Chapter 30 – Inductors and self Inductance

Inductance is to Capacitance what current is to a stationary charge. They are both defined relative to the voltage produced.

Goals for Chapter 30

- Mutual inductance
- Self-inductance
- Magnetic-field energy
- R-L circuits
- L-C circuits
- L-R-C circuits

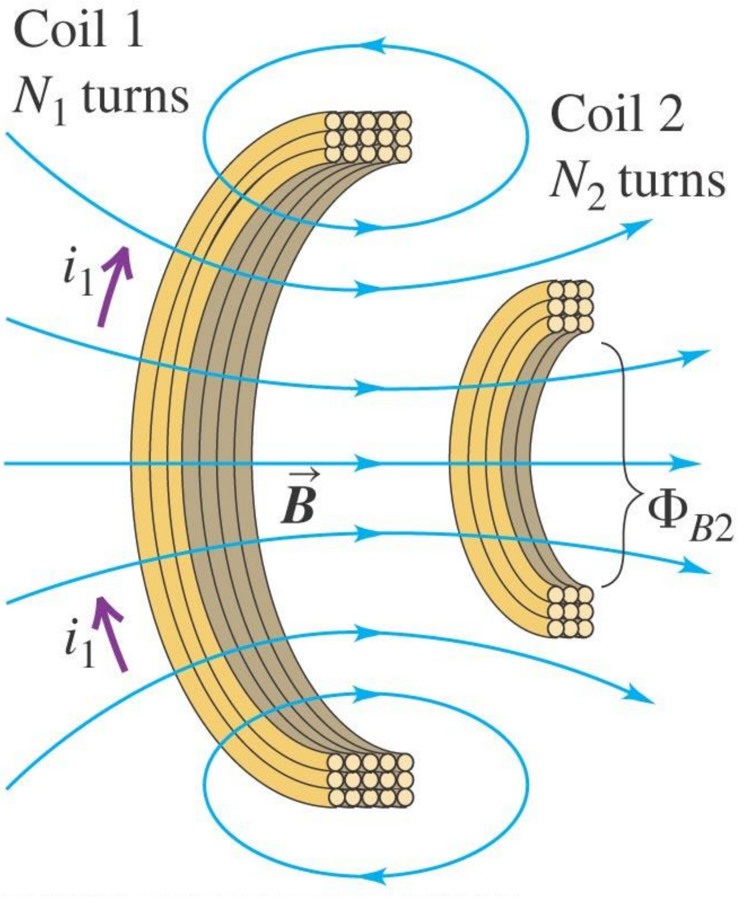
Introduction

- A charged coil can create a field that will induce a current in a neighboring coil.
- Inductance can allow a sensor to trigger the traffic light to change when the car arrives at an intersection..

