Chapter 8 Transformers and Mutual Inductance

8.1 Ideal Transformers

- Transformers consist of magnetically coupled coils the respond only when driven by time-varying excitations.
- Ideal transformers vs. real transformers.

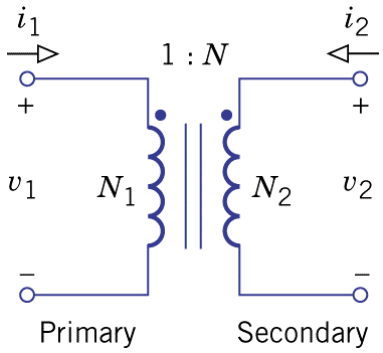


Figure 8.1.

- N_1: primary winding, N_2: secondary winding.
- Reference direction: both currents into the upper terminals.
- Dot convention: polarity is the same at both windings.
- Coils have zero resistance and are short circuits at dc steady state.
- Time varying excitation activates magnetic coupling and induces voltage at the other winding.
- Turns ratio: $N \equiv N_2 / N_1$.
- $v_2(t) = Nv_1(t)$, $i_2(t) = -i_1(t)/N$.
- With $N = 1$, a transformer becomes an isolation transformer that decouples the dc potential levels without affecting the time-varying quantities.
- An ideal transformer does not dissipate power ($p = v_1 i_1 + v_2 i_2 = 0$).
- Controlled source models (VCVS and CCCS)

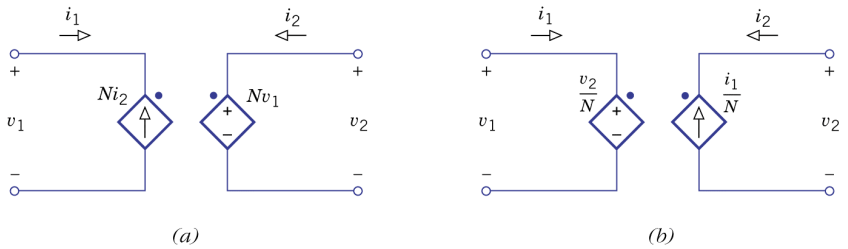
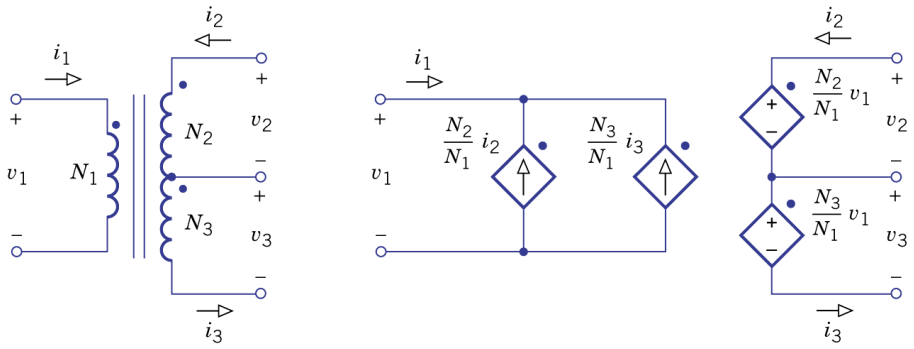


Figure 8.2.

- Transformers with three or more windings:

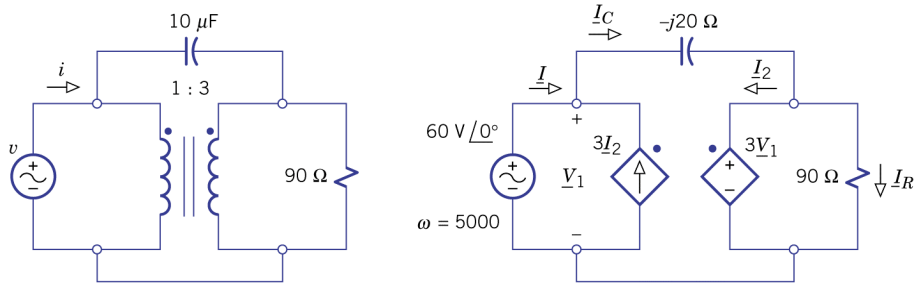


(a) Ideal transformer with tapped secondary

(b) Controled–source model

Figure 8.3.

