Self and mutual inductance

Dr. Gamal Sowilam

OBJECTIVES

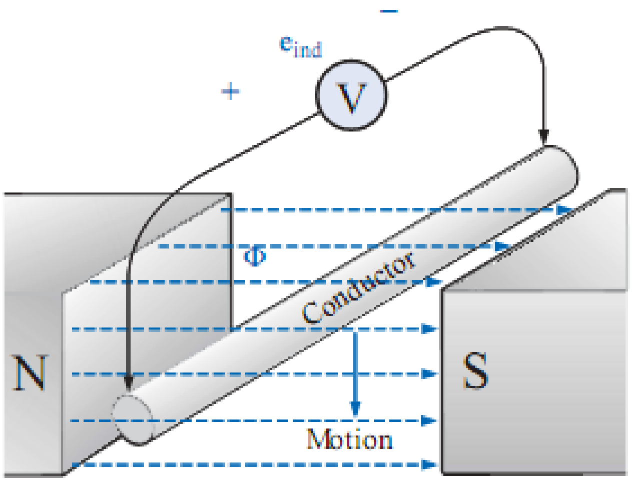
- Become familiar with the flux linkages that exist between the coils of a transformer and how the voltages across the primary and secondary are established.
- Understand the operation of an iron-core and air-core transformer and how to calculate the currents and voltages of the primary and secondary circuits.
- Be aware of how the transformer is used for impedance matching purposes to ensure a high level of power transfer.
- Become aware of all the components that make up the equivalent circuit of a transformer and how they affect its performance and frequency response.
- Understand how to use and interpret the dot convention of mutually coupled coils in a network.

INTRODUCTION

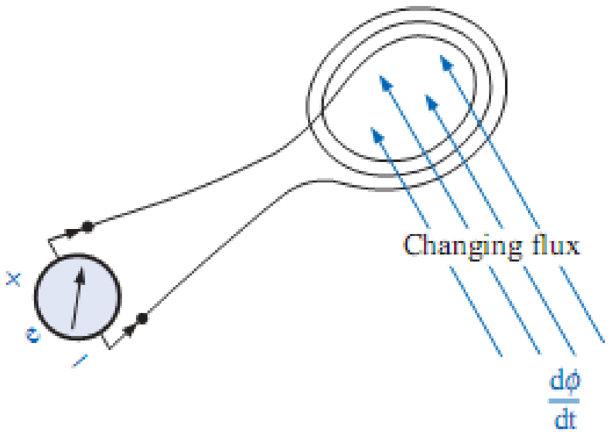
- Mutual inductance is a phenomenon basic to the operation of the *transformer,* an electrical device used today in almost every field of electrical engineering.
- This device plays an integral part in power distribution systems and can be found in many electronic circuits and measuring instruments.
- In this chapter, we discuss three of the basic applications of a transformer: to build up or step down the voltage or current, to act as an impedance matching device, and to isolate (no physical connection) one portion of a circuit from another.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Generating an induced voltage by moving a conductor through a magnetic field.



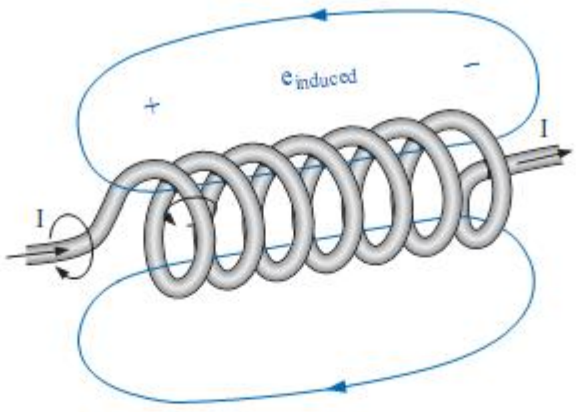
If a coil of N turns is placed in the region of a changing flux, as in Fig. 12.2, a voltage will be induced across the coil as determined by Faradays law:



$$e = N\frac{d\phi}{dt} \quad \text{(volts, V)}$$

LENZ'S LAW

If the current increases in magnitude, the flux linking the coil also increase. However, that a changing flux linking a coil induces a voltage across the coil.



For this coil, therefore, an induced voltage is developed across the coil due to the change in current through the coil. The polarity of this induced voltage tends to establish a current in the coil that produces a flux that will oppose any change in the original flux. In other words, the induced effect (e_{ind}) is a result of the increasing current through the coil.

Lenz law state that: an induced effect is always such as to oppose the cause that produced it.

SELF-INDUCTANCE

The ability of a coil to oppose any change in current is a measure of the self-inductance L of the coil.

Inductance is measured in henries (H)

Inductors are coils of various dimensions designed to introduce specified amounts of inductance into a circuit. The inductance of a coil varies directly with the magnetic properties of the coil. Ferromagnetic materials, therefore, are frequently employed to increase the inductance by increasing the flux linking the coil.

$$L = \frac{\mu N^2 A}{l}$$

μ = permeability (Wb/A · m)
N = number of turns (t)
A = m^2
l = m
L = henries (H)

Inductor Construction

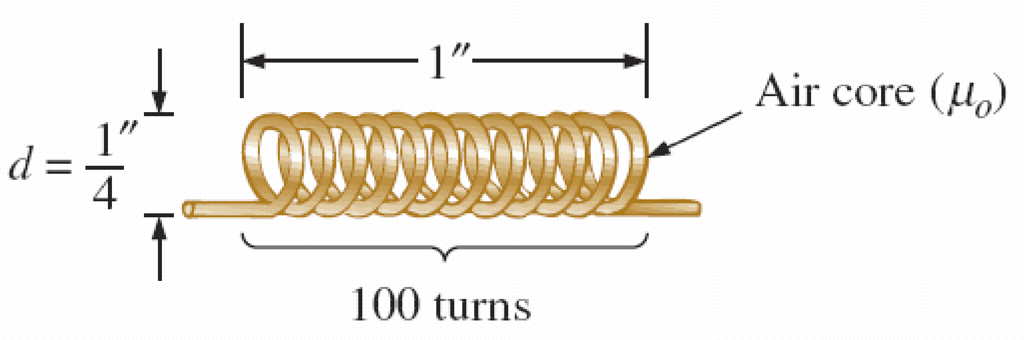


FIG. 11.18 *Air-core coil for Example 11.1.*

Types of Inductors

- Inductors, like capacitors and resistors, can be categorized under the general headings **fixed** or **variable.**
- The symbol for a fixed air-core inductor is provided in Fig. 11.20(a), for an inductor with a ferromagnetic core in Fig. 11.20(b), for a tapped coil in Fig. 11.20(c), and for a variable inductor in Fig. 11.20(d).

Types of Inductors

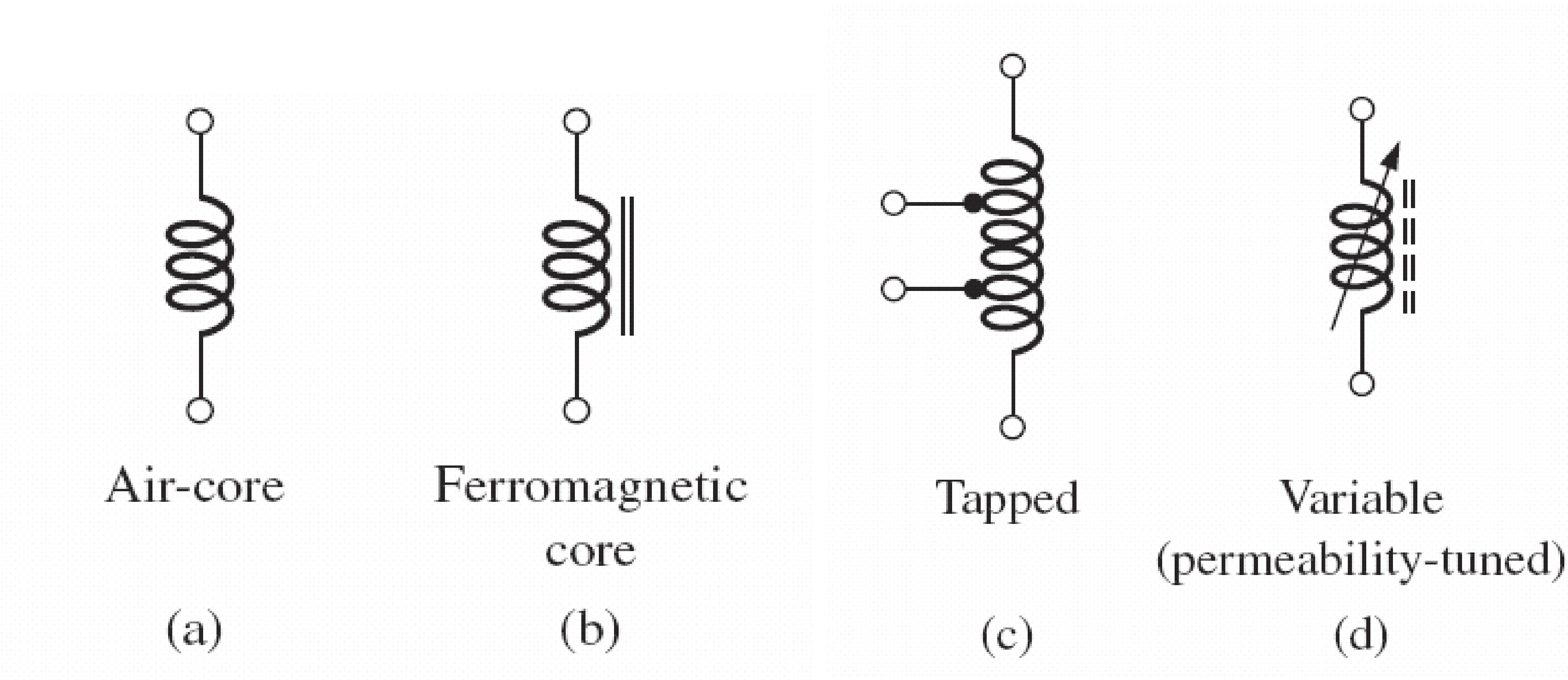


FIG. 11.20 *Inductor (coil) symbols.*

- **Fixed**
  - Fixed-type inductors come in all shapes and sizes.
  - However, ***in general, the size of an inductor is determined primarily by the type of construction, the core used, and the current rating.***

Write an expression for inductance using the current and construction of coil.