Mutual inductance

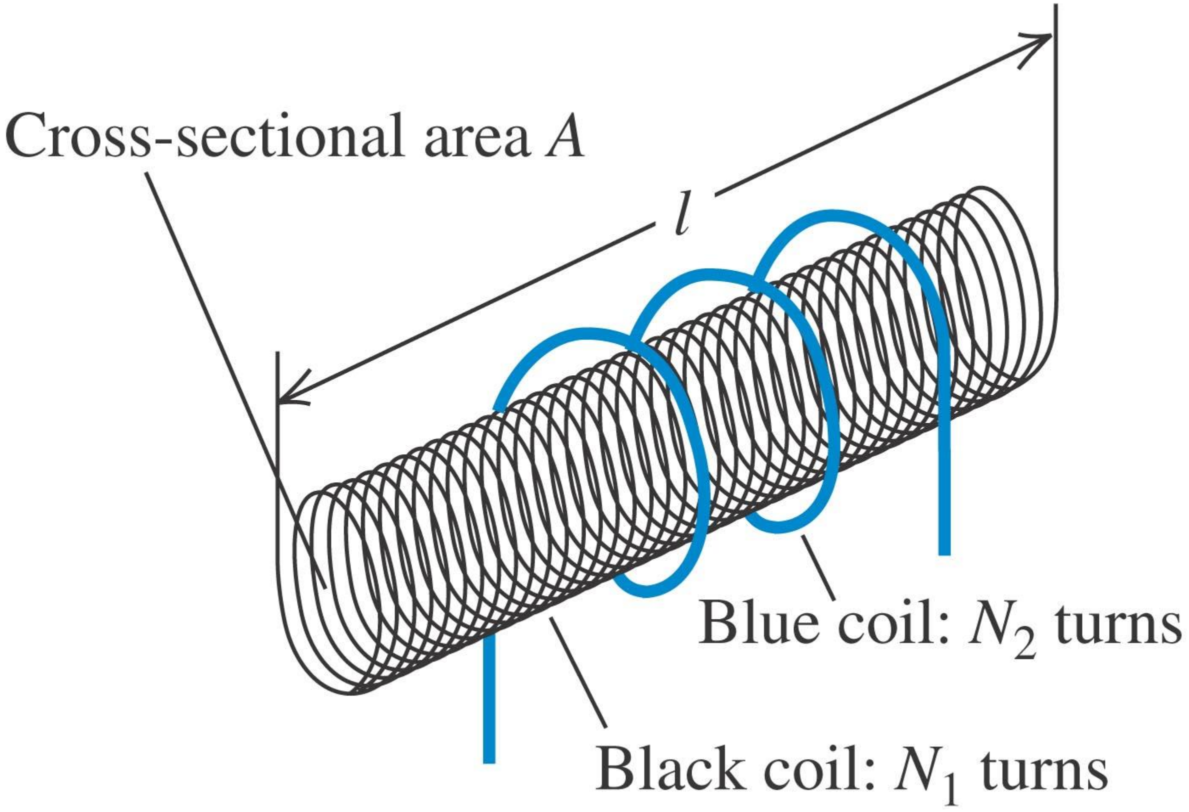
- A coil in one device generates a field that creates a current in a neighboring coil. This is the basis for a transformer.

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



Mutual inductance—examples

- Two solenoid coil one with N1 turns and the other with N2 turns
- How do they interact?



Self and Mutual Inductance

- We define inductance L as magnetic flux/current
- Here N is the number of coil turns

$$L = \frac{N\Phi}{i} \qquad N\Phi = Li$$

- In multiple coil systems there is magnetic coupling between the coils – hence Mutual inductance M

$$N_1\Phi_1 = L_{11}i_1 + L_{12}i_2,$$

$$N_2\Phi_2 = L_{21}i_1 + L_{22}i_2.$$

- Here $L_{12} = L_{21} = M$
- Energy stored in multiple coils $W = \frac{1}{2}\sum_{m,n=1}^{K} L_{m,n}i_m i_n$

EMF and Flux change

- The time derivative of the magnetic flux = EMF

$$N\frac{d\Phi}{dt} = L\frac{di}{dt} + \frac{dL}{dt}i$$

- In general dL/dt = 0 (the inductance does not change) – This is NOT always true – rail gun example
- If L = constant then:

$$N\frac{d\Phi}{dt} = L\frac{di}{dt}$$

$$N\frac{d\Phi}{dt} = -\mathcal{E} = v$$

$$\frac{di}{dt} = \frac{v}{L}$$

$$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau + i(0)$$

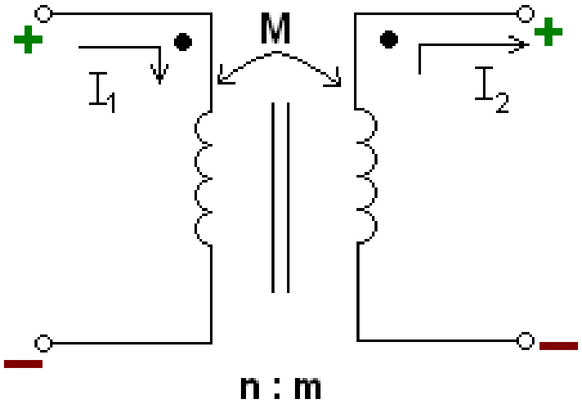
Energy in inductors

- We can related the power (I*V) to inductance and current change

$$\frac{di}{dt} = \frac{v}{L} \qquad Li\frac{di}{dt} = \frac{d}{dt}\tfrac{1}{2}Li^2 = iv$$

- Hence we can equate W_L = energy = ½ L I^2
- Note the similarity to energy in a capacitor
- W_C = ½ C V^2
- Where does the energy reside?
- In the magnetic and electric fields

Mutual Inductance and Self Inductance



$$M = k\sqrt{L_1 L_2}$$

- k is the *coupling coefficient* and $0 \leq k \leq 1$,
- L_1 is the inductance of the first coil
- L_2 is the inductance of the second coil.