Example 8.1: Analysis of a Transformer Circuit

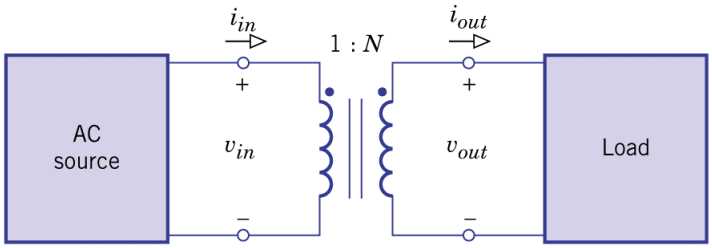


(a) Circuit with an ideal transformer

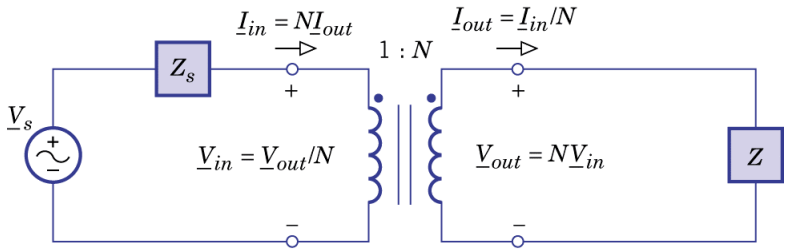
(b) Frequency–domain diagram

Figure 8.4.

- Referred networks:



(a) Ideal transformer interfacing a source and a load

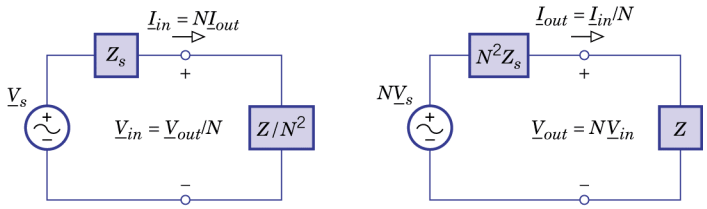


(b) Frequency–domain diagram

Figure 8.6.

- Referred load network and referred source network.

- Referred load network: R and L divided by N^2 and C multiplied by N^2.



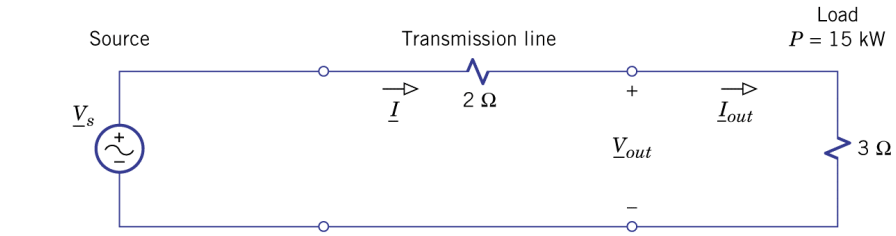
(a) Referred load impedance in the primary

(b) Referred source network in the secondary

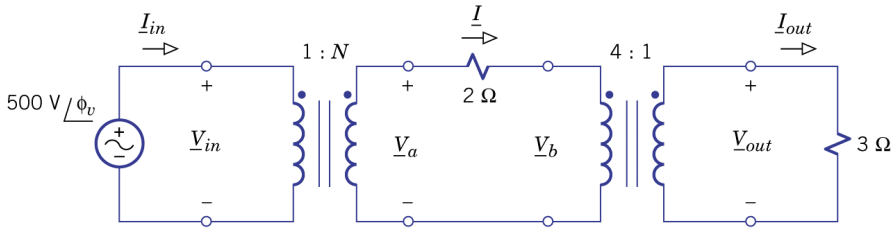
Figure 8.7.

- Referred source network: voltage multiplied by N, impedance multiplied by N^2 and current divided by N.

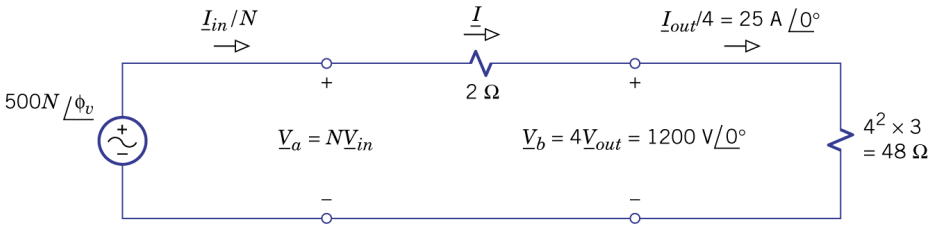
Example 8.2 Power Transmission with Transformers