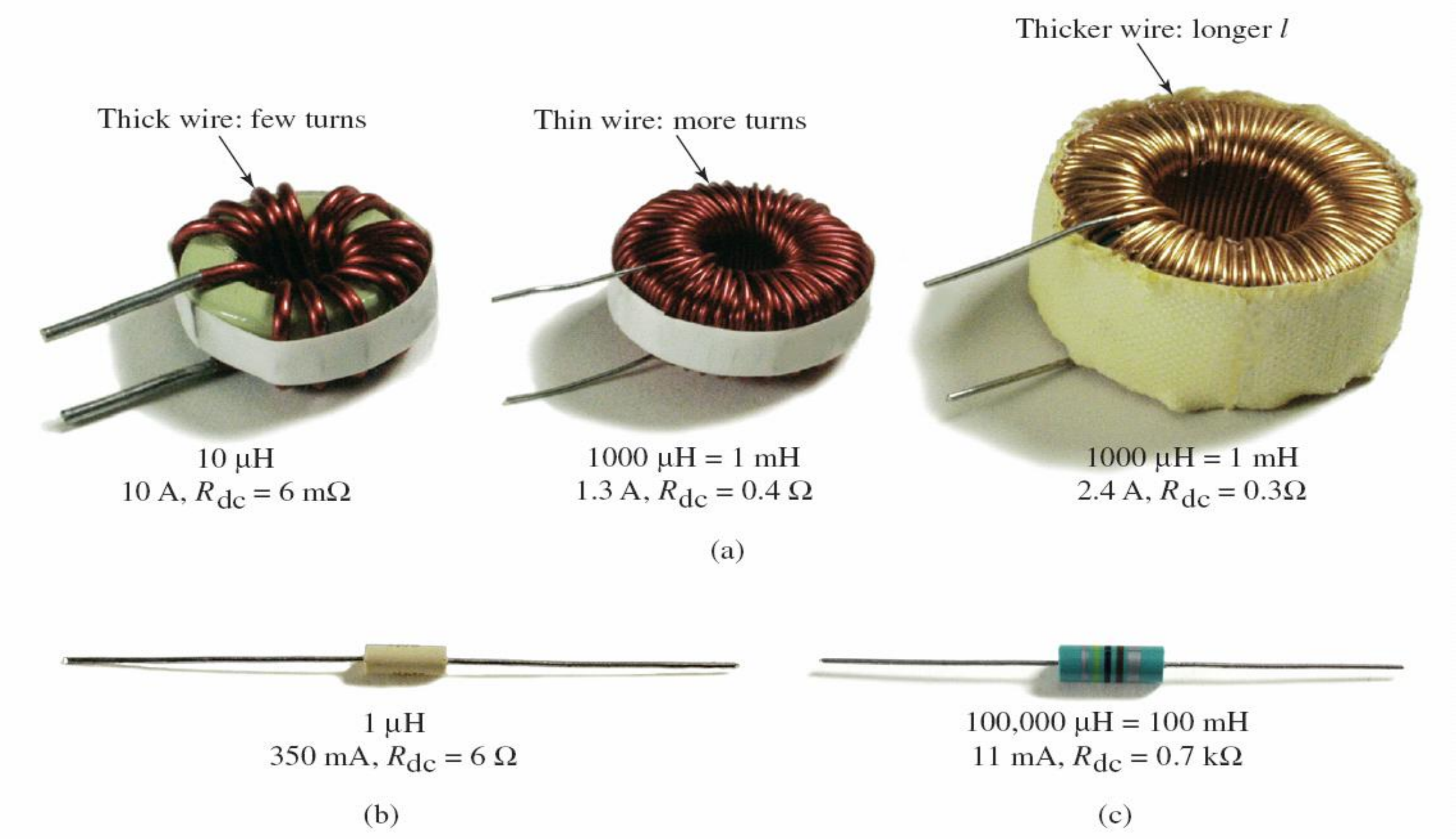
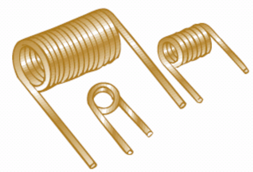


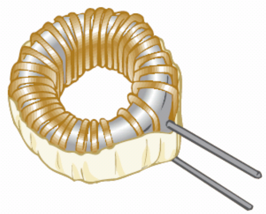
FIG. 11.21 *Relative sizes of different types of inductors: (a) toroid, high-current; (b) phenolic (resin or plastic core); (c) ferrite core.*

Types of Inductors

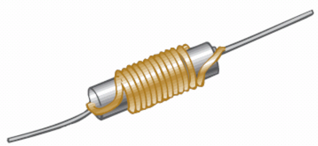
Type: Air-core inductors (1–32 turns)
Typical values: 2.5 nH–1 μH
Applications: High-frequency applications



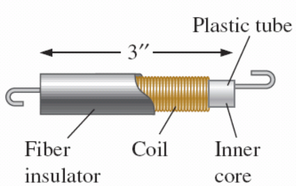
Type: Toroid coil
Typical values: 10 μH–30 mH
Applications: Used as a choke in ac power line circuits to filter transient and reduce EMI interference. This coil is found in many electronic appliances.



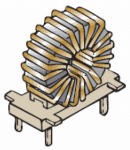
Type: Hash choke coil
Typical values: 3 μH–1 mH
Applications: Used in ac supply lines that deliver high currents.



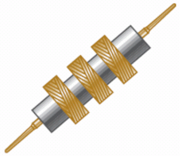
Type: Delay line coil
Typical values: 10 μH–50 μH
Applications: Used in color televisions to correct for timing differences between the color signal and the black-and-white signal.



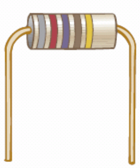
Type: Common-mode choke coil
Typical values: 0.6 mH–50 mH
Applications: Used in ac line filters, switching power supplies, battery chargers, and other electronic equipment.



Type: RF chokes
Typical values: 10 μH–470 mH
Applications: Used in radio, television, and communication circuits. Found in AM, FM, and UHF circuits.



Type: Molded coils
Typical values: 0.1 μH–100 mH
Applications: Used in a wide variety of circuits such as oscillators, filters, pass-band filters, and others.



Type: Surface-mount inductors
Typical values: 0.01 μH–250 μH
Applications: Found in many electronic circuits that require miniature components on multilayered PCBs (printed circuit boards).

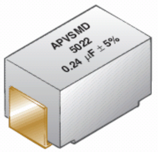


FIG. 11.22 *Typical areas of application for inductive elements.*

Types of Inductors

- **Variable**
  - A number of variable inductors are depicted in Fig. 11.23.
  - In each case, the inductance is changed by turning the slot at the end of the core to move it in and out of the unit.
  - The farther in the core is, the more the ferromagnetic material is part of the magnetic circuit, and the higher is the magnetic field strength and the inductance level.

Types of Inductors

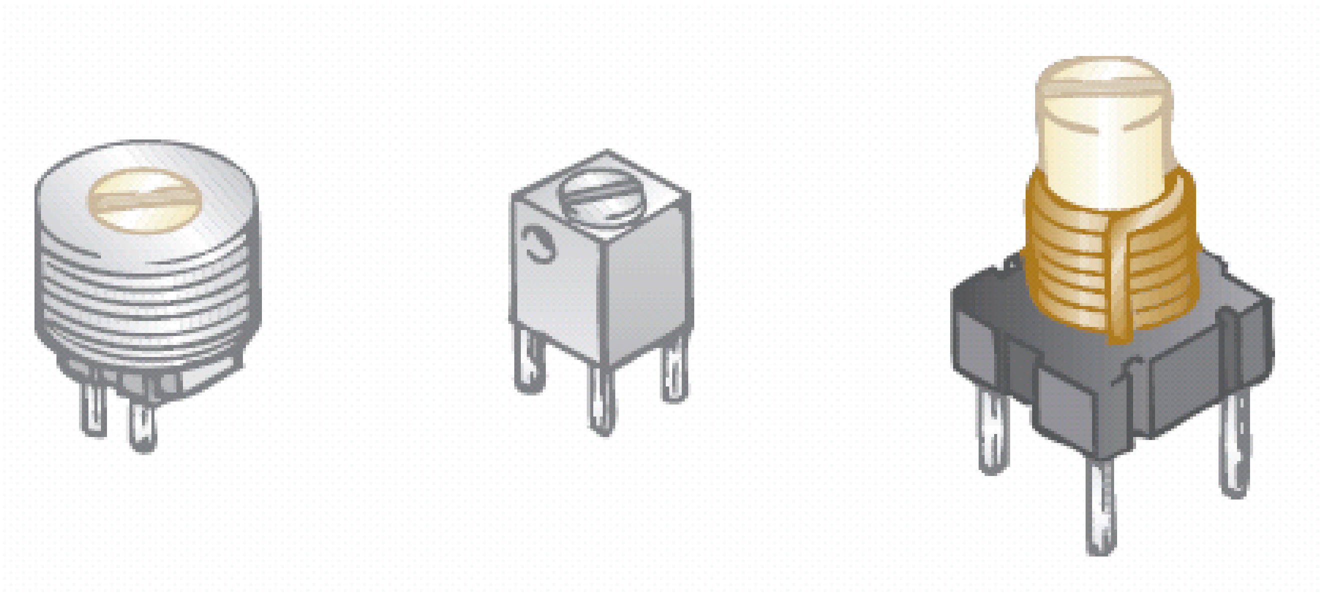


FIG. 11.23 *Variable inductors with a typical range of values from 1* m*H to 100* m*H; commonly used in oscillators and various RF circuits such as CB transceivers, televisions, and radios.*

Practical Equivalent Inductors

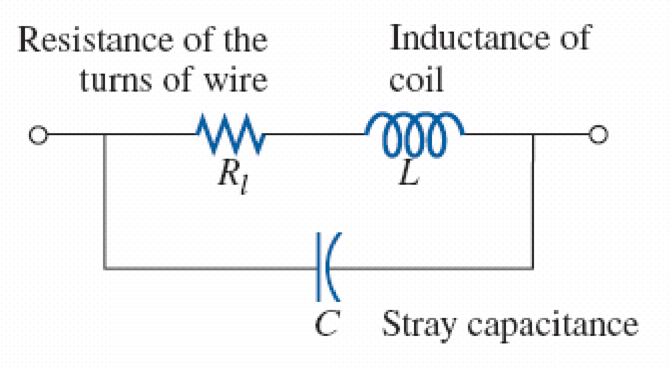


FIG. 11.24 *Complete equivalent model for an inductor.*

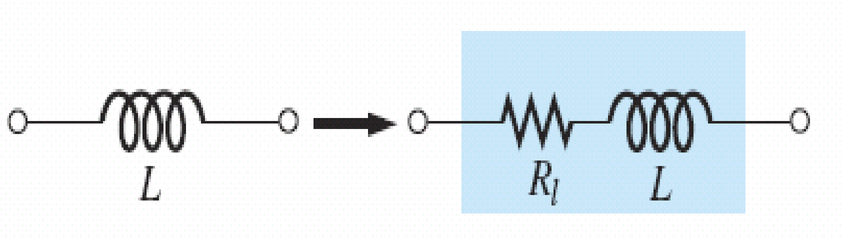


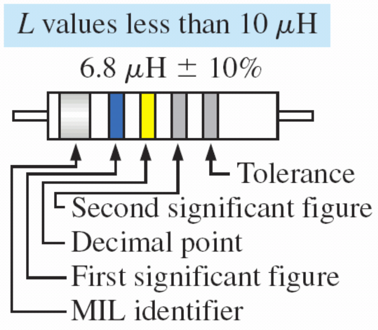
FIG. 11.25 *Practical equivalent model for an inductor.*

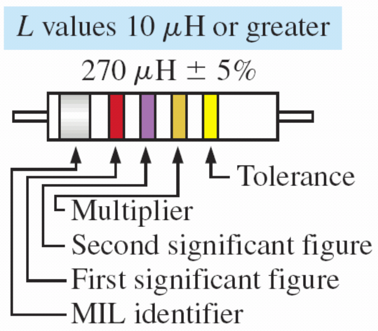
Labeling

Color Code Table

Color[1]	Significant Figure	Multiplier[2]	Inductance Tolerance (%)
Black	0	1	
Brown	1	10	
Red	2	100	
Orange	3	1000	
Yellow	4		
Green	5		
Blue	6		
Violet	7		
Gray	8		
White	9		
None[2]			±20
Silver			±10
Gold	Decimal point		±5

[1] Indicates body color.
[2] The multiplier is the factor by which the two significant figures are multiplied to yield the nominal inductance value.





