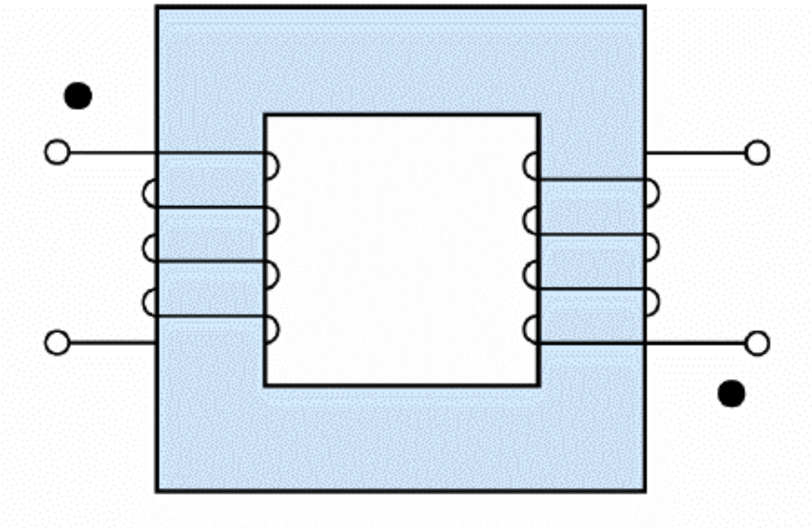
Voltage is induced in a coil when its flux linkages change:

$$e = \frac{d\lambda}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt}$$

Mutual Inductance

Self inductance for coil 1

$$L_1 = \frac{\lambda_{11}}{i_1}$$



Self inductance for coil 2

$$L_2 = \frac{\lambda_{22}}{i_2}$$

Mutual inductance between coils 1 and 2:

$$M = \frac{\lambda_{21}}{i_1} = \frac{\lambda_{12}}{i_2}$$

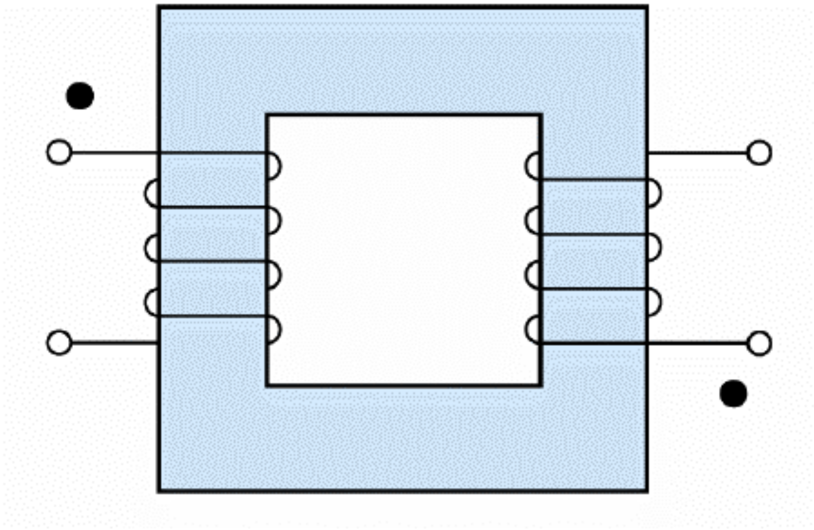
Mutual Inductance

Total fluxes linking the coils:

$$\lambda_1 = \lambda_{11} \pm \lambda_{12}$$

$$\lambda_2 = \lambda_{22} \pm \lambda_{21}$$

Mutual Inductance

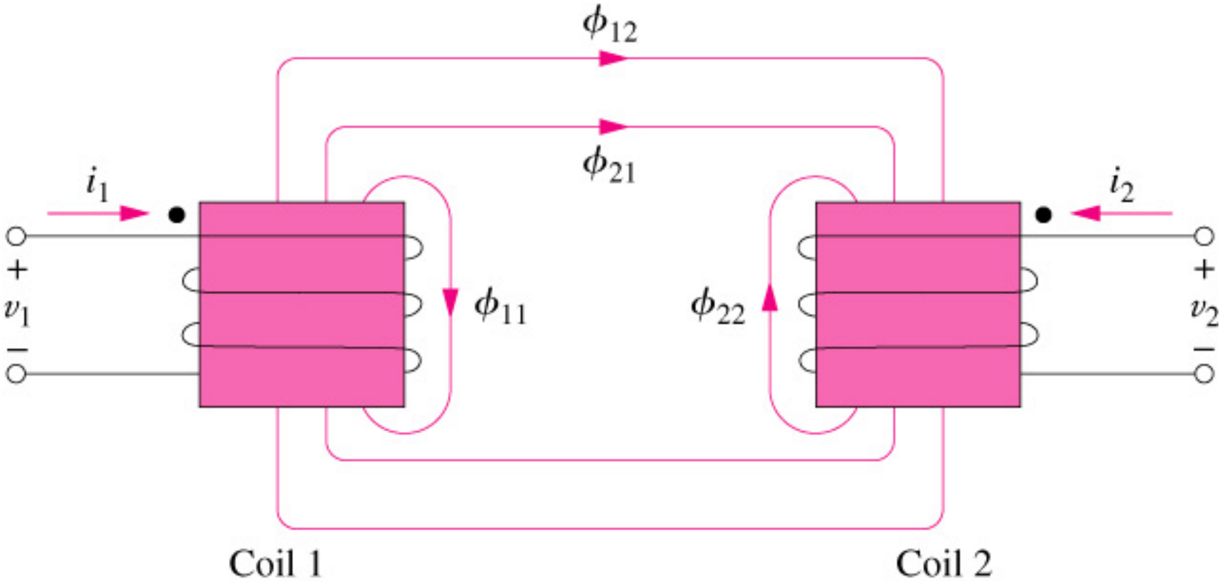


Currents entering the dotted terminals produce aiding fluxes

Check the right hand rule!

Mutual Inductance: Dot Convention

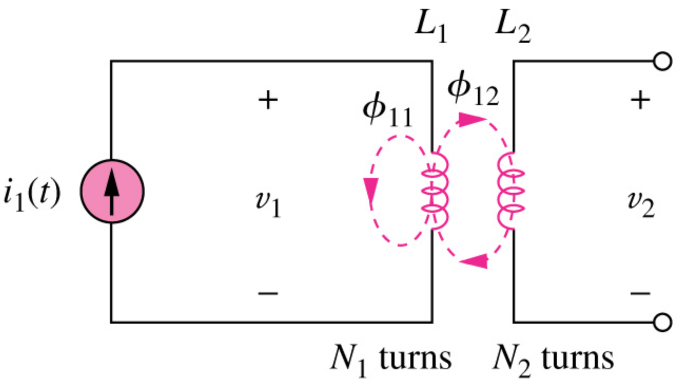
- If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

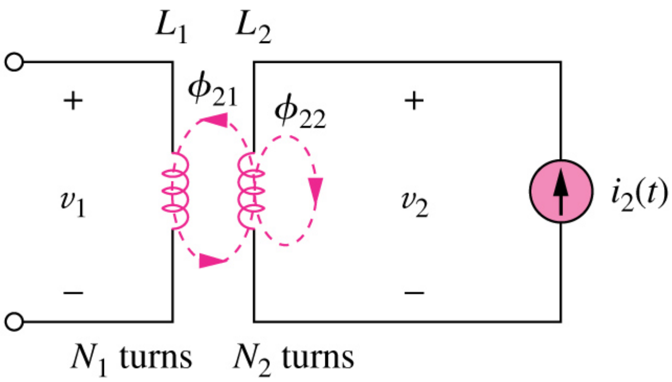


Check the right hand rule again!