Induced voltage with self and mutual inductance

$$V_1 = L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt}$$

- V_1 is the voltage across the inductor of interest
- L_1 is the inductance of the inductor of interest
- dI_1 / dt is through the inductor of interest
- dI_2 / dt is through the inductor that is coupled to the first inductor
- M is the mutual inductance.

Transformers – Voltage Ratios

- Basically a mutual inductance device between two inductors – primary and secondary

$$V_s = V_p \frac{N_s}{N_p}$$

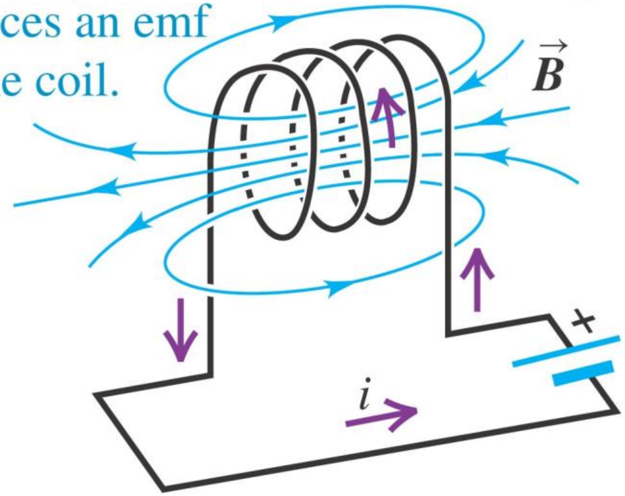
- V_s is the voltage across the secondary inductor
- V_p is the voltage across the primary inductor (the one connected to a power source)
- N_s is the number of turns in the secondary inductor
- N_p is the number of turns in the primary inductor.