Cylindrical molded choke coils are marked with five colored bands. A wide silver band, located at one end of the coil, identifies military radio-frequency coils. The next three bands indicate the inductance in microhenries, and the fourth band is the tolerance.

Color coding is in accordance with the color code table, shown on the left. If the first or second band is gold, it represents the decimal point for inductance values less than 10. Then the following two bands are significant figures. For inductance values of 10 or more, the first two bands represent significant figures, and the third is the multiplier.

FIG. 11.26 *Molded inductor color coding.*

INDUCTORS IN SERIES AND IN PARALLEL

- Inductors, like resistors and capacitors, can be placed in series or in parallel.

- Increasing levels of inductance can be obtained by placing inductors in series, while decreasing levels can be obtained by placing inductors in parallel.

INDUCTORS IN SERIES

- For inductors in series, the total inductance is found in the same manner as the total resistance of resistors in series (Fig. 11.56):

$$L_T = L_1 + L_2 + L_3 + \cdots + L_N$$

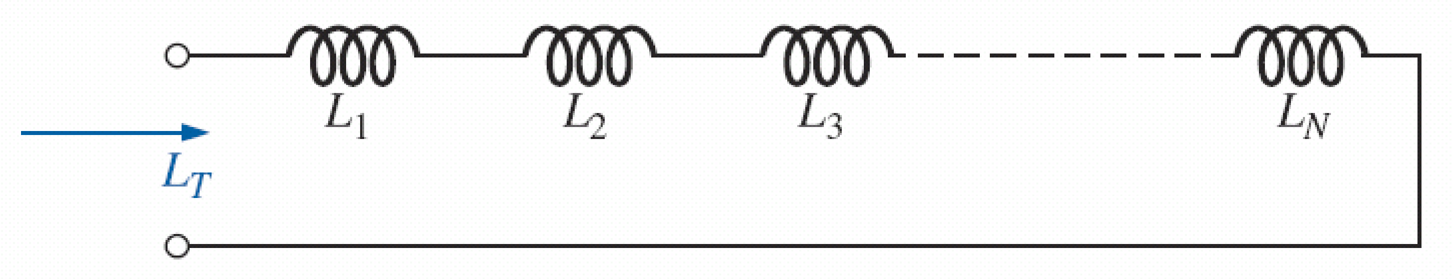


FIG. 11.56 *Inductors in series.*

INDUCTORS IN PARALLEL

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

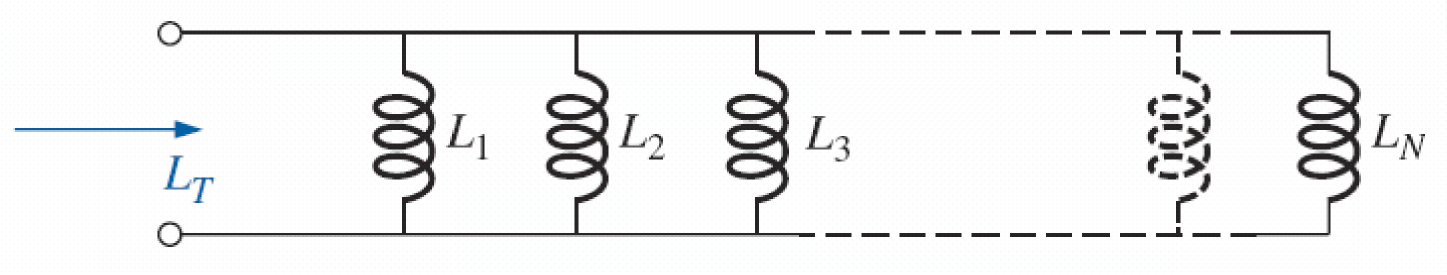


FIG. 11.57 *Inductors in parallel.*

INDUCTORS IN SERIES AND IN PARALLEL

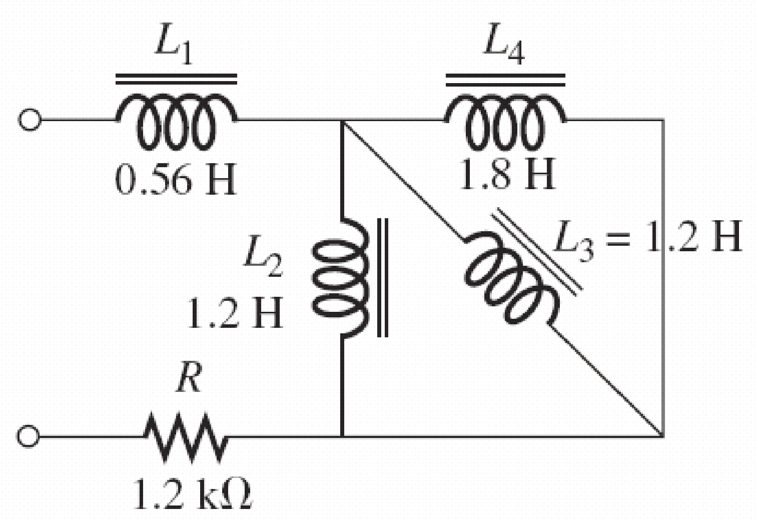


FIG. 11.58 *Example 11.9.*

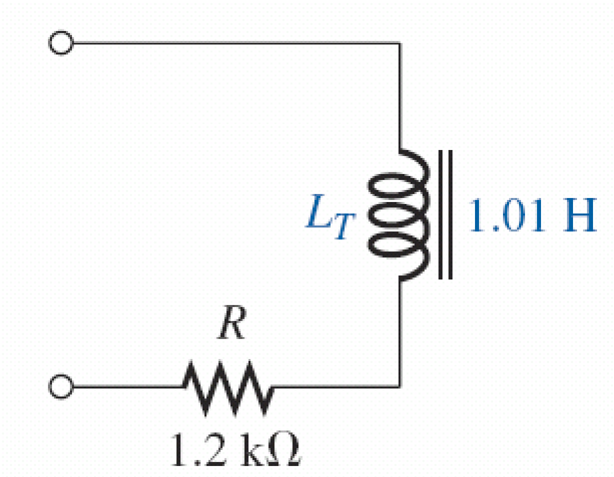


FIG. 11.59 *Terminal equivalent of the network in Fig. 11.58.*

MUTUAL INDUCTANCE

the coil to which the source is applied is called the primary, and the coil to which the load is applied is called the secondary.

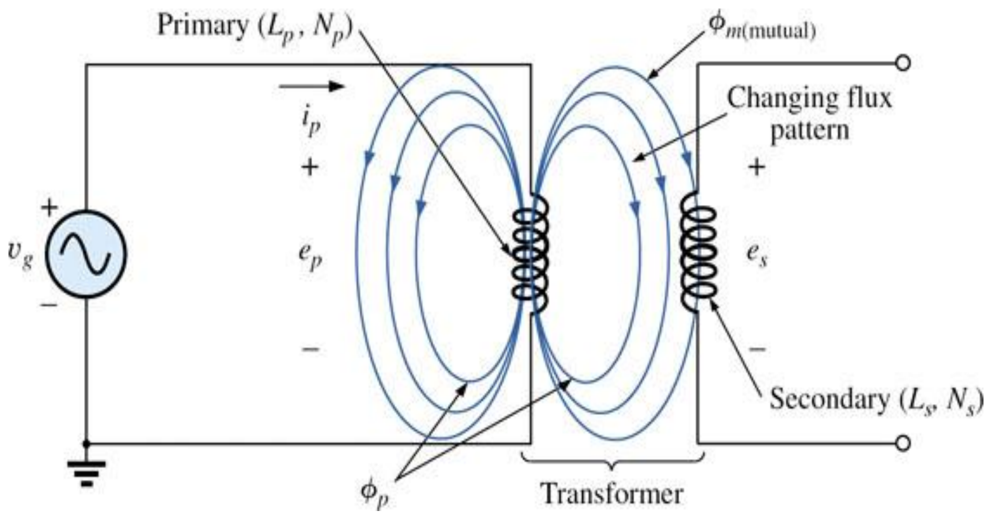


FIG. 22.1 *Defining the components of a transformer.*

The Primary voltage

$$e_p = N_p \frac{d\phi_p}{dt} \quad \text{(volts, V)}$$

$$e_p = L_p \frac{di_p}{dt} \quad \text{(volts, V)}$$

The voltage induced across the secondary, is

$$e_s = N_s \frac{d\phi_m}{dt} \quad \text{(volts, V)}$$

$$k \text{ (coefficient of coupling)} = \frac{\phi_m}{\phi_p}$$

Since the maximum level of ϕ_m is ϕ_p, the coefficient of coupling between two coils can never be greater than 1.