Mutual Inductance

- It is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).





$$v_2 = M_{21} \frac{di_1}{dt}$$

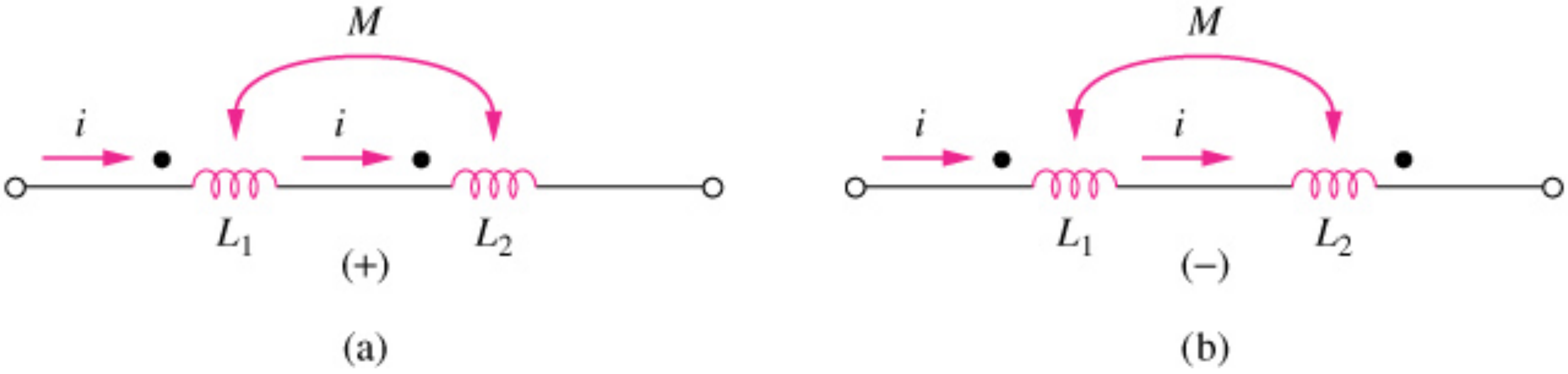
$$v_1 = M_{12} \frac{di_2}{dt}$$

The open-circuit mutual voltage across coil 2

The open-circuit mutual voltage across coil 1

Mutual Inductance

Dot convention for coils in series; the sign indicates the polarity of the mutual voltage; (a) series-aiding connection, (b) series-opposing connection.

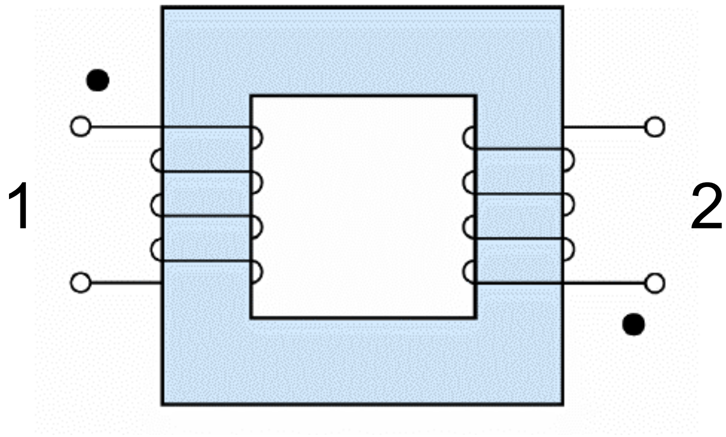


$$L = L_1 + L_2 + 2M$$

(series - aiding connection)

$$L = L_1 + L_2 - 2M$$

(series - opposing connection)



Circuit Equations for Mutual Inductance

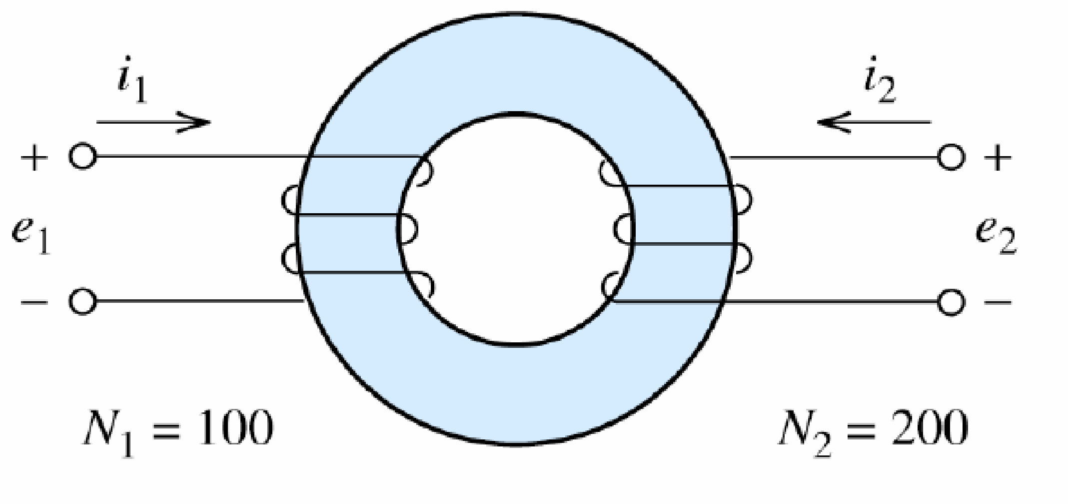
$$\lambda_1 = L_1 i_1 \pm M i_2$$

$$\lambda_2 = \pm M i_1 + L_2 i_2$$

$$e_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$e_2 = \frac{d\lambda_2}{dt} = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

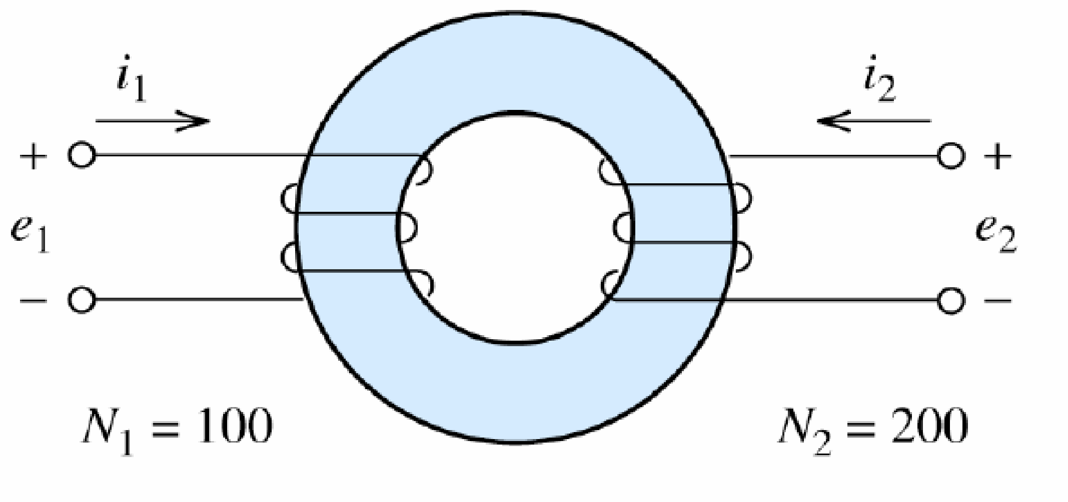
Example



$$\phi_1 = \frac{N_1 i_1}{\mathcal{R}} = \frac{100 i_1}{10^7} = 10^{-5} i_1 \qquad \lambda_{21} = N_2 \phi_1 = 200 \times 10^{-5} i_1$$