Transformers – Current Ratios

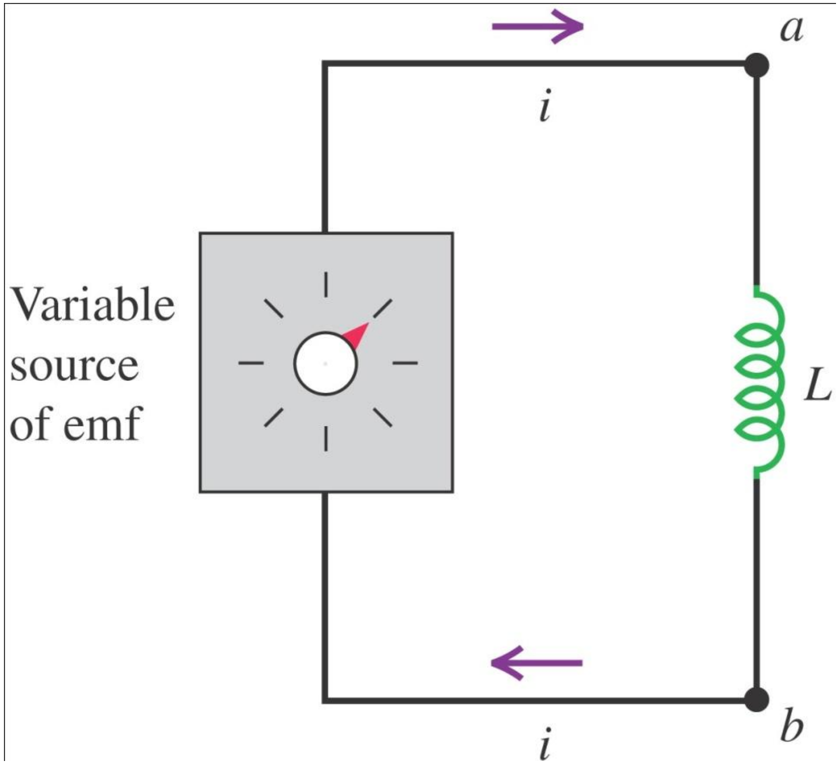
$$I_s = I_p \frac{N_p}{N_s}$$

- I_s is the current through the secondary inductor
- I_p is the current through the primary inductor (the one connected to a power source)
- N_s is the number of turns in the secondary inductor
- N_p is the number of turns in the primary inductor
- Note – Power $I_P V_P = I_S V_S$ is conserved in an IDEAL transformer
- In real transformers there is loss - heat

Self-inductance

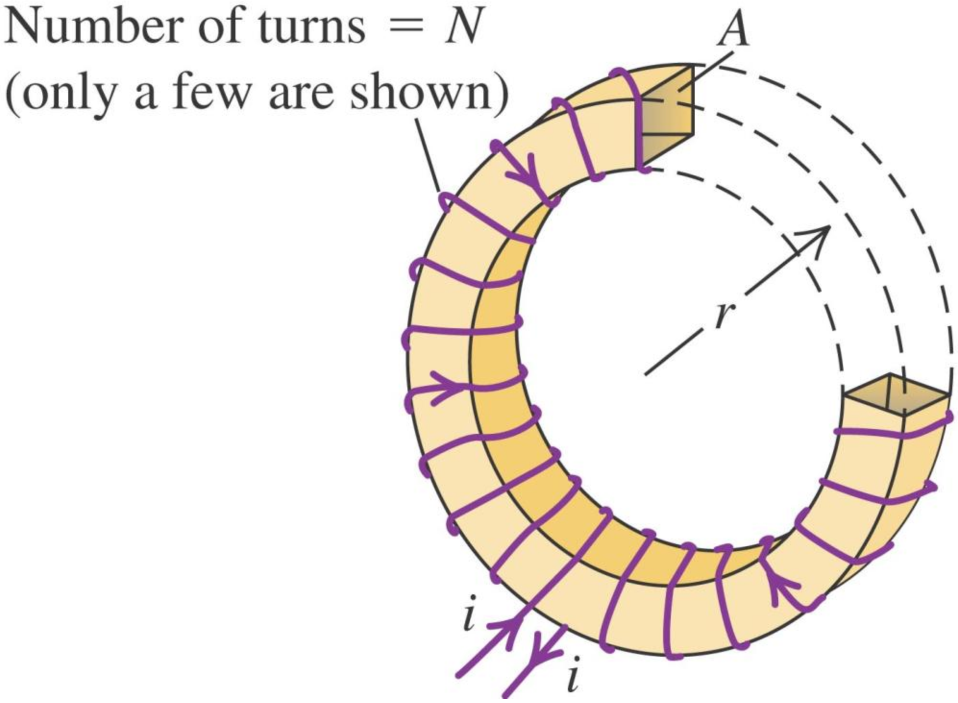


Self-inductance: If the current *i* in the coil is changing, the changing flux through the coil induces an emf in the coil.

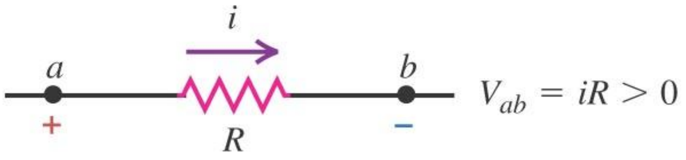


Applications and calculations

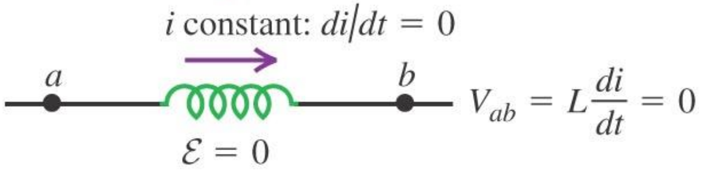
- There are many cases where self and mutual inductance are important.



