Time varying Electromagnetic Fields

Inductance and Transformers

* Self-inductance
* Mutual inductance
* Transformers

Inductance

According to Faraday's Electromagnetic Induction law,
induction of an electromotive force occurs in a circuit
by varying the magnetic flux linked with the circuit.

From this law, the concept of Inductance may be derived
as the property of an electric circuit by which
an electromotive force is induced in it
as the result of a changing magnetic flux.

Inductance

Inductance L may be defined in terms of the electromotive force generated to oppose a change in current ΔI in the given time duration Δt according to Faraday's law as follows:

$$Emf = -L\frac{\Delta I}{\Delta t}$$

Unit for L: $\dfrac{\text{volt second}}{\text{ampere}} = \text{Henry}$

Inductance is measured in S.I. unit called henry (H):

Inductance

$$Emf = -L\frac{\Delta I}{\Delta t}$$

Unit for L: $\frac{\text{volt second}}{\text{ampere}} = \text{Henry}$

1 henry is 1 volt-second / ampere.

If the rate of change of current in a circuit $\Delta I / \Delta t$ is one ampere per second and the resulting electromotive force is one volt, then the inductance of the circuit is one henry.

Inductance

Since emf results due to
the rate of change of magnetic flux ,
inductance L may also be defined as a measure of
the amount of magnetic flux φ produced
for a given electric current I as
$L = \varphi / I$
where the inductance L is one henry,
if current I of one ampere,
produces magnetic flux φ of one weber.

There are two types of inductances:
1 Self –inductance
2. Mutual Inductance

Self-inductance

Self-inductance in terms of Magnetic Flux

A coil carrying current has magnetic flux associated with it.
The flux Φ is directly proportional to the current *I*

$$\Phi = L\,I,$$

L is the constant of proportionality.

L is called the self inductance.
Thus the ratio of the magnetic flux to the current is the **self-inductance**