Mutual inductance: $M = \frac{\lambda_{21}}{i_1} = 2mH$

Example



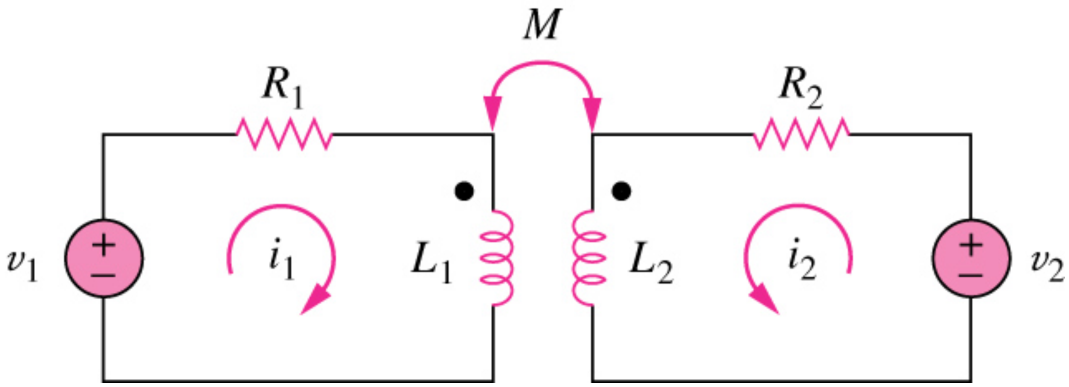
Does the flux produced by i_2 aid or oppose the flux produced by i_1?

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

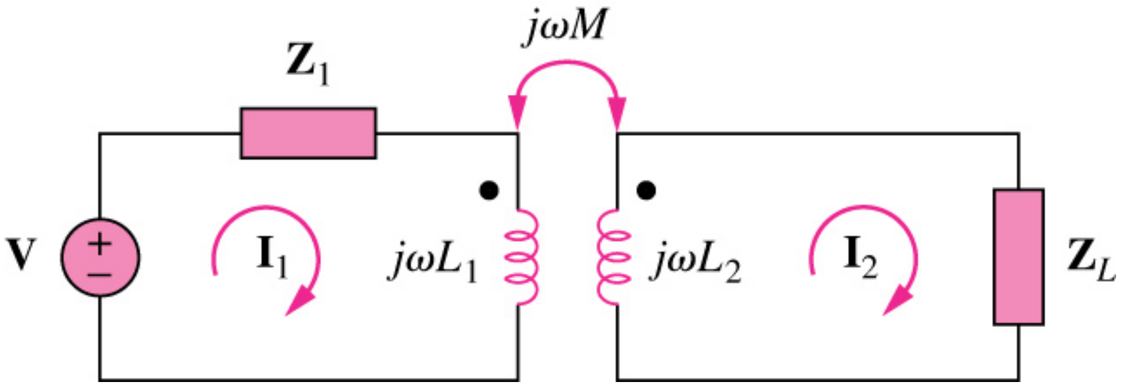
$$e_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

Right Hand Rule!

Mutual Inductance



Time-domain analysis of a circuit containing coupled coils.

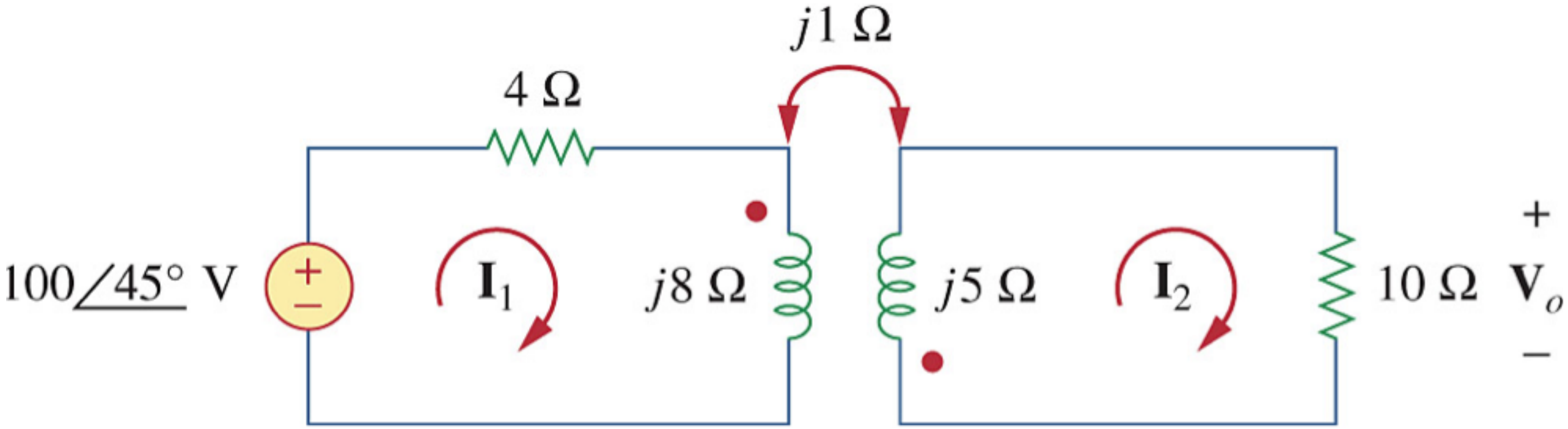


Frequency-domain analysis of a circuit containing coupled coils

Mutual Inductance

Example

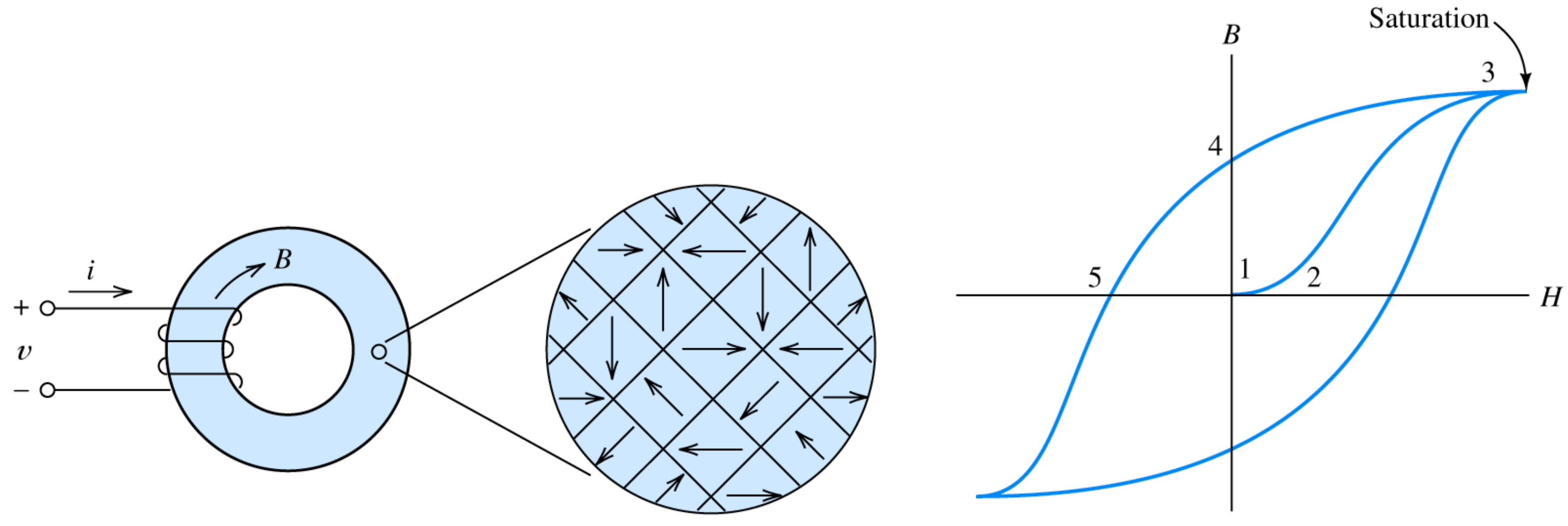
Calculate the phasor voltage V_o



Ans:

$$V_o = 10\angle -135^\circ \text{ V}$$

Magnetic Materials



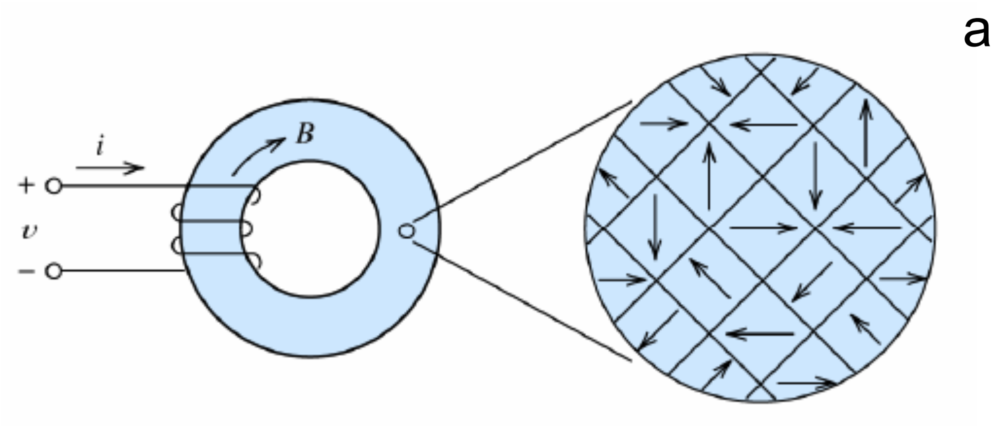
(a) Sample and coil for applying *H*

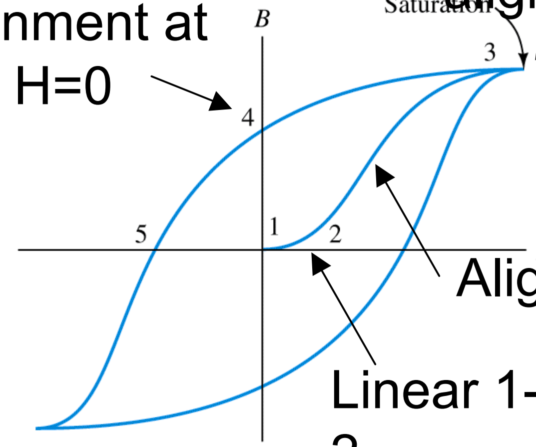
(b) Magnetic domains

(c) Hysteresis loop in the *B* – *H* plane

Figure 15.18 Materials such as iron display a $B\text{–}H$ relationship with hysteresis and saturation.

Magnetic Materials





Residual alignment at H=0

Total alignment

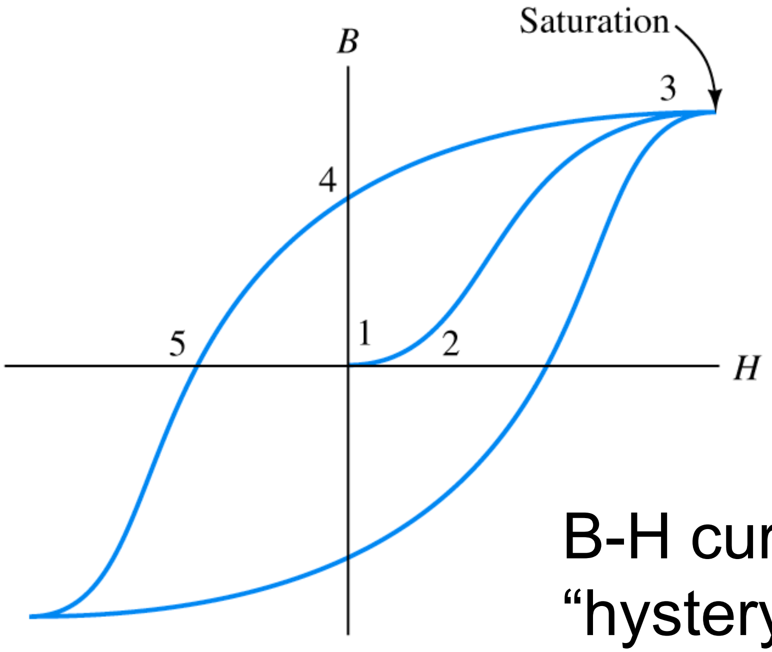
Alignment 2-3

Linear 1-2

- Magnetic field of atoms within small domains are aligned
- Magnetic fields of the small domains are initially randomly oriented
- As the magnetic field intensity increases, the domains tend to align, leaving a residual alignment even when the applied field is reduced to zero

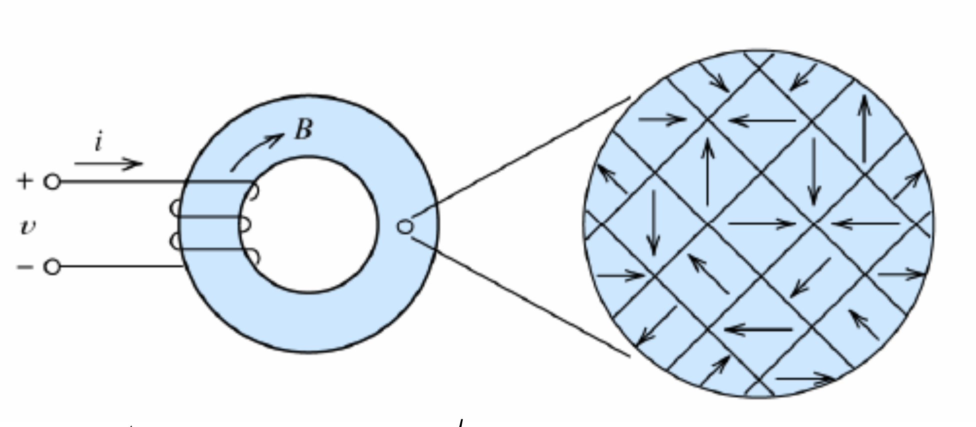
Magnetic Materials

The relationship between *B* and *H* is not linear for the types of iron used in motors and transformers.