MUTUAL INDUCTANCE

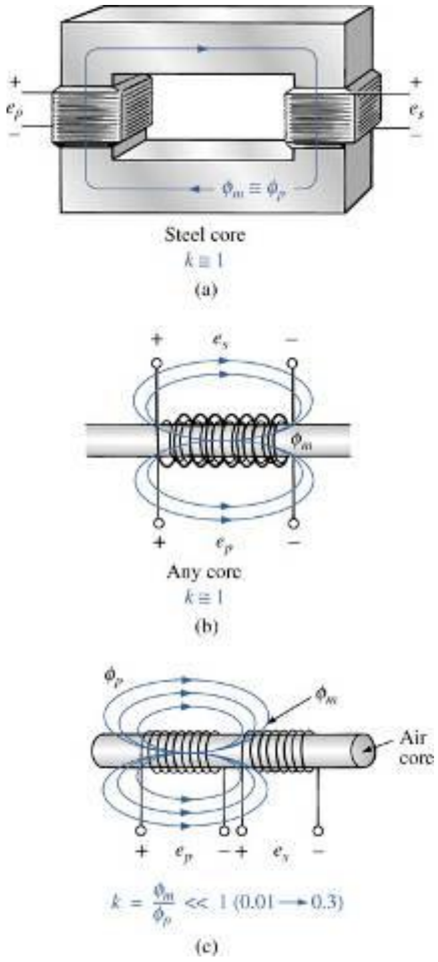


FIG. 22.2 *Windings having different coefficients of coupling.*

it will never approach a level of 1. Those coils with low coefficients of coupling are said to be loosely coupled. For the secondary, we have

$$e_s = N_s \frac{d\phi_m}{dt} = N_s \frac{dk\phi_p}{dt}$$

$$e_s = kN_s \frac{d\phi_p}{dt} \quad \text{(volts, V)}$$

The mutual inductance between the two coils is determined by

$$M = N_s \frac{d\phi_m}{di_p} \quad \text{(henries, H)}$$

$$M = N_p \frac{d\phi_p}{di_s} \quad \text{(henries, H)}$$

mutual inductance between two coils is proportional to the instantaneous change in flux linking one coil due to an instantaneous change in current through the other coil.

In terms of the inductance of each coil and the coefficient of coupling, the mutual inductance is determined by

$$M = k\sqrt{L_p L_s} \quad \text{(henries, H)}$$

The secondary voltage e_s can also be found in terms of the mutual inductance if we rewrite this equation as:

$$e_s = N_s \frac{d\phi_m}{dt} \quad \text{(volts, V)}$$

$$e_s = N_s \left(\frac{d\phi_m}{di_p}\right)\left(\frac{di_p}{dt}\right)$$

and, since $M = N_s(d\phi_m/di_p)$, it can also be written

$$e_s = M \frac{di_p}{dt} \quad \text{(volts, V)}$$

Similarly,

$$e_p = M \frac{di_s}{dt} \quad \text{(volts, V)}$$

EXAMPLE 21.1 For the transformer in Fig. 21.3

a. Find the mutual inductance M.
b. Find the induced voltage e_p if the flux ϕ_p changes at the rate of 450 mWb/s.
c. Find the induced voltage e_s for the same rate of change indicated in part (b).
d. Find the induced voltages e_p and e_s if the current i_p changes at the rate of 0.2 A/ms.

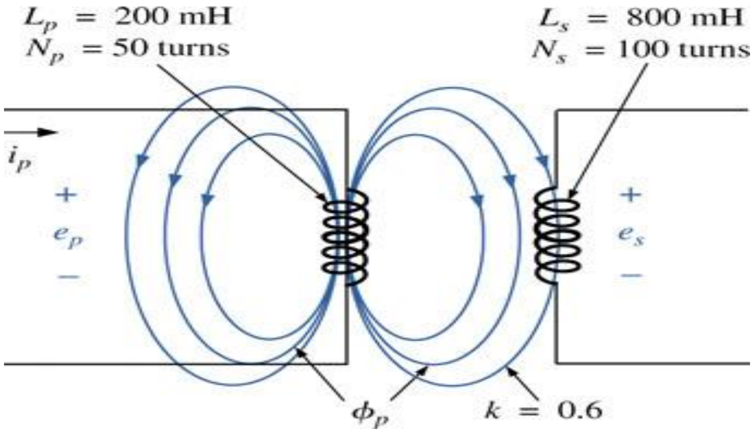


FIG. 22.3 *Example 22.1*

Solutions:

a. $M = k\sqrt{L_p L_s} = 0.6\sqrt{(200\text{ mH})(800\text{ mH})}$
$= 0.6\sqrt{16 \times 10^{-2}} = (0.6)(400 \times 10^{-3}) = \mathbf{240\text{ mH}}$

b. $e_p = N_p \dfrac{d\phi_p}{dt} = (50)(450\text{ mWb/s}) = \mathbf{22.5\text{ V}}$

c. $e_s = kN_s \dfrac{d\phi_p}{dt} = (0.6)(100)(450\text{ mWb/s}) = \mathbf{27\text{ V}}$

d. $e_p = L_p \dfrac{di_p}{dt} = (200\text{ mH})(0.2\text{ A/ms}) = (200\text{ mH})(200\text{ A/s}) = \mathbf{40\text{ V}}$

$e_s = M \dfrac{di_p}{dt} = (240\text{ mH})(200\text{ A/s}) = \mathbf{48\text{ V}}$

THE IRON-CORE TRANSFORMER

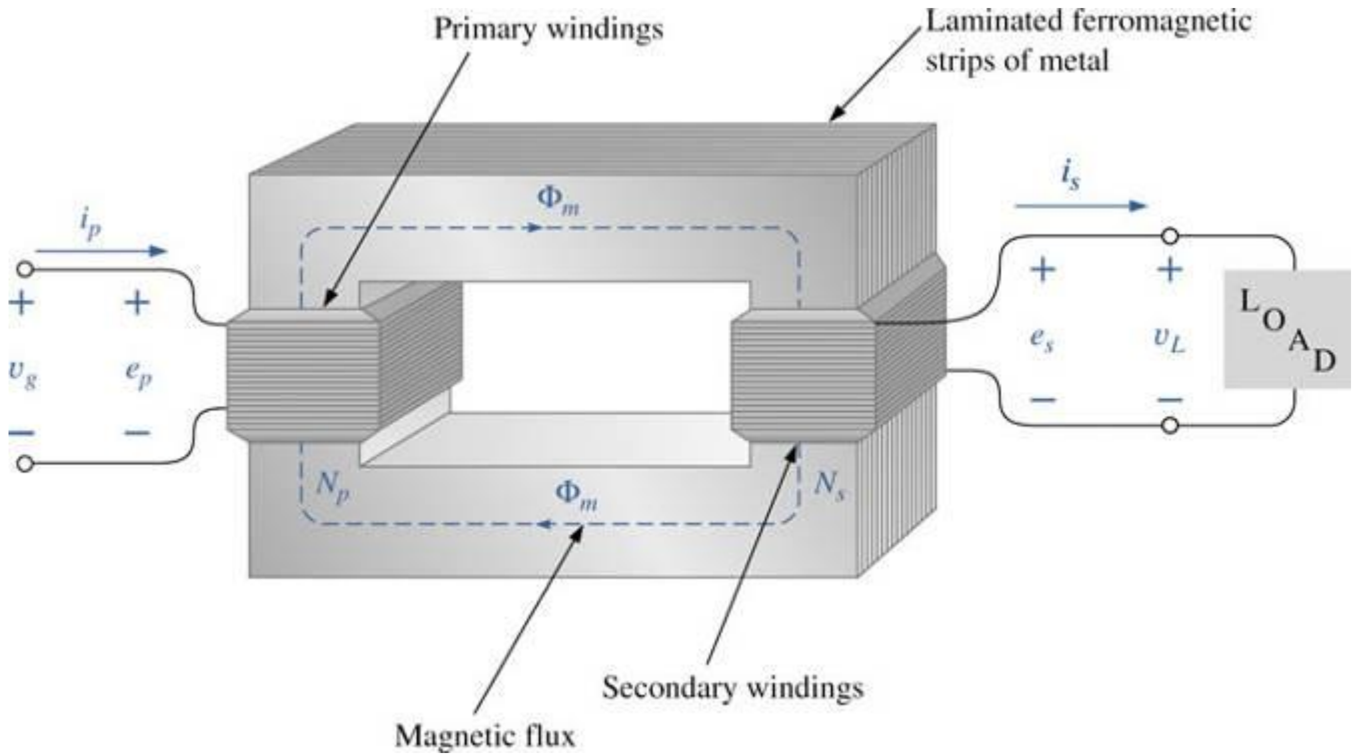


FIG. 22.4 *Iron-core transformer.*

The Ideal Iron-core Transformer

The coefficient of coupling is its maximum value ,1,
Ideal Transformer ➔ neglect losses, leakage reactance, the hysteresis and eddy current losses

In fact, the magnitude of the flux is directly proportional to the current through the primary windings. Therefore, the two are in phase, and for sinusoidal inputs, the magnitude of the flux will vary as a sinusoid also. That is, if

$$i_p = \sqrt{2} I_p \sin \omega t$$

$$\phi_m = \Phi_m \sin \omega t$$

The induced voltage across the primary due to a sinusoidal input can be determined by Faradays law:

$$e_p = N_p \frac{d\phi_p}{dt} = N_p \frac{d\phi_m}{dt}$$

$$e_p = N_p \frac{d}{dt}(\Phi_m \sin \omega t)$$

$$\Rightarrow$$

$$e_p = \omega N_p \Phi_m \cos \omega t$$

$$e_p = \omega N_p \Phi_m \sin(\omega t + 90°)$$

The effective value (rms) of e_p is

$$E_p = \frac{\omega N_p \Phi_m}{\sqrt{2}} = \frac{2\pi f N_p \Phi_m}{\sqrt{2}}$$

$$\boxed{E_p = 4.44 f N_p \Phi_m}$$

For the case under discussion, where the flux linking the secondary equals that of the primary, if we repeat the procedure just described for the induced voltage across the secondary, we get

$$E_s = 4.44 f N_s \Phi_m$$

$$\frac{E_p}{E_s} = \frac{4.44 f N_p \Phi_m}{4.44 f N_s \Phi_m} \quad \Rightarrow \quad \frac{E_p}{E_s} = \frac{N_p}{N_s}$$

The ratio of the magnitudes of the induced voltages is the same as the ratio of the corresponding turns.

If we consider that

$$e_p = N_p \frac{d\phi_m}{dt} \quad \text{and} \quad e_s = N_s \frac{d\phi_m}{dt}$$

and divide one by the other, that is,

$$\frac{e_p}{e_s} = \frac{N_p(d\phi_m/dt)}{N_s(d\phi_m/dt)}$$

then

$$\frac{e_p}{e_s} = \frac{N_p}{N_s}$$

The instantaneous values of e_1 and e_2 are therefore related by a constant determined by the turns ratio.

since $\mathbf{V}_g = \mathbf{E}_1$ and $\mathbf{V}_L = \mathbf{E}_2$ for the ideal situation,

$$\frac{\mathbf{V}_g}{\mathbf{V}_L} = \frac{N_p}{N_s}$$

The ratio N_p/N_s, usually represented by the lowercase letter a, is referred to as the **transformation ratio:**

$$a = \frac{N_p}{N_s}$$

$$N_p i_p = N_s i_s$$

Since the instantaneous values of i_p and i_s are related by the turns ratio, the phasor quantities $\mathbf{I}_p$ and $\mathbf{I}_s$ are also related by the same ratio:

$$N_p \mathbf{I}_p = N_s \mathbf{I}_s$$

$$\frac{\mathbf{I}_p}{\mathbf{I}_s} = \frac{N_s}{N_p}$$

EXAMPLE 21.2 For the iron-core transformer of Fig. 21.5:

a. Find the maximum flux Φ_m.

b. Find the secondary turns N_s.