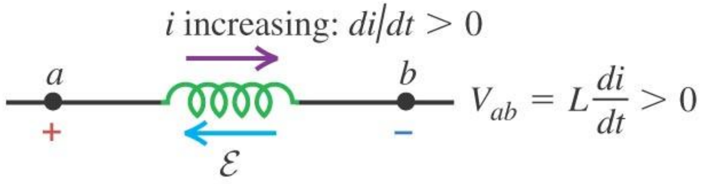
(a) Resisitor with current i flowing from a to b: potential drops from a to b.



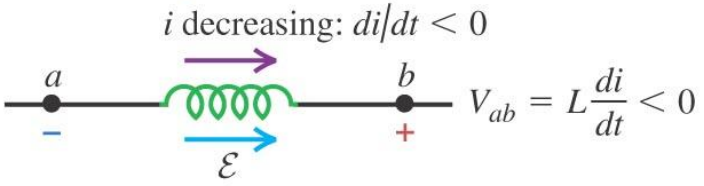
(b) Inductor with *constant* current i flowing from a to b: no potential difference.



(c) Inductor with *increasing* current i flowing from a to b: potential drops from a to b.

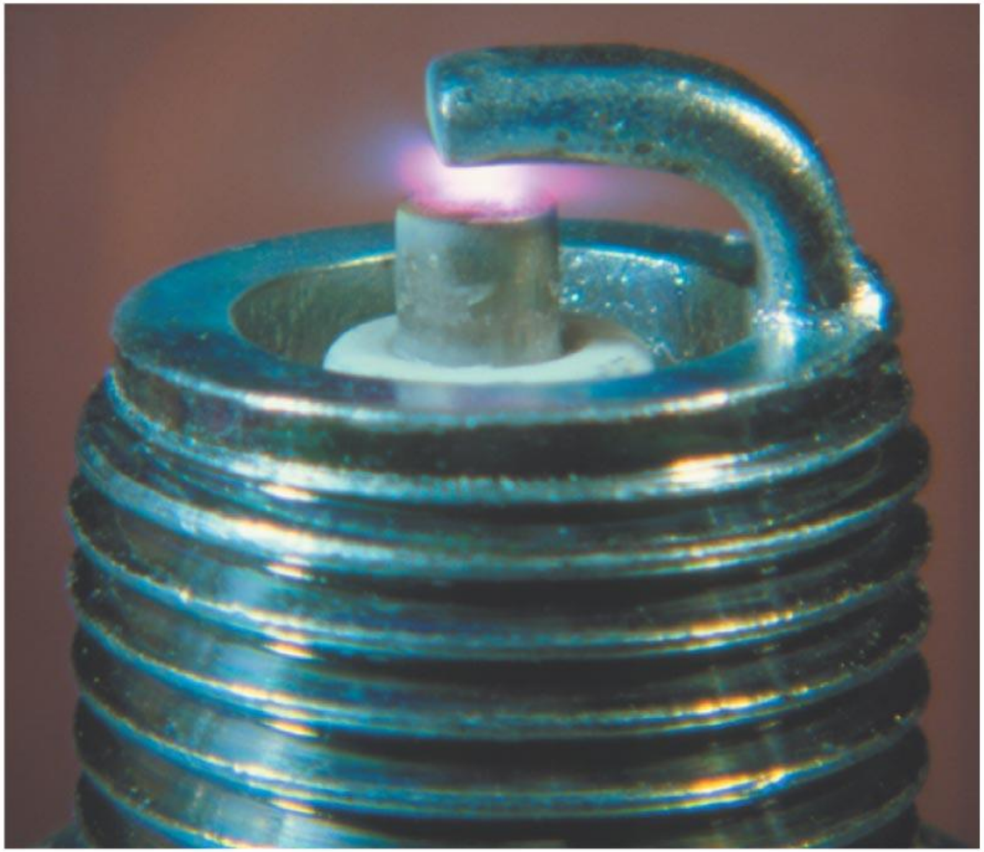


(d) Inductor with *decreasing* current i flowing from a to b: potential increases from a to b.