B-H curves exhibits "hysterysis"

Energy Considerations



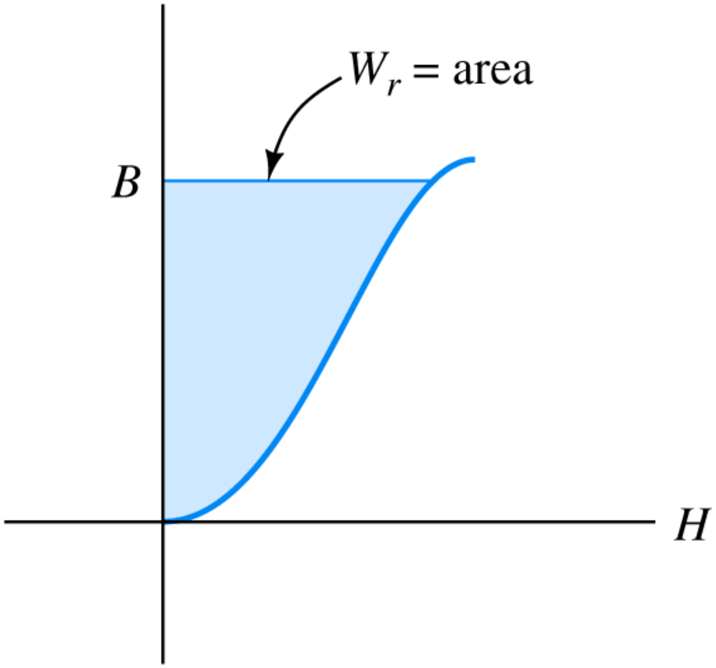
$$W = \int_0^t vi\,dt = \int_0^t N\frac{d\phi}{dt}i\,dt = \int_0^\phi Ni\,d\phi$$

$$Ni = Hl \ and \ d\phi = AdB$$

$$W = \int_0^B AlH\,dB = V_{core}\int_0^B H\,dB$$

$$\frac{W}{V} = W_V = \int_0^B H\,dB$$

Energy Considerations



$$W_v = \frac{W}{Al} = \int_0^B H\,dB$$

The area between the B-H curve and the B axis represents the volumetric energy supplied to the core

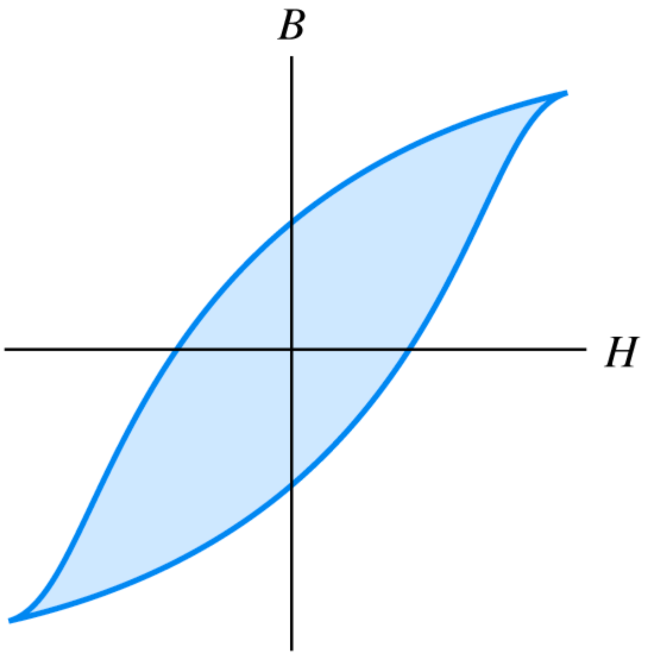
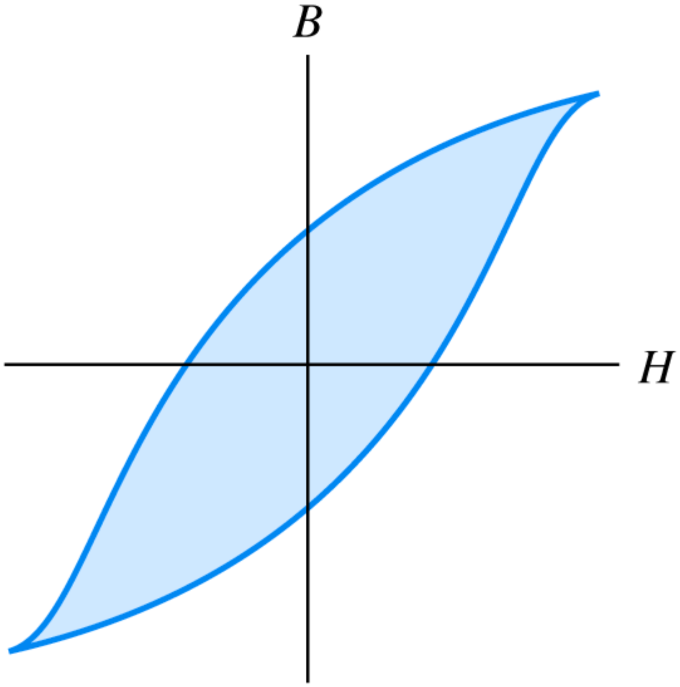


Figure 15.20 The area of the hysteresis loop is the volumetric energy converted to heat per cycle.

Core Loss

Power loss due to hysteresis is proportional to frequency, assuming constant peak flux.