(a) Power transfer via transmission line



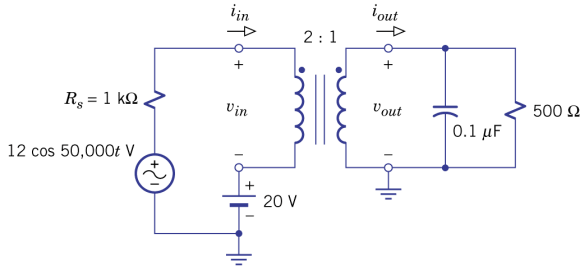
(b) Circuit with transformers at each end



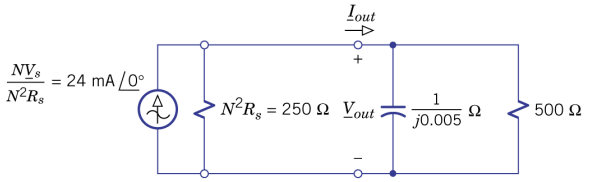
(c) Frequency–domain diagram with both ends referred into the middle

Figure 8.8.

Example 8.3 Transformer-Coupled Oscillator



(a) Transformer circuit with ac and dc sources

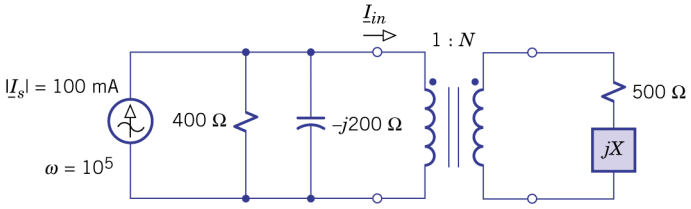


(b) Frequency–domain diagram with referred source network in the secondary

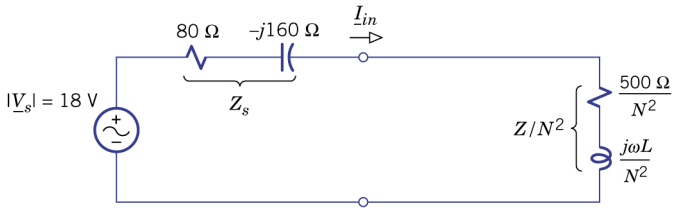
Figure 8.9.

- Impedance matching (refer to Figure 8.6b).

Example 8.4 Impedance Matching with a Transformer



(a) Model of amplifier with impedance–matching transformer

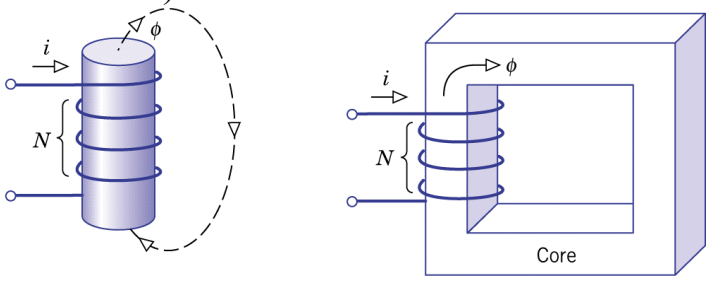


(b) Referred load in the primary

Figure 8.10.

8.2 Magnetic Coupling and Mutual Inductance

- Magnetic circuits: N turns, magnetic flux ϕ and current i (with right-hand rule).



(a) Coil with cylindrical core *(b)* Magnetic circuit Figure 8.11.

- Magnetomotive force (mmf): F=Ni.
- Reluctance: R=$l/\mu_r\mu_0 A$.
- F=Rϕ.