Self-inductance

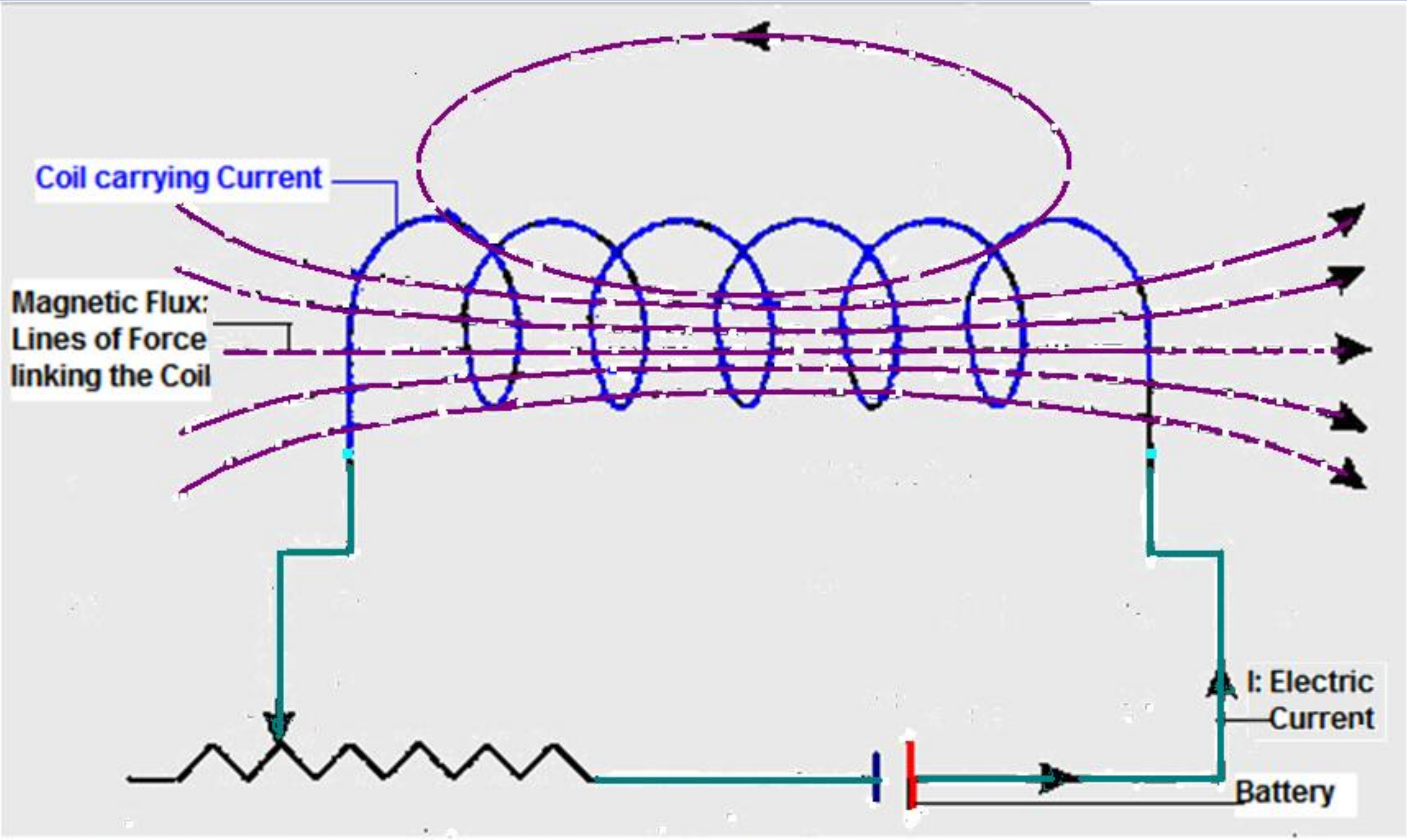


Fig. Self-Inductance: Magnetic Flux linking a Coil carrying Electric Current

Self-inductance

Self-inductance in terms of emf

According to Faraday's laws of electromagnetic induction
when the electric current flowing through a wire changes,
magnetic flux associated with it changes,
electromotive force (emf) is induced
that opposes the change in flux and current

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

The opposing induced emf is called as the "back emf".
EMF is due to a change in flux in that circuit (wire) itself,
hence it is called as Self inductance.

Self-inductance

Self-inductance in terms of emf

Suppose the current flowing in the circuit increases from *0* to *I.*
It changes by an amount *dI* in a time duration *dt* then the magnetic flux linking the circuit changes by an amount

$$d\Phi = L\,dI$$

$$\mathcal{E} = -L\,\frac{dI}{dt}.$$

If the current is increasing then the induced emf always acts to reduce the current, and *vice versa*.
the self inductance ***L*** of a circuit is necessarily a positive number

Self-inductance of A solenoid

Consider a helically wound solenoid of length **ℓ**
and cross-sectional area ***A*.**
Length is much greater than its diameter.
n is the number of turns per unit length.
Current *I* is flowing through the solenoid.
hence the field within the solenoid is
approximately constant

Self-inductance of A solenoid

The magnetic flux across each turn of area *A* is

$= A\,\mu_0 n\, I$

The total number of turns = $n\,\ell$ turns,

The total magnetic flux is $\varphi = A\,\mu_0 n^2 I\,\ell$

(Using the definition of Self inductance: $L = \varphi / I$

$\Rightarrow L = A\mu_0 n^2\,\ell$

Mutual Inductance

Mutual Inductance in Terms of Magnetic Flux

Consider two coils in proximity of each other.
When a steady current flows in the I coil,
its magnetic field is linked to the II coil also and
vise a versa.
They are called as inductively coupled coils.

Though there is no direct physical coupling
between the two coils,
coupling is provided entirely due to
the magnetic field generated
by the current flowing in the I coil.

Mutual Inductance

The flux φ_2 through the second coil is
directly proportional to the current I1 flowing in the I coil.
Hence, $\varphi_2 = M_{21} I_1$ (M_{21}= constant of proportionality)
M_{21} is called the mutual inductance
of coil 2 with respect to coil 1.