400 Hz vs 60 Hz transformers



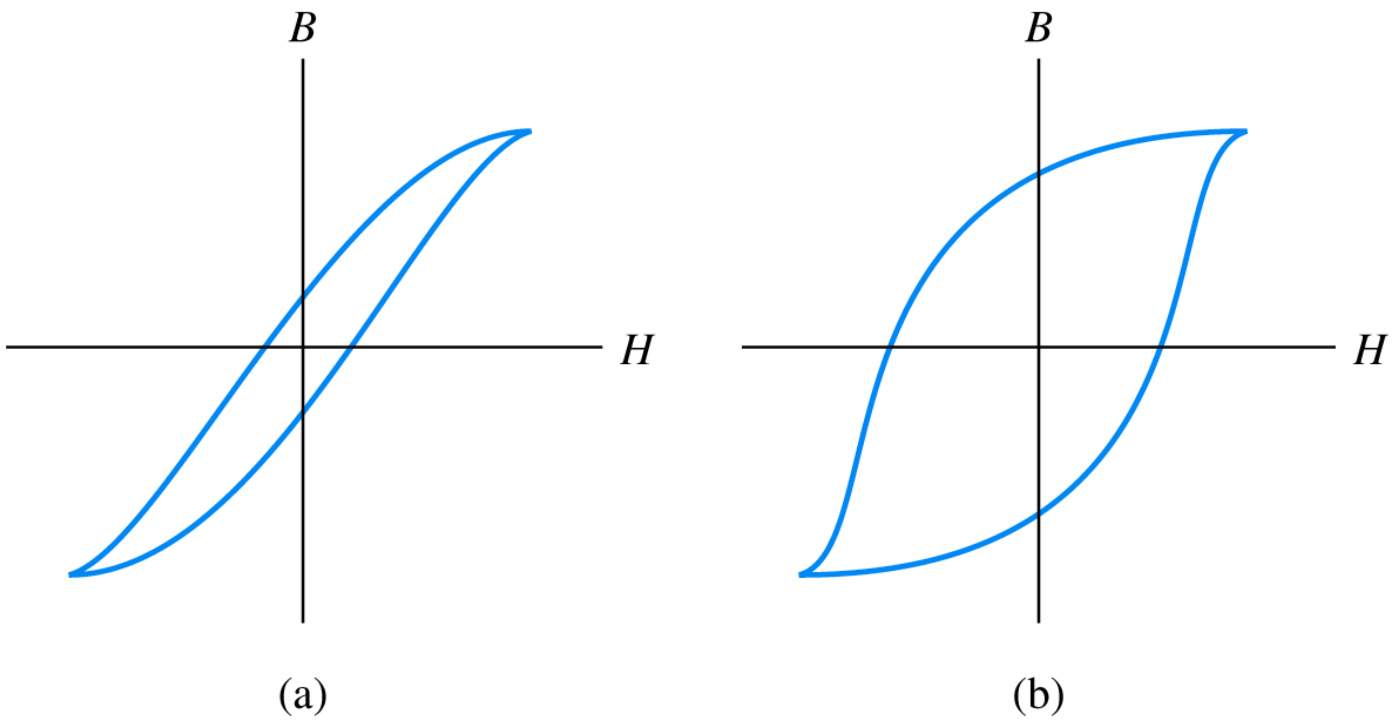


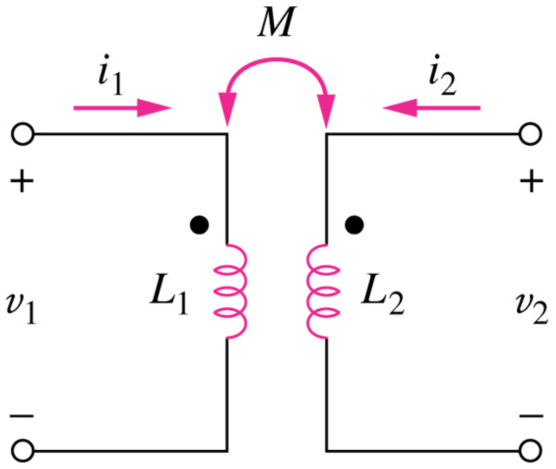
Figure 15.21 When we want to minimize core loss (as in a transformer or motor), we choose a material having a thin hysteresis loop. On the other hand, for a permanent magnet, we should choose a material with a wide loop.

Energy Stored in the Magnetic Field

$$W_v = \int_0^B \frac{B}{\mu} dB = \frac{B^2}{2\mu}$$

Energy in a Coupled Circuit

- The coupling coefficient, k, is a measure of the magnetic coupling between two coils; 0≤k≤1.



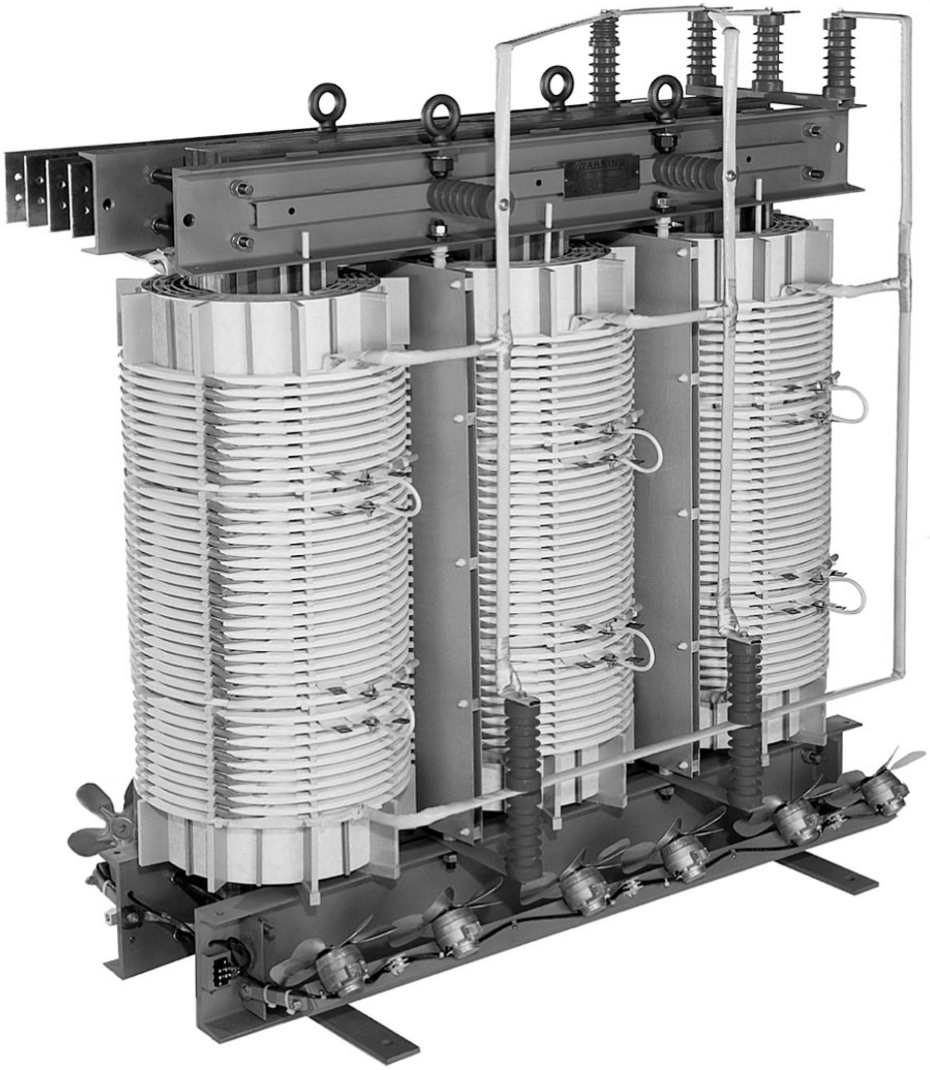
$$\boxed{M = k\sqrt{L_1 L_2}}$$

- The instantaneous energy stored in the circuit is given by

$$\boxed{w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M I_1 I_2}$$

What is a transformer?

- It is an electrical device designed on the basis of the concept of magnetic coupling
- It uses magnetically coupled coils to transfer energy from one circuit to another
- It is the key circuit elements for stepping up or stepping down ac voltages or currents, impedance matching, isolation, etc.



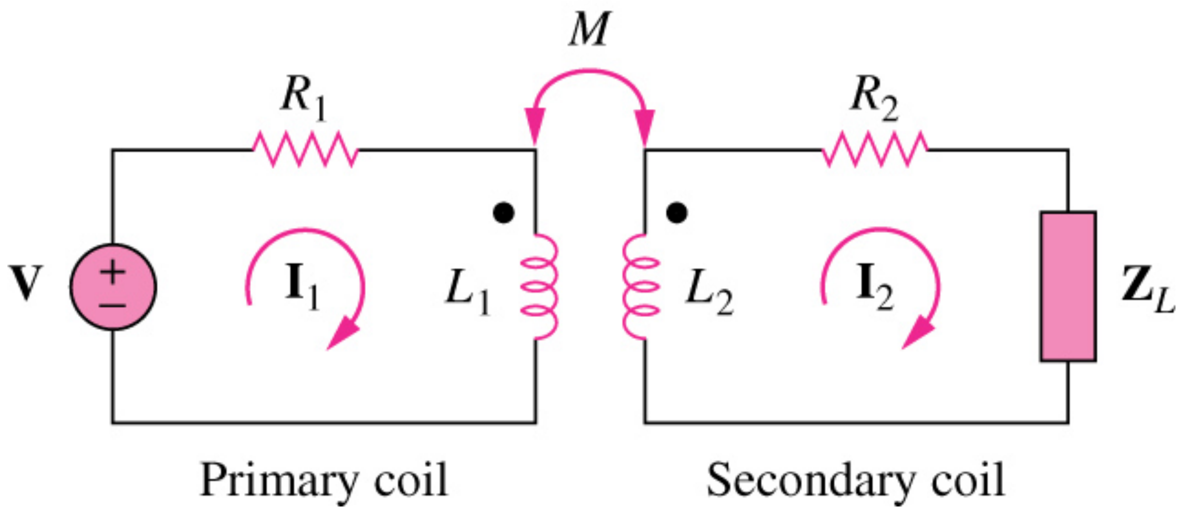
(a)