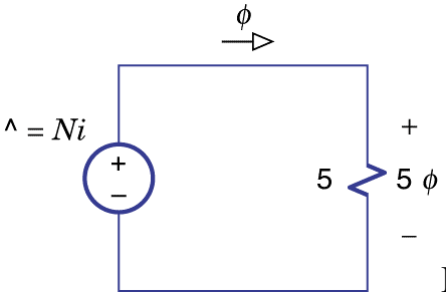
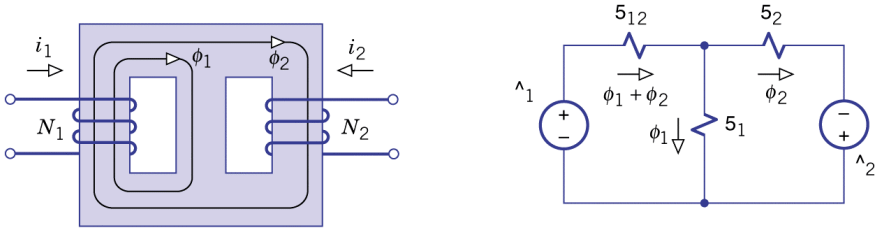
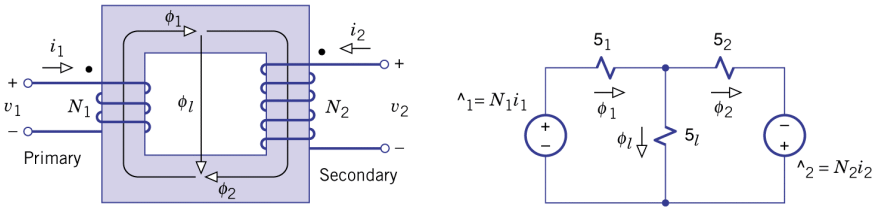


Figure 8.12.



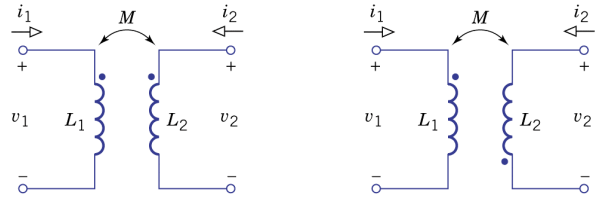
(a) Magnetic core with two flux loops *(b)* Magnetic circuit diagram Figure 8.13.

- Self-inductance and mutual inductance.



(a) Magnetic core with flux leakage *(b)* Magnetic circuit diagram Figure 8.14.

- Currents entering the dotted ends of both windings produce flux in the same direction.
- Leakage flux, self-inductance, mutual inductance and coupling coefficient.



(a) The same winding sense *(b)* Opposite winding sense Figure 8.15.

Example 8.5: Series Equivalent Inductance

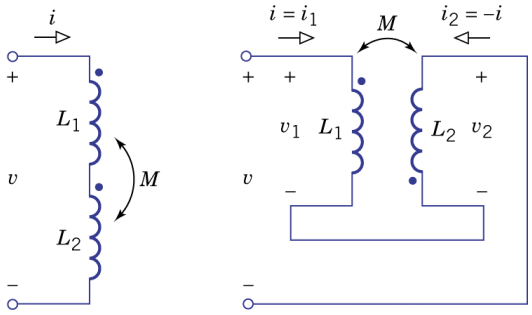


Figure 8.16.

- Stored energy:

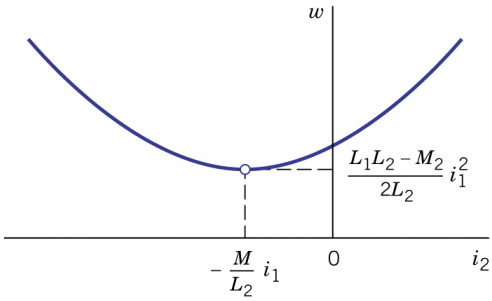


Figure 8.18.

- Unity coupling:

8.3 Circuits with Mutual Inductance

- Mutual inductance represented by controlled sources:

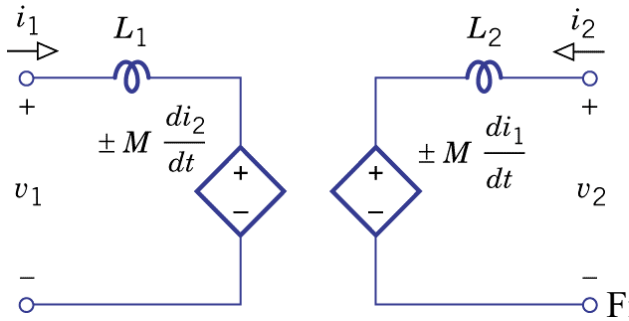


Figure 8.19.