Similarly the flux φ_1 through the I coil is
directly proportional to the current I_2 flowing in the II coil.
Hence, $\varphi_1 = M_{12} I_2$ (M_{12}= constant of proportionality).
M_{12} is called the mutual inductance
of coil 1 with respect to coil 2.

It can be proved that $M_{12} = M_{21}$

Mutual Inductance

Mutual Inductance in Terms of Magnetic Flux

The flux linking coil 2 when a current flows in coil 1
is exactly the same as the flux linking coil 1
when the same current flows in coil 2.

$M_{12} = M_{21} = M$

M is called as the "**Mutual Inductance**"
between two coils.

It implies that the mutual inductance of two coupled coils
is a purely geometric quantity, dependent on
the sizes, shapes, and relative orientations of the coils

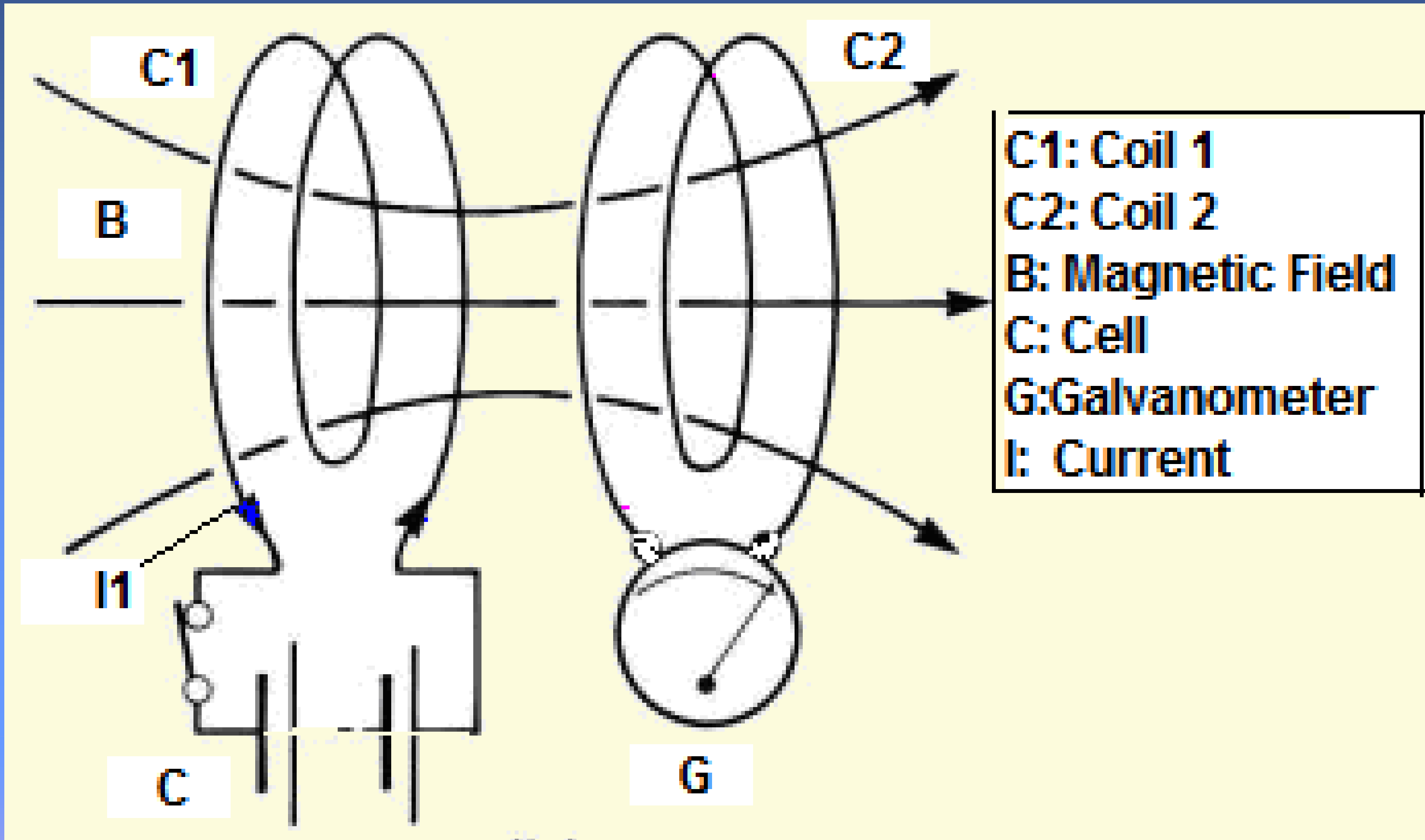
Mutual Inductance

Mutual Inductance in Terms of Magnetic Flux

If the magnetic field is steady and is not changing, there will be no induced voltage in the second coil.

However, if the current in the first coil is changing by opening/ closing a switch to stop/start the current, there will be a change in the magnetic flux associated with the second coil and a voltage will be induced in it according to Faraday's law.

Mutual Inductance



Mutual Inductance in coil2 due to current in a coupled coil1

emf induced in Coil C_2 due to the Current-flow in Coil C_1

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt}$$

Mutual Inductance

Mutual Inductance in Terms of Magnetic Flux

If current I_1 flowing in coil 1 changes by an amount dI_1 in time duration dt, the flux linking coil 2 changes by $d\varphi_2 = M\, dI_1$ during same duration, then an emf is produced in second coil because of the change in flux linking it

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt}$$

It can be written as $$\mathcal{E}_2 = -M\frac{dI_1}{dt}.$$

$(\varphi_2 = M_{21} I_1 = M I_1)$

Mutual Inductance