JT-11SSP-6M
JT-69-HMX
JT-123-SLPC
JT-11-GMPC
JT-DB-E
JT-6110K-B
JT-112-LPC
JT-115K-MAX
JT-10KB-DPC
JT-110K-HPC
JT-11-FLPCH
JT-11-EL
JT-16-B
JT-11-DM
JT-123-BL
JT-11-FL

(b)

Linear Transformer

- It is generally a four-terminal device comprising two (or more) magnetically coupled coils

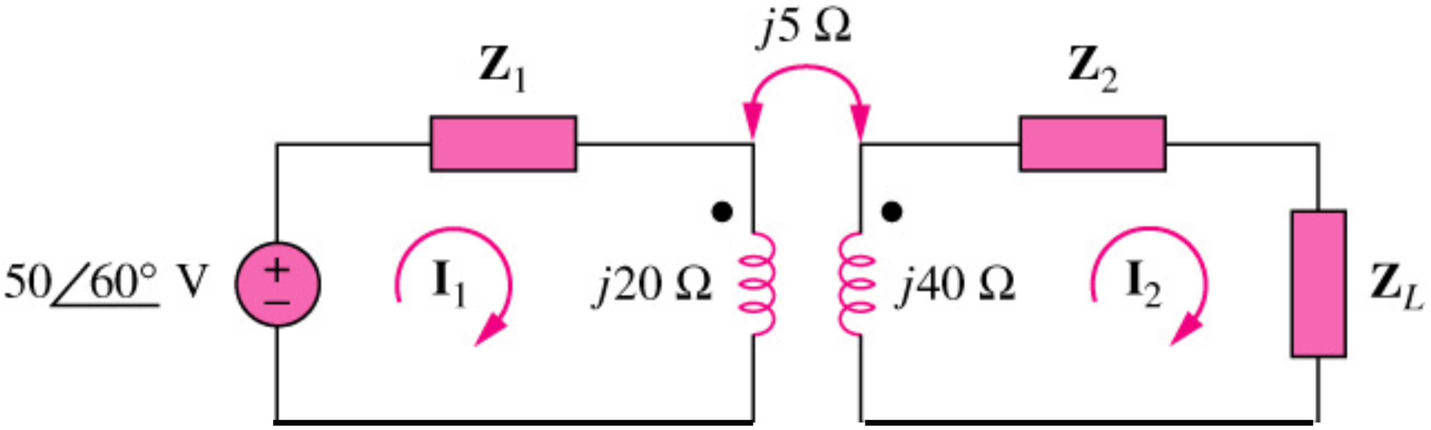


$$Z_{in} = \frac{V}{I_1} = R_1 + j\omega L_1 + Z_R \ , \ Z_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L} \text{ is } \textit{reflected} \quad \textit{impedance}$$

Linear Transformer

Example

In the circuit below, calculate the input impedance and current I_1. Take Z_1=60-*j*100Ω, Z_2=30+*j*40Ω, & Z_L=80+j60Ω.



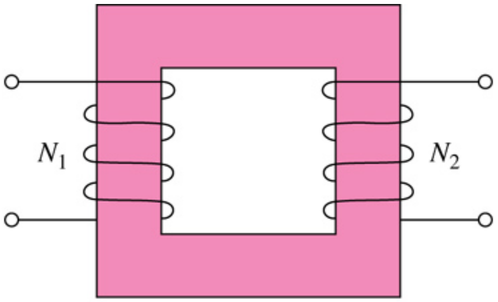
$$\vec{Z}_{in} = \vec{Z}_1 + j20 + \frac{(M^2 = 5^2)}{(j40 + \vec{Z}_2 + \vec{Z}_L)}$$

Ans: $\mathrm{Z_{in}} = 100.14\angle -53.1°\Omega;\ \mathrm{I_1} = 0.5\angle 113.1°\mathrm{A}$

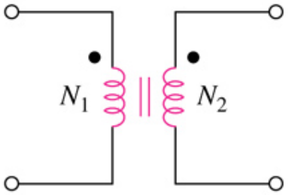
*Worked out as Example 13.4 in textbook

Ideal Transformer

- An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.

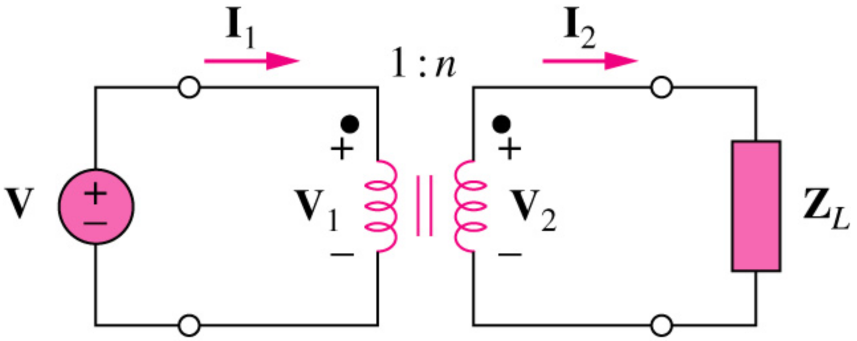


(a)



(b)

(a) Ideal Transformer
(b) Circuit symbol



$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

V2>V1→ step-up transformer
V2<V1→ step-down transformer

Ideal Transformer

Example

An ideal transformer is rated at 2400/120V, 9.6 kVA, and has 50 turns on the secondary side.

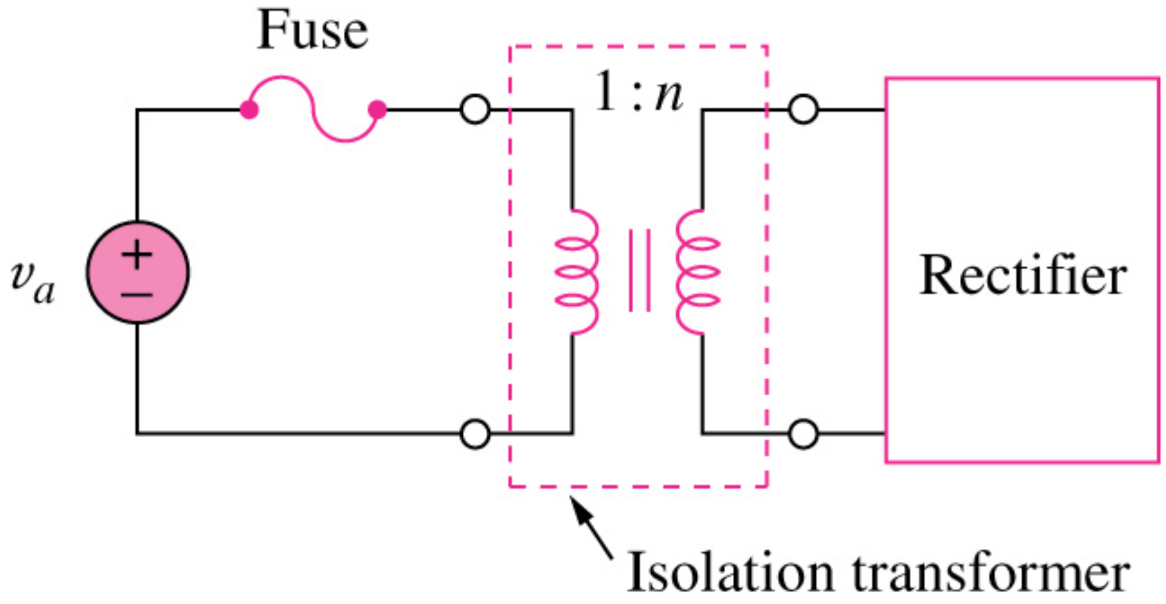
Calculate:

(a) the turns ratio,
(b) the number of turns on the primary side, and
(c) the current ratings for the primary and secondary windings.