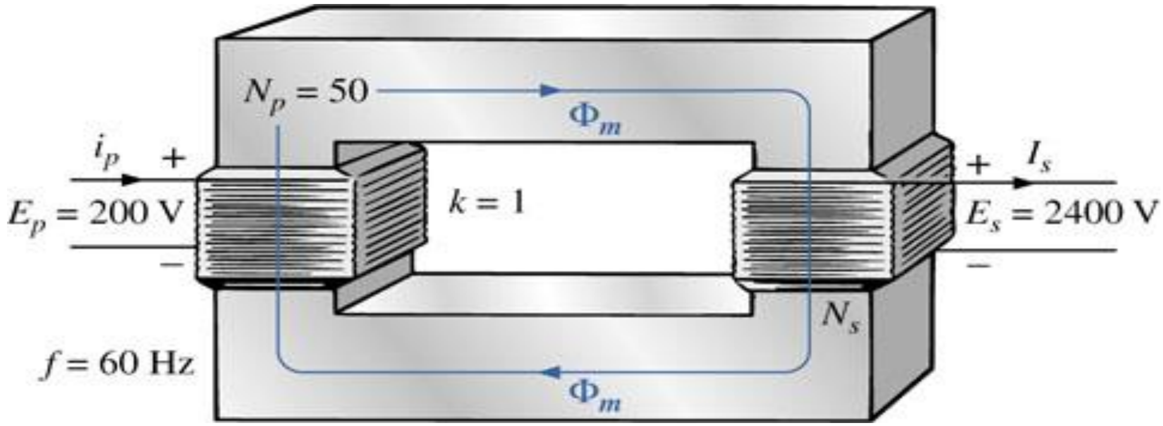


FIG. 22.5 *Example 22.2.*

Solutions:

a. $E_p = 4.44 N_p f \Phi_m$

Therefore, $$\Phi_m = \frac{E_p}{4.44 N_p f} = \frac{200\text{ V}}{(4.44)(50\text{ t})(60\text{ Hz})}$$

and $$\Phi_m = \mathbf{15.02\ mWb}$$

b. $$\frac{E_p}{E_s} = \frac{N_p}{N_s}$$

Therefore, $$N_s = \frac{N_p E_s}{E_p} = \frac{(50\text{ t})(2400\text{ V})}{200\text{ V}}$$

$$= \mathbf{600\ turns}$$

REFLECTED IMPEDANCE AND POWER

In the previous section we found that

$$\frac{\mathbf{V}_g}{\mathbf{V}_L} = \frac{N_p}{N_s} = a \quad \text{and} \quad \frac{\mathbf{I}_p}{\mathbf{I}_s} = \frac{N_s}{N_p} = \frac{1}{a}$$

Dividing the first by the second, we have

$$\frac{\mathbf{V}_g/\mathbf{V}_L}{\mathbf{I}_p/\mathbf{I}_s} = \frac{a}{1/a}$$

or

$$\frac{\mathbf{V}_g/\mathbf{I}_p}{\mathbf{V}_L/\mathbf{I}_s} = a^2 \quad \text{and} \quad \frac{\mathbf{V}_g}{\mathbf{I}_p} = a^2\frac{\mathbf{V}_L}{\mathbf{I}_s}$$

However, since

$$\mathbf{Z}_p = \frac{\mathbf{V}_g}{\mathbf{I}_p} \quad \text{and} \quad \mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}_s}$$

then

$$\boxed{\mathbf{Z}_p = a^2\mathbf{Z}_L}$$

$$\frac{E_p}{E_s} = a = \frac{I_s}{I_p}$$

$$\boxed{E_pI_p = E_sI_s}$$

$$\boxed{P_{in} = P_{out}} \quad \text{(ideal conditions)}$$

EXAMPLE 21.3 For the iron-core transformer of Fig. 21.6:

a. Find the magnitude of the current in the primary and the impressed voltage across the primary.

b. Find the input resistance of the transformer.

Solutions:

a. $$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

$$I_p = \frac{N_s}{N_p} I_s = \left(\frac{5\ \text{t}}{40\ \text{t}}\right)(0.1\ \text{A}) = \mathbf{12.5\ mA}$$

$$V_L = I_s Z_L = (0.1\ \text{A})(2\ \text{k}\Omega) = 200\ \text{V}$$

Also, $$\frac{V_g}{V_L} = \frac{N_p}{N_s}$$

$$V_g = \frac{N_p}{N_s} V_L = \left(\frac{40\ \text{t}}{5\ \text{t}}\right)(200\ \text{V}) = \mathbf{1600\ V}$$

b. $$Z_p = a^2 Z_L$$

$$a = \frac{N_p}{N_s} = 8$$

$$Z_p = (8)^2(2\ \text{k}\Omega) = R_p = \mathbf{128\ k\Omega}$$

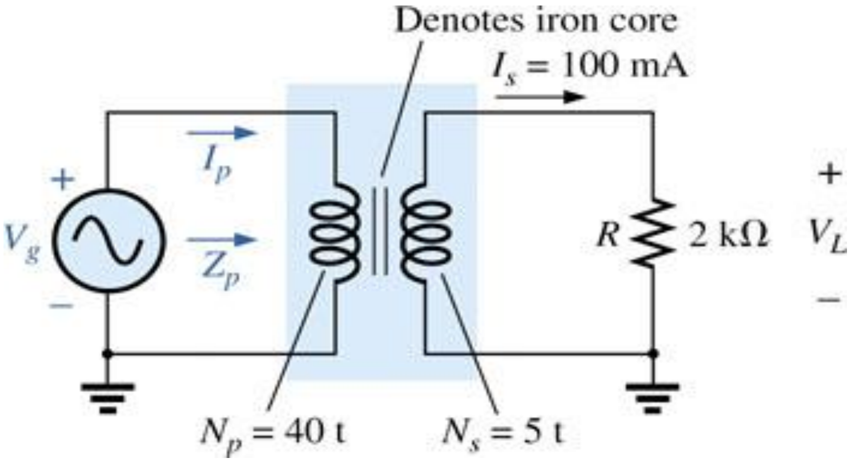


FIG. 22.6 *Example 22.3.*

EXAMPLE 21.4 For the residential supply appearing in Fig. 21.7, determine (assuming a totally resistive load) the following:

a. the value of R to ensure a balanced load
b. the magnitude of I_1 and I_2
c. the line voltage V_L
d. the total power delivered
e. the turns ratio $a = N_p/N_s$

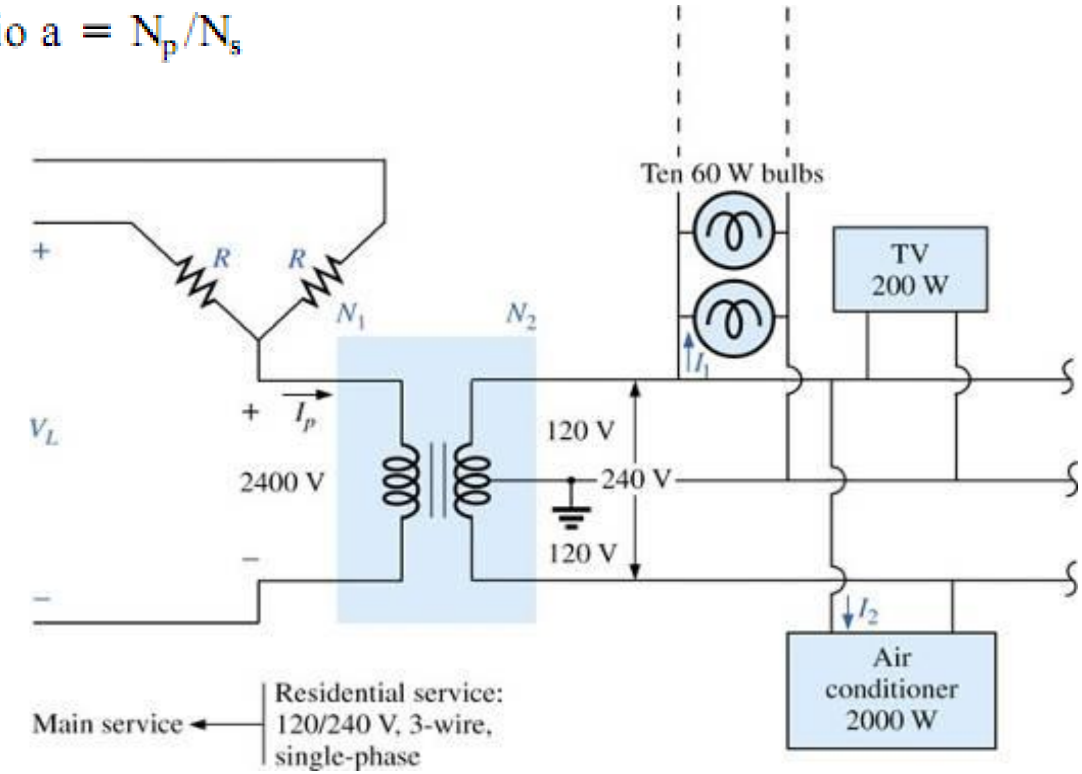


FIG. 22.7 *Single-phase residential supply.*

Solutions:

a. $P_T = (10)(60\text{ W}) + 400\text{ W} + 2000\text{ W}$

$= 600\text{ W} + 400\text{ W} + 2000\text{ W} = 3000\text{ W}$

$P_{in} = P_{out}$

$V_p I_p = V_s I_s = 3000\text{ W}$ (purely resistive load)

$(2400\text{ V})I_p = 3000\text{ W}$ and $I_p = 1.25\text{ A}$

$$R = \frac{V_\phi}{I_p} = \frac{2400\text{ V}}{1.25\text{ A}} = \mathbf{1920\ \Omega}$$

b. $P_1 = 600\text{ W} = VI_1 = (120\text{ V})I_1$

and $I_1 = \mathbf{5\ A}$

$P_2 = 2000\text{ W} = VI_2 = (240\text{ V})I_2$

and $I_2 = \mathbf{8.33\ A}$

c. $V_L = \sqrt{3}V_\phi = 1.73(2400\text{ V}) = \mathbf{4152\ V}$

d. $P_T = 3P_\phi = 3(3000\text{ W}) = \mathbf{9\ kW}$

e. $$a = \frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{2400\text{ V}}{240\text{ V}} = \mathbf{10}$$

EQUIVALENT CIRCUIT (IRON-CORE TRANSFORMER)