Thus, the emf generated in the II coil due to the current I_1 flowing in the I coil is directly proportional to the rate at which that current changes.

$$\mathcal{E}_2 = -M\,\frac{dI_1}{dt}.$$

Similarly for the emf generated in the I coil due to the current I_2 flowing in the II coil is directly proportional to the rate at which that current changes.

$$\mathcal{E}_1 = -M\,\frac{dI_2}{dt}.$$

Mutual Inductance

M is a measure of the mutual induction between two magnetically coupled coils.

It is the ratio of the induced electromotive force to the rate of change of current producing it.

It is also called as the **coefficient of mutual induction.**

The SI unit of mutual inductance is called henry (H).
One henry is equivalent to a volt-second per ampere.

M is a purely geometric quantity.
It depends only on the size, number of turns, relative position, and relative orientation of the two coils.

Mutual Inductance

Mutual Inductance between Two Wires

Suppose two insulated wires
wound on the same core have the parameters
A: cross-sectional area *A*
ℓ: the length ℓ of the core,
N_1 and N_2: the number of turns of two wires
I_1 and I_2: the currents flowing in the wires, then
the flux Φ_2 linking the II wire is given by

$$\Phi_2 = N_2 \, B_1 \, A = \mu_0 \, N_1 \, N_2 \, A \, I_1 / l$$

Mutual Inductance

Mutual Inductance between Two Wires

The mutual inductance of the II wire with respect to the I

$$M_{21} = \frac{\Phi_2}{I_1} = \frac{\mu_0 N_1 N_2 A}{l}$$

The mutual inductance of the I wire with respect to the II

$$M_{12} = \frac{\Phi_1}{I_2} = \frac{\mu_0 N_1 N_2 A}{l}.$$

$$M_{21} = M_{12}$$

The mutual inductance between the two wires is given by

$$M = \frac{\mu_0 N_1 N_2 A}{l}.$$

Mutual Inductance

Mutual Inductance between Two Wires

M is a geometric quantity which depends on
the dimensions
material of the core and
the manner of winding around the core.

However it does not depend on
the value of currents flowing through the wires.