- Frequency domain model of self- and mutual inductances:

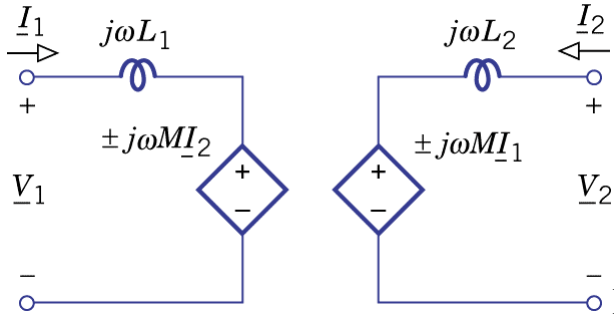
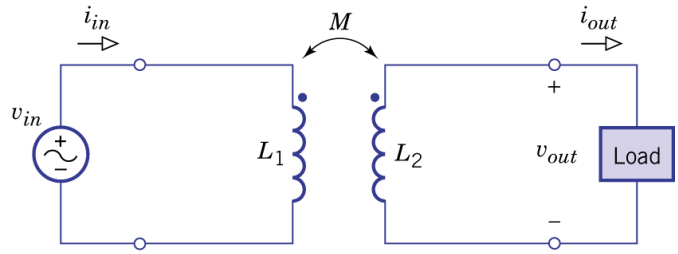
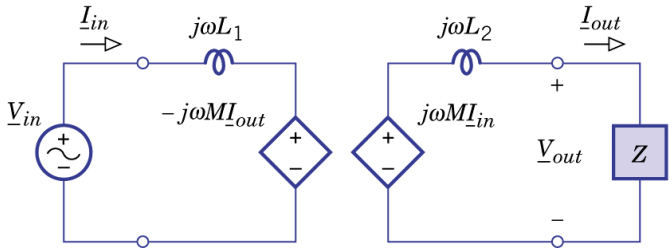


Figure 8.20.



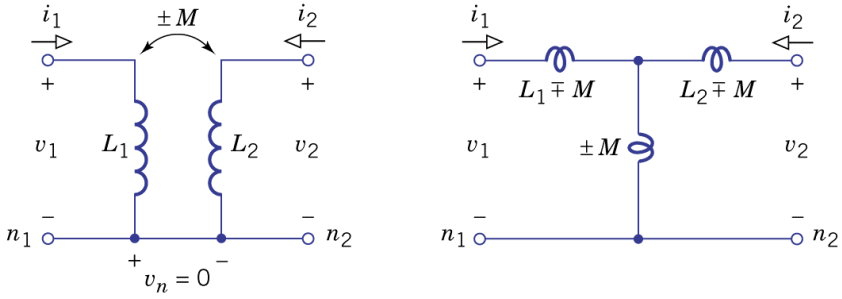
(a) Transformer interfacing a source and a load



(b) Frequency–domain diagram with controlled sources Figure 8.21.

Example 8.6: Comparison of a real and ideal transformer

- Equivalent tee (T) networks:



(a) Magnetically coupled coils *(b)* Equivalent tee (T) network Figure 8.23.

- Equivalent pi (Π) networks:

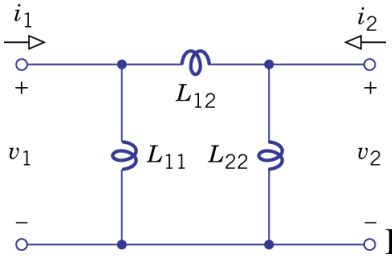
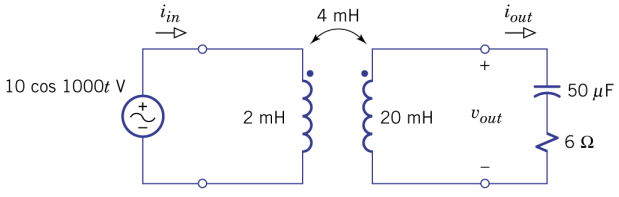
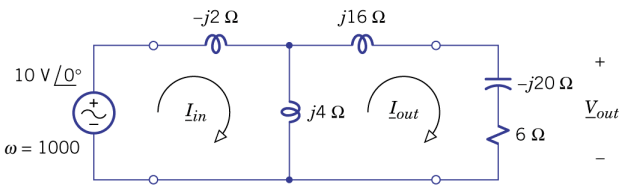


Figure 8.24.