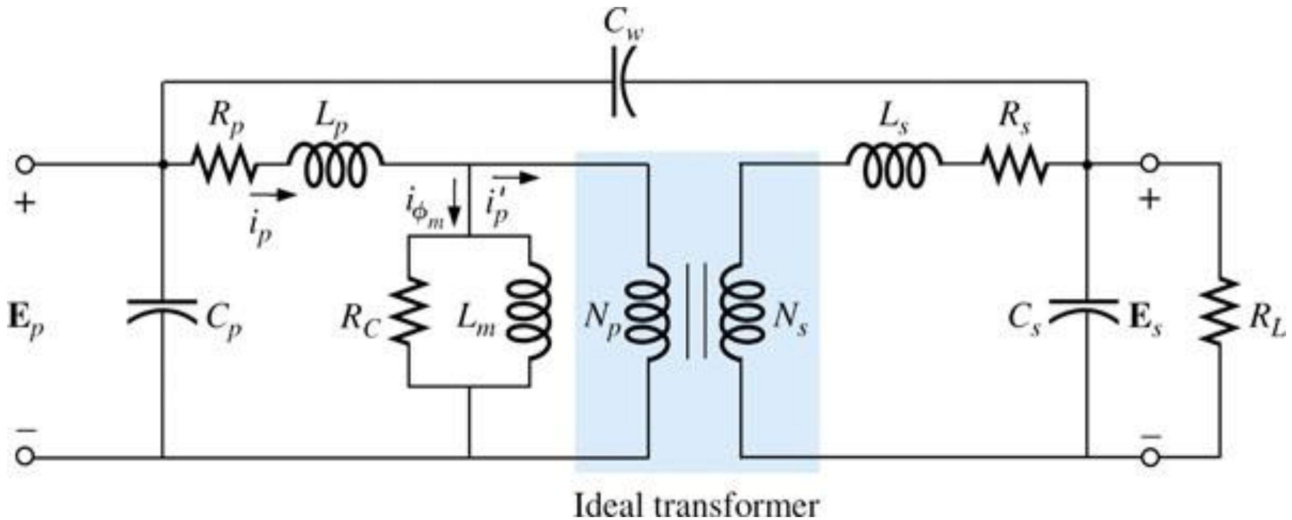


FIG. 22.15 *Equivalent circuit for the practical iron-core transformer.*

R_p and R_s are simply the dc resistance of the primary and secondary windings.

R_c represents the hysteresis and eddy current losses (core losses) within the core due to an ac flux through the core.

The inductance L_m (magnetizing inductance) is the inductance associated with the magnetization of the core, that is, the establishing of the flux Φ_m in the core.

C_p the lumped capacitances of the primary
C_s the lumped capacitances of secondary circuits.
C_w represents the equivalent lumped capacitances between the windings of the transformer.

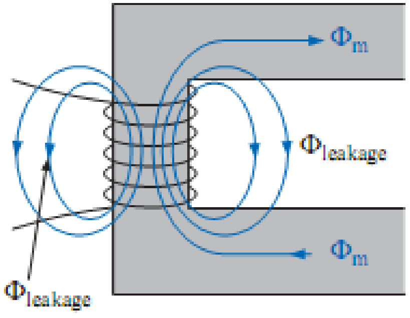


FIG. 21.16

Identifying the leakage flux of the primary.

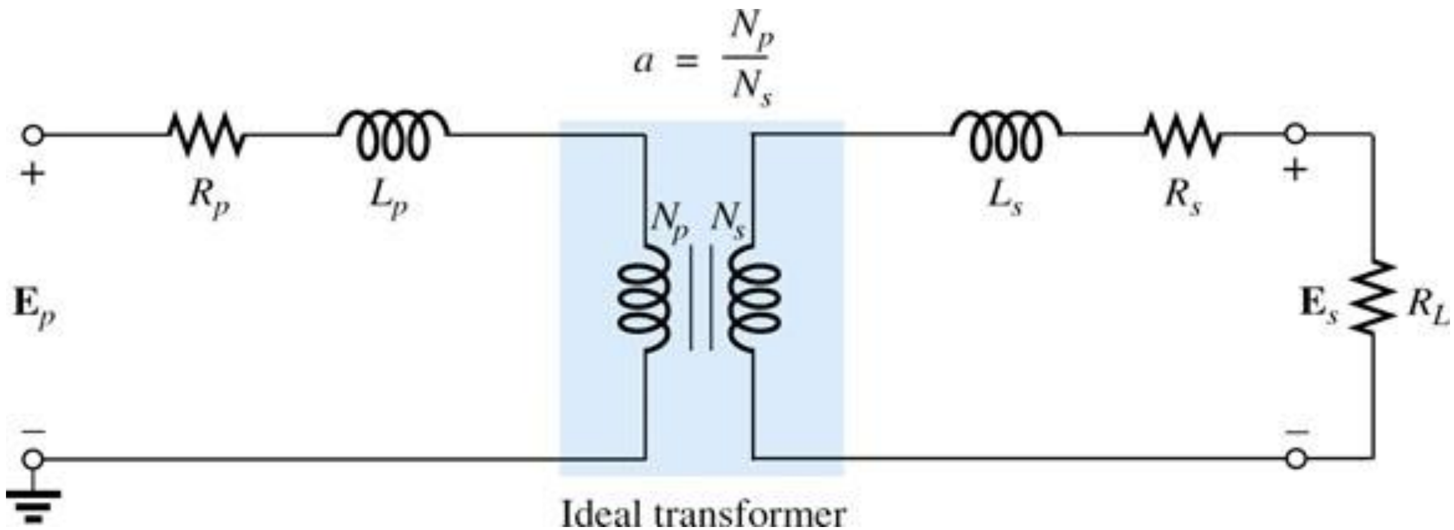


FIG. 22.17 *Reduced equivalent circuit for the nonideal iron-core transformer.*

If we now reflect the secondary circuit through the ideal transformer, as shown in Fig. 21.18(a), we will have the load and generator voltage in the same continuous circuit.

$$\mathbf{Z}_p = a^2\mathbf{Z}_L$$

$$R_{equivalent} = R_e = R_p + a^2R_s$$

$$X_{equivalent} = X_e = X_p + a^2X_s$$

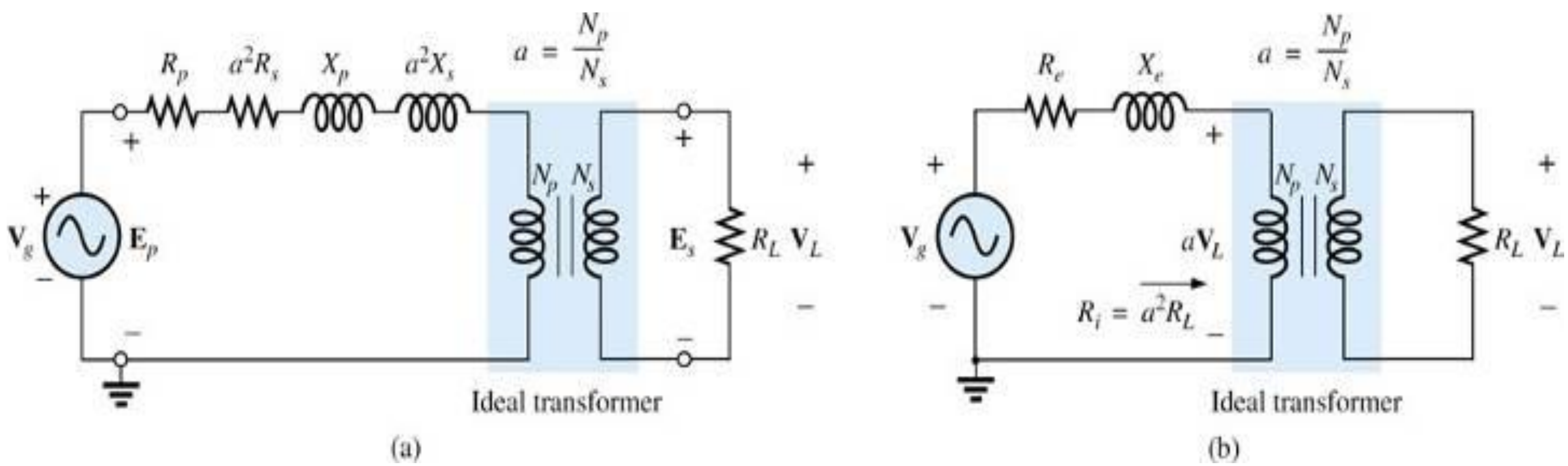


FIG. 22.18 *Reflecting the secondary circuit into the primary side of the iron-core transformer.*

$$a\mathbf{V}_L = \frac{(R_i)\mathbf{V}_g}{(R_e + R_i) + j\,X_e} \quad\Rightarrow\quad \mathbf{V}_L = \frac{a^2R_L\mathbf{V}_g}{(R_e + a^2R_L) + j\,X_e}$$

Phasor diagram for the iron-core transformer

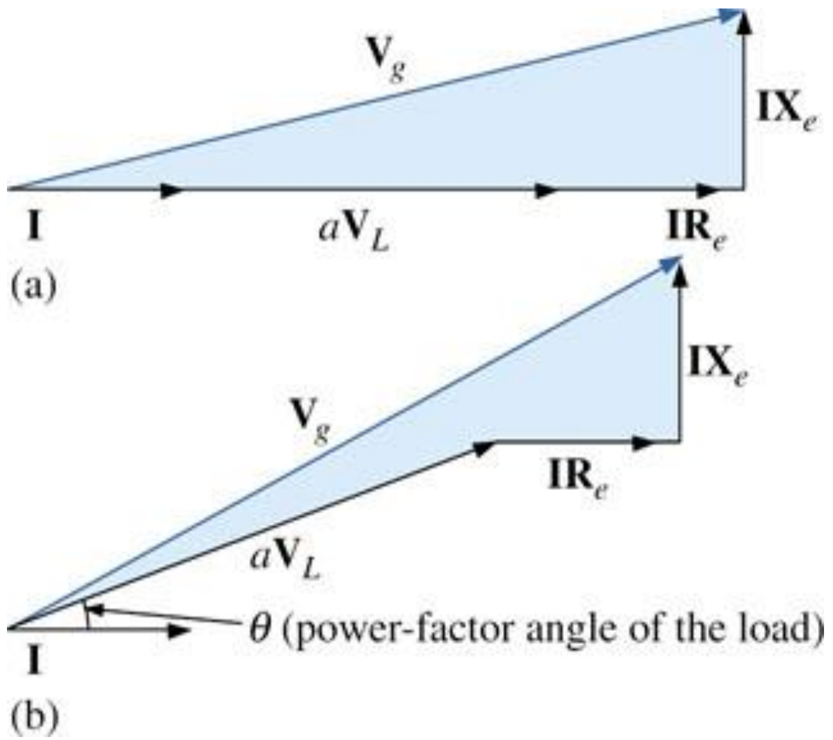


FIG. 22.19 *Phasor diagram for the iron-core transformer with (a) unity power-factor load (resistive) and (b) lagging power-factor load (inductive).*

Draw phasor diagram with load power factor

EXAMPLE 21.7 For a transformer having the equivalent circuit of Fig. 21.20:

a. Determine R_e and X_e.
b. Determine the magnitude of the voltages V_L and V_g.
c. Determine the magnitude of the voltage V_g to establish the same load voltage in part (b) if R_e and $X_e = 0\ \Omega$. Compare with the result of part (b).