Applications of Mutual Inductance

One of the important application
of Mutual Inductance is
in a machine like Transformer.

Transformer is a device that
transforms alternating electrical voltage
from one level to another level and
transfers electrical energy from one circuit to
another through inductively coupled conductors.

Transformer

Principle of Transformer

Transformer operates on the principle of
electromagnetic induction -Mutual Induction

When the electric current through a circuit changes,
the magnetic flux associated with the circuit in proximity
(i.e. magnetically coupled circuit) changes and
an electromotive force (EMF) is induced in coupled circuit.

Transformer

Construction of Transformer

The principle parts of
a hypothetical ideal transformer are as follows:

The Primary: A coil or winding
connected to an AC source serves as an Input.

The Secondary: A coil or winding in close proximity to
the primary winding, but electrically isolated from it.
It is connected to a load
to which electrical power is to be delivered.
Ideally, primary and secondary windings
have zero resistance

Transformer

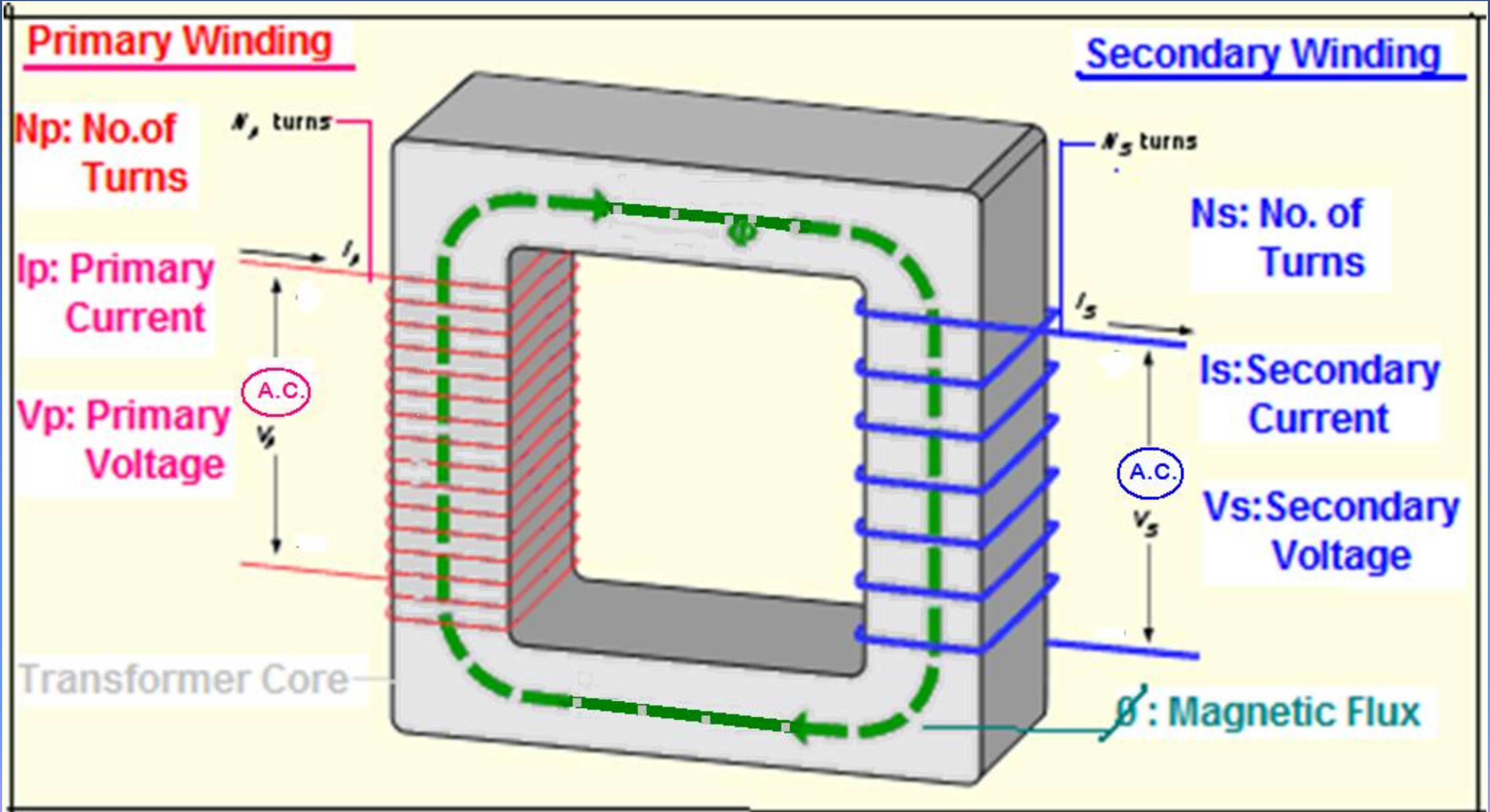


Fig: Transformer Construction