Example 8.7: Transformer circuit analysis with a tee network

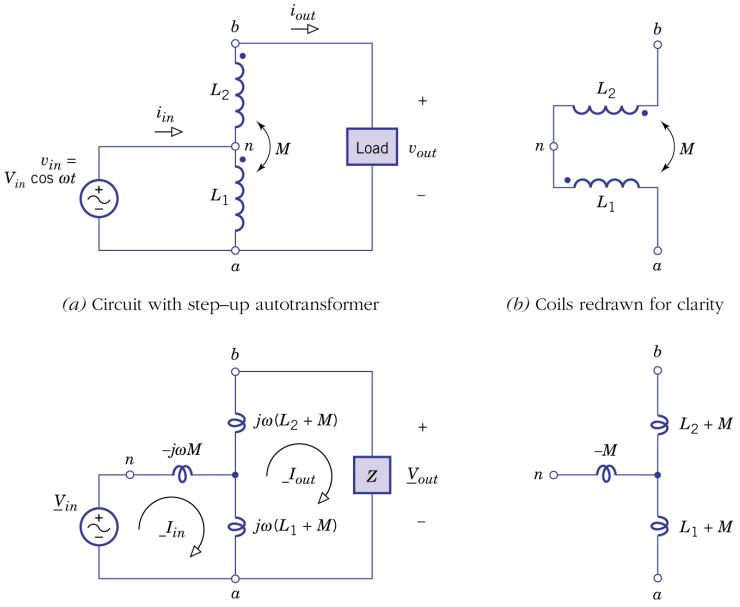


(a) Circuit with magnetic coupling



(b) Frequency–domain diagram with the equivalent tee network Figure 8.25.

Example 8.8: Step-Up Autotransformer



(a) Circuit with step–up autotransformer

(b) Coils redrawn for clarity

(d) Frequency–domain diagram

(c) Equivalent tee network

Figure 8.26.

- Equivalent networks with an ideal transformer to represent mutual inductance.

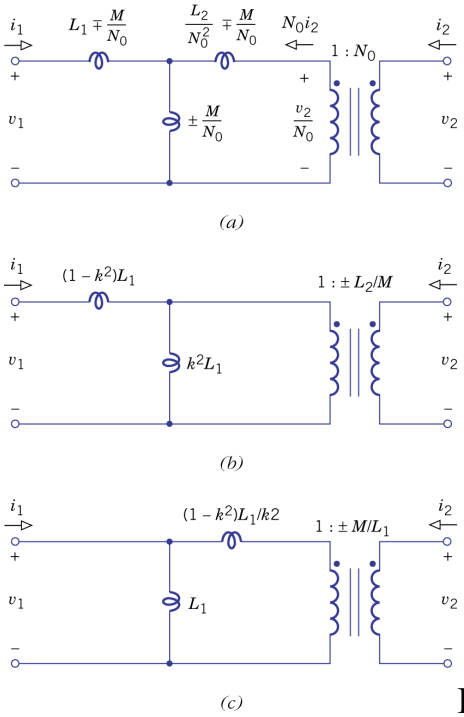
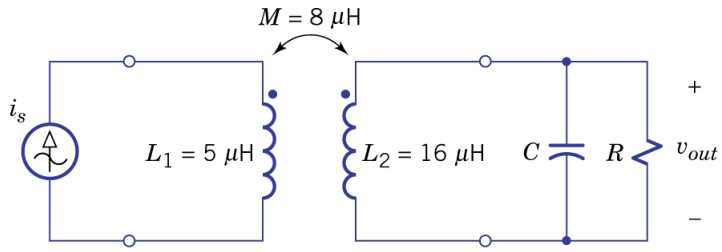
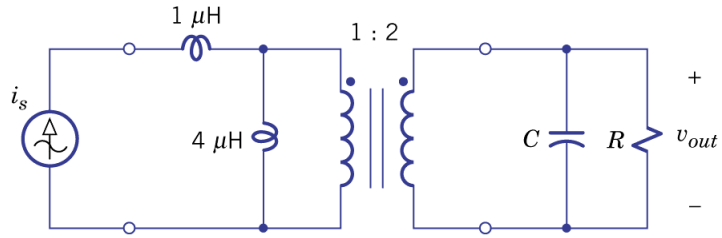


Figure 8.27.

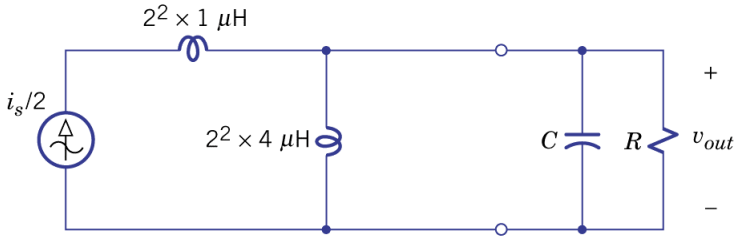
Example 8.9: Design of a Tuned Amplifier



(a) Model of "tuned" transistor amplifier



(b) Equivalent circuit with an ideal transformer



(c) Referred source network in the secondary

Figure 8.28.