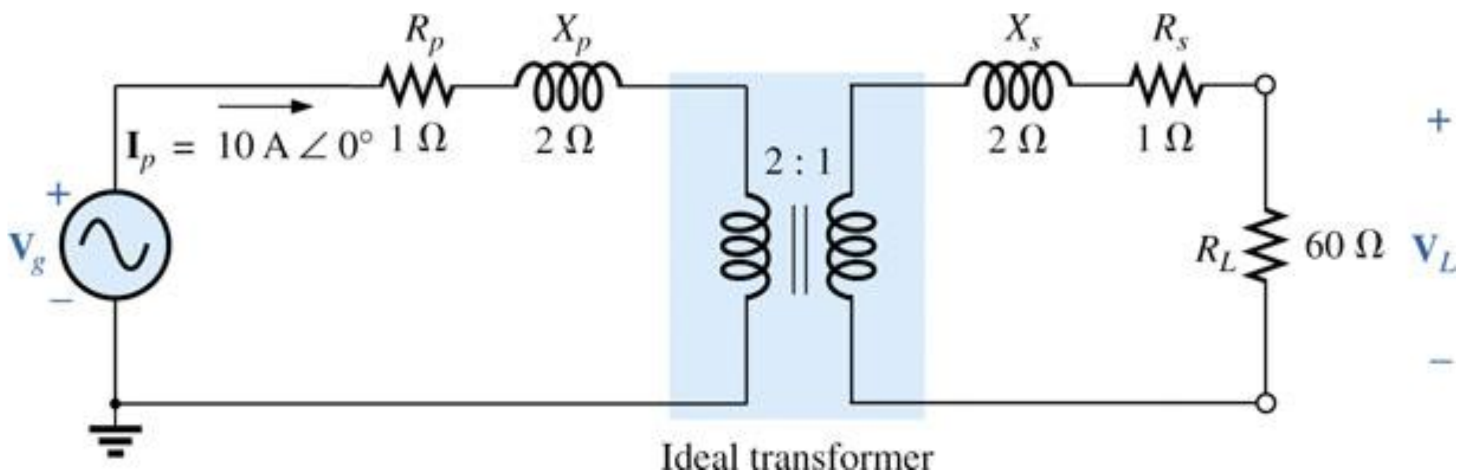


FIG. 22.20 *Example 22.7*

Solutions:

a. $R_e = R_p + a^2R_s = 1\ \Omega + (2)^2(1\ \Omega) = \mathbf{5\ \Omega}$

$X_e = X_p + a^2X_s = 2\ \Omega + (2)^2(2\ \Omega) = \mathbf{10\ \Omega}$

b. The transformed equivalent circuit appears in Fig. 21.21.

$$aV_L = (I_p)(a^2R_L) = 2400\text{ V}$$

Thus,

$$V_L = \frac{2400\text{ V}}{a} = \frac{2400\text{ V}}{2} = \mathbf{1200\text{ V}}$$

and

$$\mathbf{V}_g = \mathbf{I}_p(R_e + a^2R_L + j\,X_e)$$
$$= 10\text{ A}(5\ \Omega + 240\ \Omega + j\ 10\ \Omega) = 10\text{ A}(245\ \Omega + j\ 10\ \Omega)$$
$$\mathbf{V}_g = 2450\text{ V} + j\ 100\text{ V} = 2452.04\text{ V}\ \angle 2.34° = \mathbf{2452.04\text{ V}\ \angle 2.34°}$$

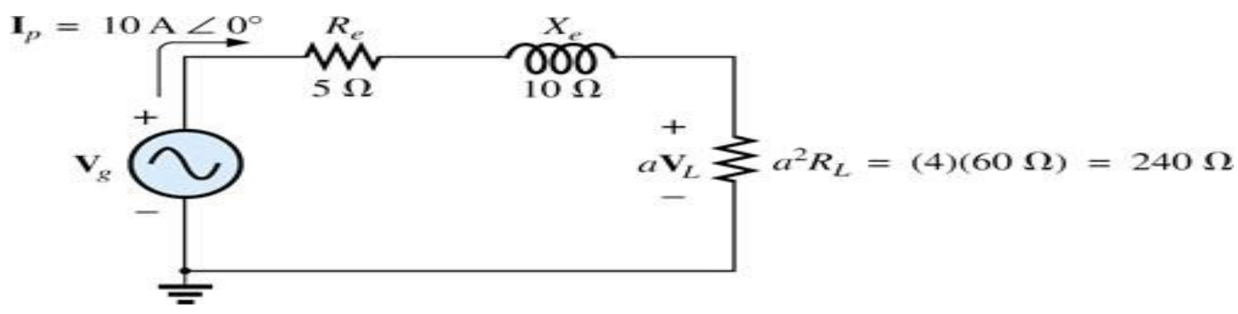


FIG. 22.21 *Transformed equivalent circuit of Fig. 22.20.*

c. For R_e and $X_e = 0$, $V_g = aV_L = (2)(1200\text{ V}) = 2400\text{ V}$.

Therefore, it is necessary to increase the generator voltage by 52.04 V (due to R_e and X_e) to obtain the same load voltage.

SERIES CONNECTION OF MUTUALLY COUPLED COILS

1.Mutually coupled coils connected in series with positive mutual inductance.

$$e_1 = L_1 \frac{di_1}{dt} + M_{12}\frac{di_2}{dt}$$

However, since $i_1 = i_2 = i$,

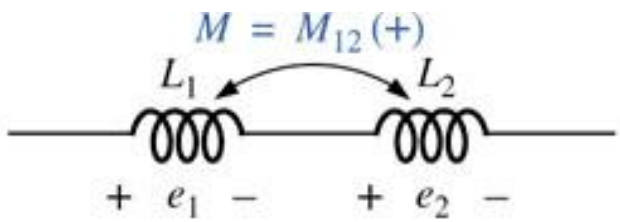
$$e_1 = L_1 \frac{di}{dt} + M_{12}\frac{di}{dt}$$

or

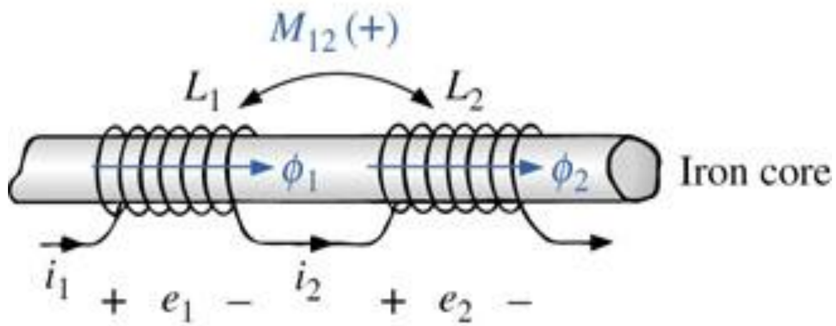
$$e_1 = (L_1 + M_{12})\frac{di}{dt} \qquad \text{(volts, V)}$$

and, similarly,

$$e_2 = (L_2 + M_{12})\frac{di}{dt} \qquad \text{(volts, V)}$$



(a)



(b)

For the series connection, the total induced voltage across the series coils, represented by e_T, is

$$e_T = e_1 + e_2 = (L_1 + M_{12})\frac{di}{dt} + (L_2 + M_{12})\frac{di}{dt}$$

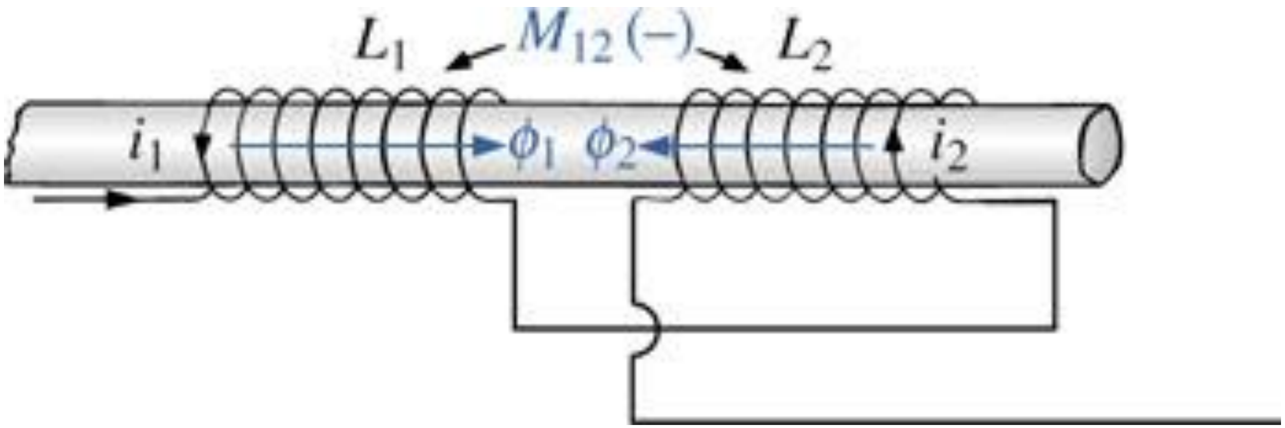
or

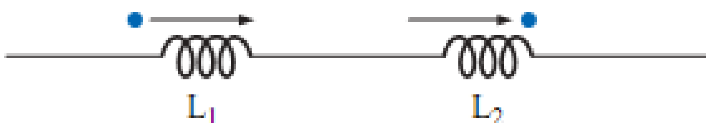
$$e_T = (L_1 + L_2 + M_{12} + M_{12})\frac{di}{dt}$$

and the total effective inductance is

$$L_{T(+)} = L_1 + L_2 + 2M_{12} \qquad \text{(henries, H)} \qquad \textbf{(21.27)}$$

2. Mutually coupled coils connected in series with negative mutual inductance.





$$L_{T(-)} = L_1 + L_2 - 2M_{12}$$ (henries, H)