Transformer

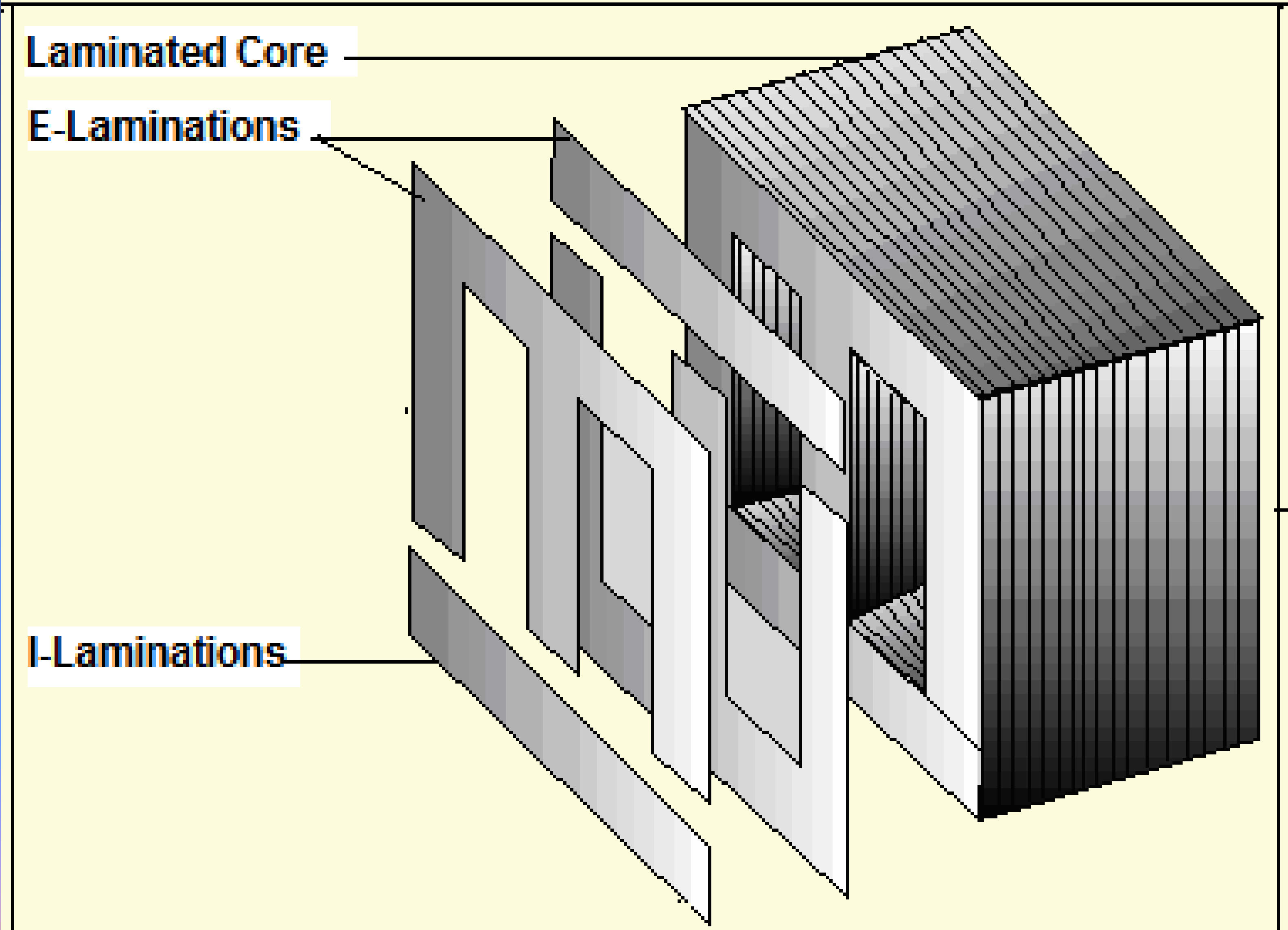


Fig. Transformer Laminated Core with E-I Laminations

Transformer

The Core,
On it both the primary and secondary coils are wound

It is made up of a material
of very high magnetic permeability, such as iron.

It provides the best path of low/ negligible reluctance
for the magnetic flux and
minimizes loss in magnetic and electrical energy.

In Iron-core transformers, core can be made by
stacking layers of thin steel/ iron laminations
insulated from each other by a coat of
non-conducting paint/ varnish
to reduce eddy current losses.

Transformer

The Core (contd)
Generally,
with high frequency (above 20 kHz) voltage sources,
air-core transformers are used while
with low frequency (below 20 kHz) voltage sources,
Iron-core transformers are used.

The iron-core transformer provides
better power transfer than the air-core transformer.

The Enclosure
It protects the above components from
dirt, moisture, and mechanical damage

Transformer

Transformer Working

The alternating current (A.C.) that flows through
the primary winding establishes
a time-varying magnetic flux in the primary and
voltage V_p is induced in the primary due to self-induction.

The flux in the primary is linked to
the secondary winding also, through the transformer core.

The varying magnetic flux in the primary induces
a varying electromotive force (emf)
or "voltage" V_s in the secondary winding
due to the effect called as mutual induction.

Transformer

Transformer Working

In accordance with Faraday's law of induction, Induced voltages are proportional to the rate of change of flux in the windings:

$$v_P = N_P \frac{d\Phi_P}{dt} \qquad v_S = N_S \frac{d\Phi_S}{dt}$$

V_p and V_s are the induced emfs in Primary & Secondary , N_p and N_s are their numbers of turns respectively,

$\frac{d\Phi_P}{dt}$ and $\frac{d\Phi_S}{dt}$

are the time derivatives of the magnetic flux φ_p and φ_s linking the primary and secondary windings respectively.

Transformer

Transformer Working

As the core of an ideal transformer has zero reluctance all the magnetic flux produced by the primary winding also links the secondary,
The emfs are equal in magnitude to the measured terminal voltages. so

$$\Phi_P = \Phi_S$$

$$\frac{v_P}{v_S} = \frac{N_P}{N_S}$$

This is the well-known basic equation for stepping up or stepping down transformer voltage.