Ans:
(a) This is a step-down transformer, n=0.05
(b) N_1 = 1000 turns
(c) I_1 = 4A and I_2 = 80A -- Maximum

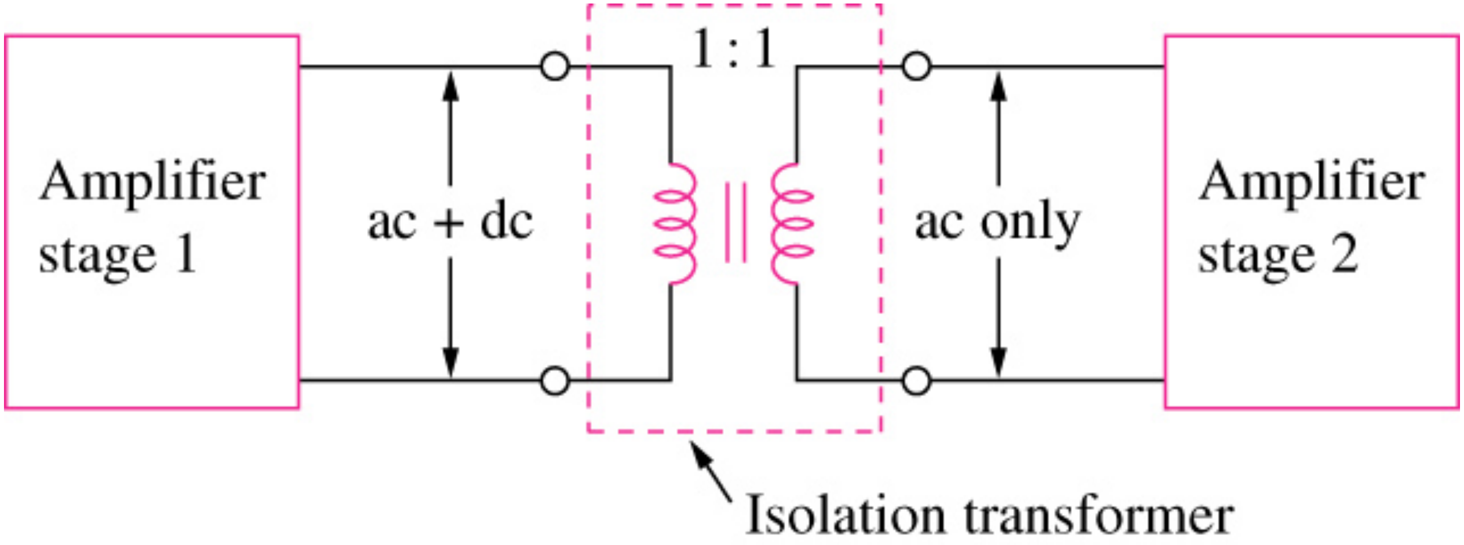
Applications

- Transformer as an Isolation Device to isolate ac supply from a rectifier



Applications

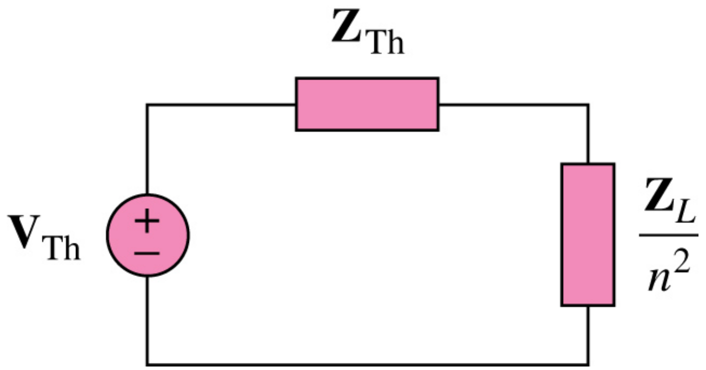
- Transformer as an Isolation Device to isolate dc between two amplifier stages.



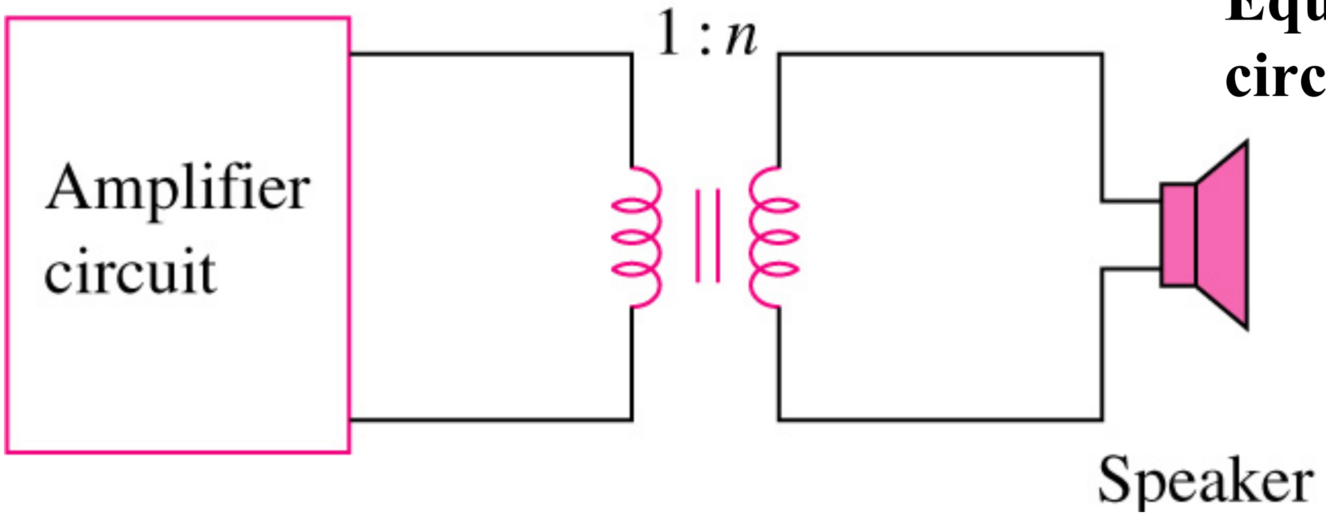
Applications

- Transformer as a Matching Device

Using an ideal transformer to match the speaker to the amplifier



Equivalent circuit



Applications

Example

Calculate the turns ratio of an ideal transformer required to match a 100Ω load to a source with internal impedance of 2.5kΩ. Find the load voltage when the source voltage is 30V.

Ans: $n = 0.2$; $V_L = 3V$

Applications

- A typical power distribution system