The total mutual inductance

$$M_{12} = \frac{1}{4}(L_{T(+)} - L_{T(-)})$$

Examples of SERIES CONNECTION OF MUTUALLY COUPLED COILS

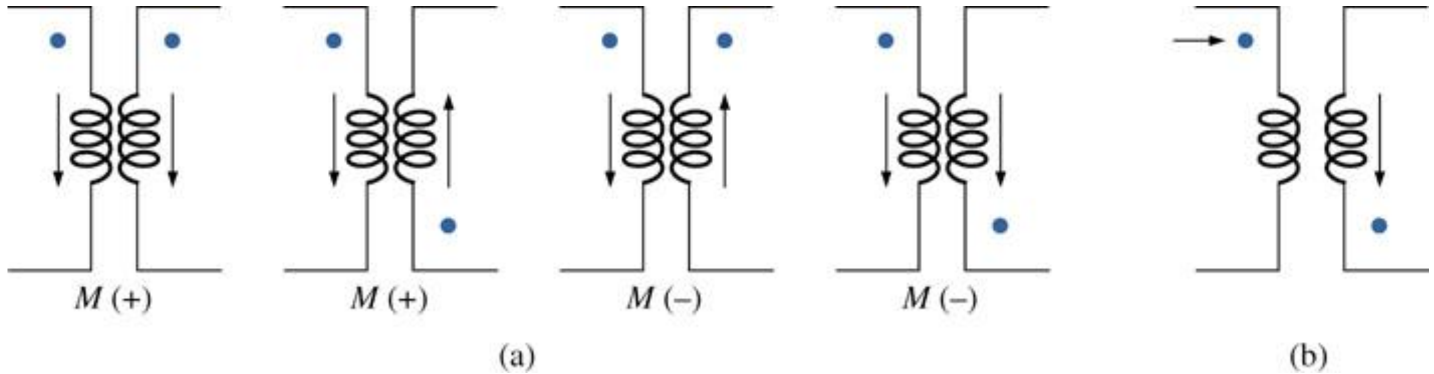


FIG. 22.28 *Defining the sign of M for mutually coupled transformer coils.*

EXAMPLE 21.8 Find the total inductance of the series coils of Fig. 21.29.

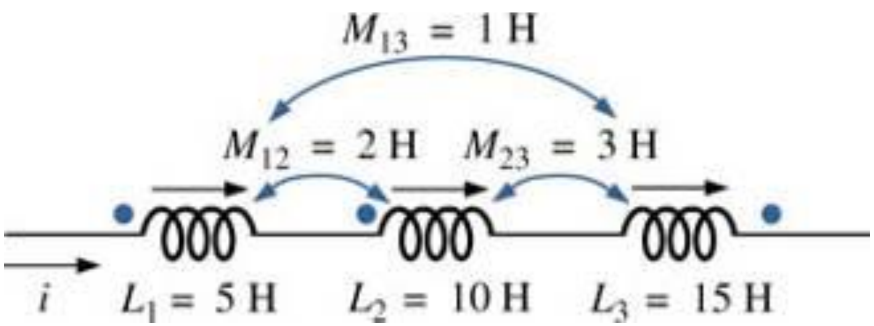


FIG. 22.29 *Example 22.8.*

Solution:

Current vectors leave dot.

Coil 1: $L_1 + M_{12} - M_{13}$

One current vector enters dot, while one leaves.

Coil 2: $L_2 + M_{12} - M_{23}$

Coil 3: $L_3 - M_{23} - M_{13}$

and

$$L_T = (L_1 + M_{12} - M_{13}) + (L_2 + M_{12} - M_{23}) + (L_3 - M_{23} - M_{13})$$
$$= L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{13}$$

Substituting values, we find

$$L_T = 5\text{ H} + 10\text{ H} + 15\text{ H} + 2(2\text{ H}) - 2(3\text{ H}) - 2(1\text{ H})$$
$$= 34\text{ H} - 8\text{ H} = \mathbf{26\text{ H}}$$

EXAMPLE 21.9 Write the mesh equations for the transformer network in Fig. 21.30.

Solution: For each coil, the mutual term is positive, and the sign of M in $\mathbf{X}_m = \omega M \angle 90°$ is positive, as determined by the direction of $\mathbf{I}_1$ and $\mathbf{I}_2$. Thus,

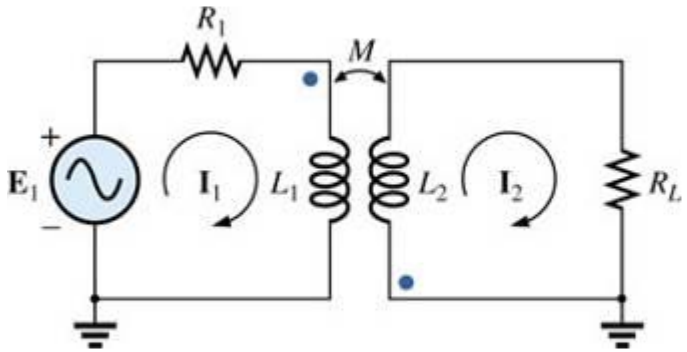


FIG. 22.30 *Example 22.9.*

$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - \mathbf{I}_1 X_{L_1} \angle 90° - \mathbf{I}_2 X_m \angle 90° = 0$$

or

$$\mathbf{E}_1 - \mathbf{I}_1(R_1 + j\,X_{L_1}) - \mathbf{I}_2 X_m \angle 90° = 0$$

For the other loop,

$$-\mathbf{I}_2 X_{L_2} \angle 90° - \mathbf{I}_1 X_m \angle 90° - \mathbf{I}_2 R_L = 0$$

or

$$\mathbf{I}_2(R_L + j\,X_{L_2}) + \mathbf{I}_1 X_m \angle 90° = 0$$

AIR-CORE TRANSFORMER

- As the name implies, the air-core transformer does not have a ferromagnetic core to link the primary and secondary coils.
- Rather, the coils are placed sufficiently close to have a mutual inductance that establishes the desired transformer action.

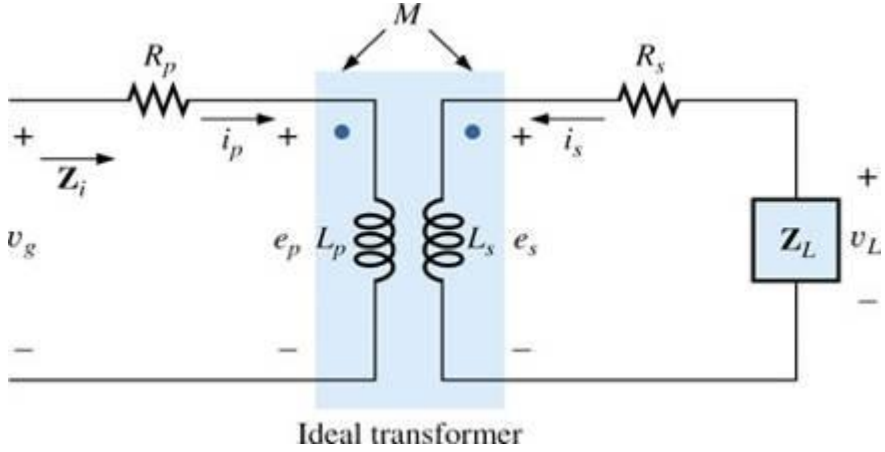


FIG. 22.31 *Air-core transformer equivalent circuit.*

AIR-CORE TRANSFORMER

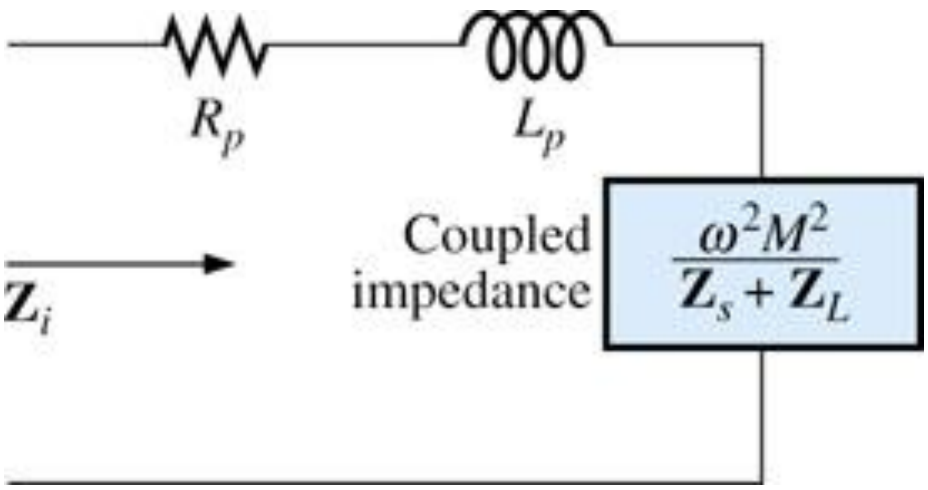


FIG. 22.32 *Input characteristics for the air-core transformer.*

AIR-CORE TRANSFORMER

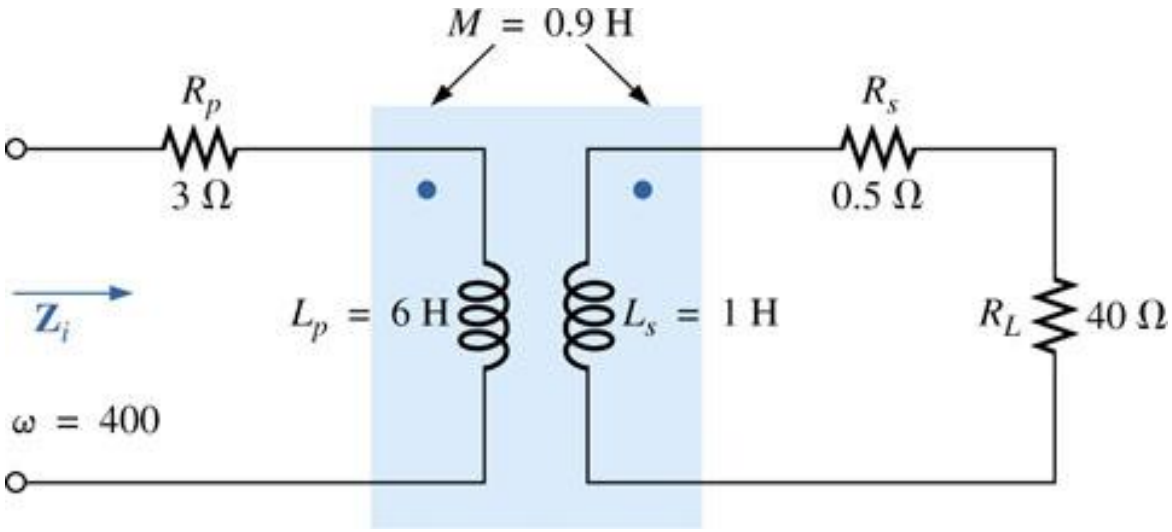


FIG. 22.33 *Example 22.10.*

APPLICATIONS

- **Soldering Gun**
- **Low-Voltage Compensation**
- **Ballast Transformer**
- **Recent Developments**