Transformer

Transformer Working

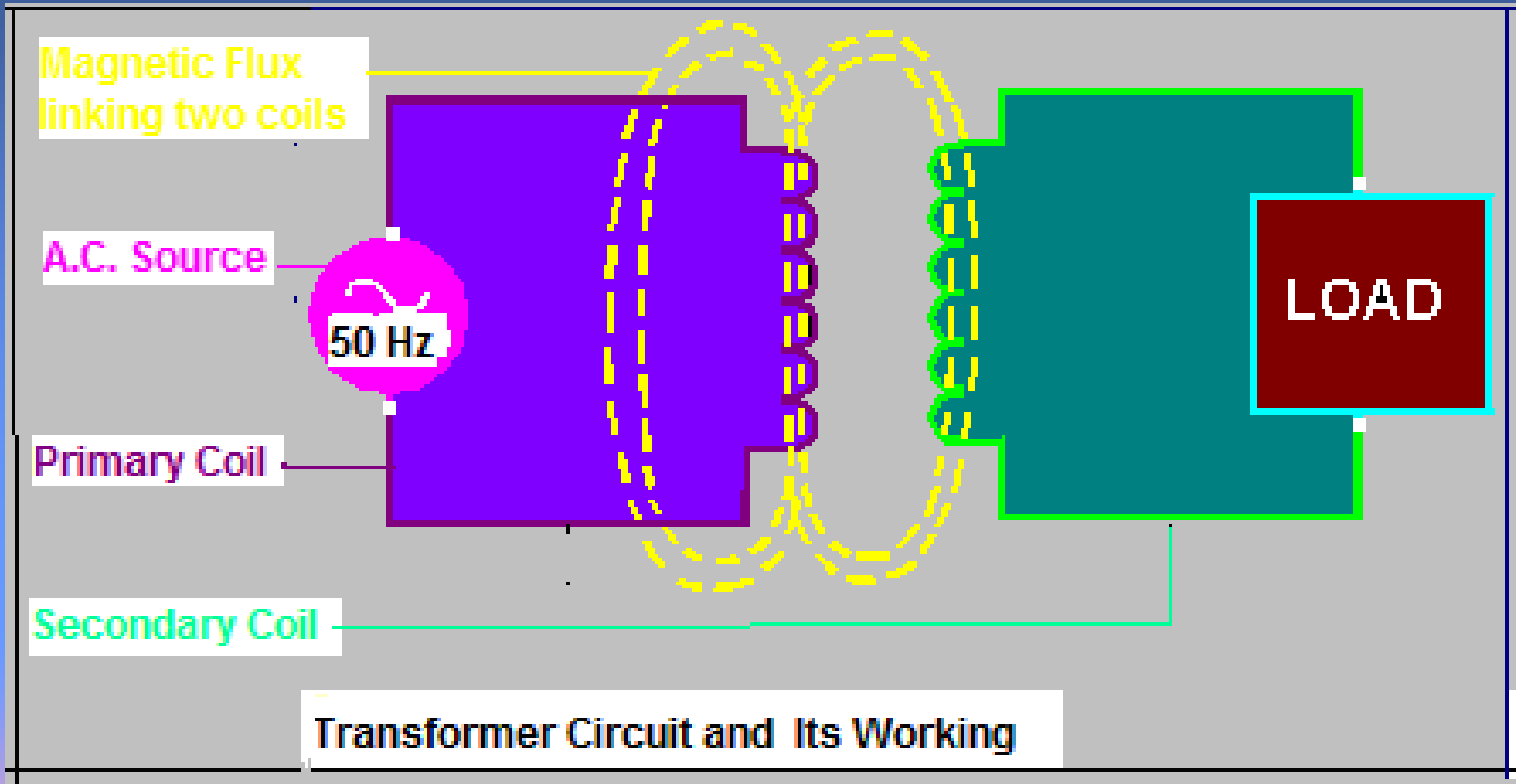
The ratio of transformation
of primary to secondary voltage is therefore the same as
the ratio of the number of turns in the windings; alternatively,
the volts-per-turn ratio is the same in both windings.

If the secondary coil is connected to a load,
current flows through it.

Thus electrical power is transferred
from the primary circuit to the secondary circuit.

Ideally, the transformer is perfectly efficient;
all the incoming energy is transformed from the primary circuit
to the magnetic field and into the secondary circuit.

Transformer



Transformer

Transformer Working

In the ideal condition, the incoming /input electric power must equal the outgoing / output power i.e.

$P_{input} = P_{output}$

$P_{input} = I_P V_P$ and $P_{output} = I_S V_S$

$P_{input} = P_{output}$

$I_P V_P = I_S V_S$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

Varying the ratio of the number of turns between the primary and the secondary windings, the ratio of the input and output voltages and currents can be controlled