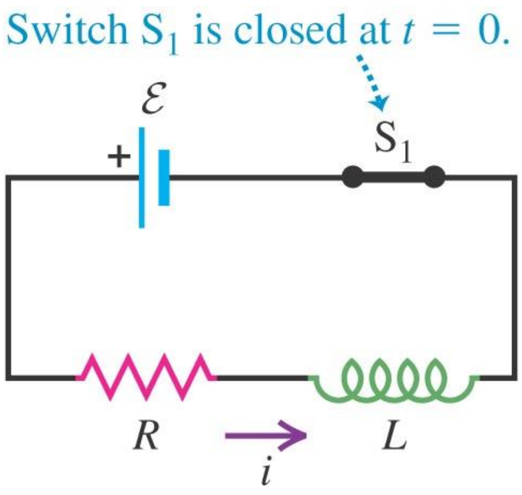
Magnetic field energy

- Your car uses the collapse of the magnetic field in a transformer to create the spark in your sparkplug.

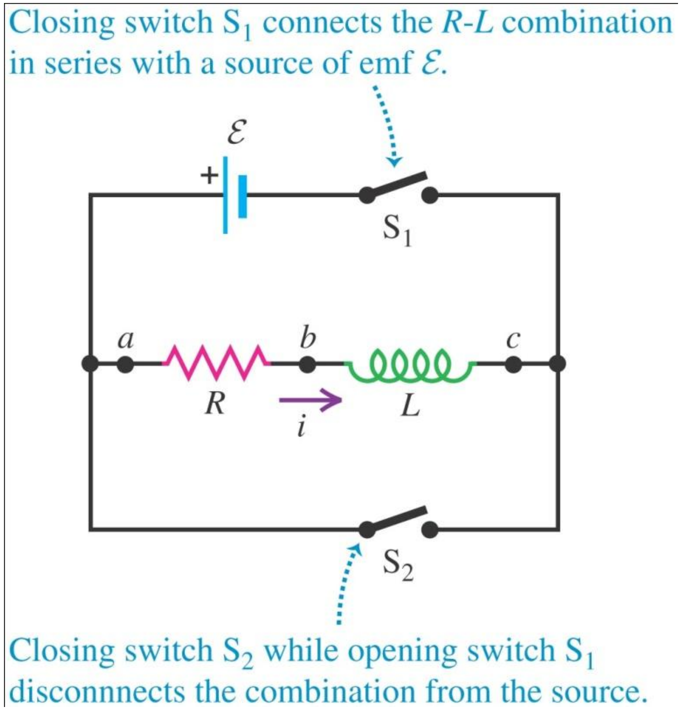


The R-L circuit

- The LR circuit is like the RC circuit from capacitance. In a capacitor energy was stored in the electric field. In an inductor energy is stored in the magnetic field.

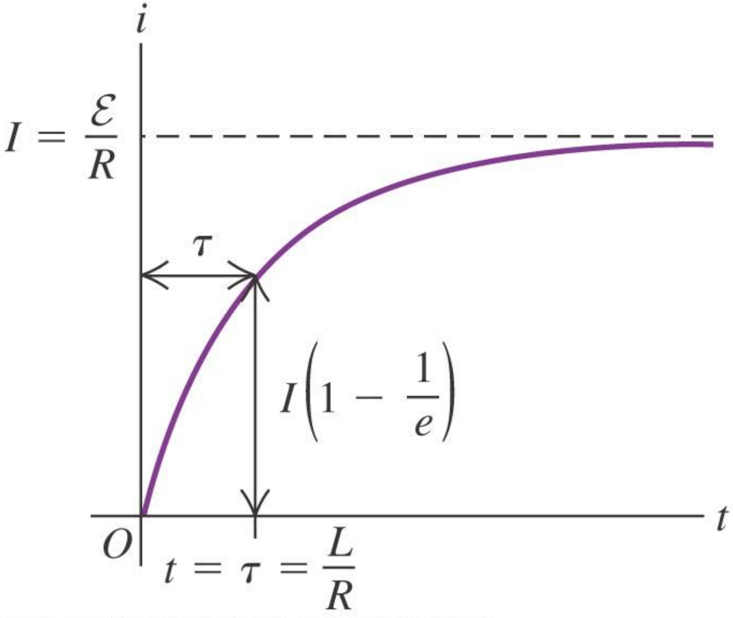


Switch S_1 is closed at $t = 0$.



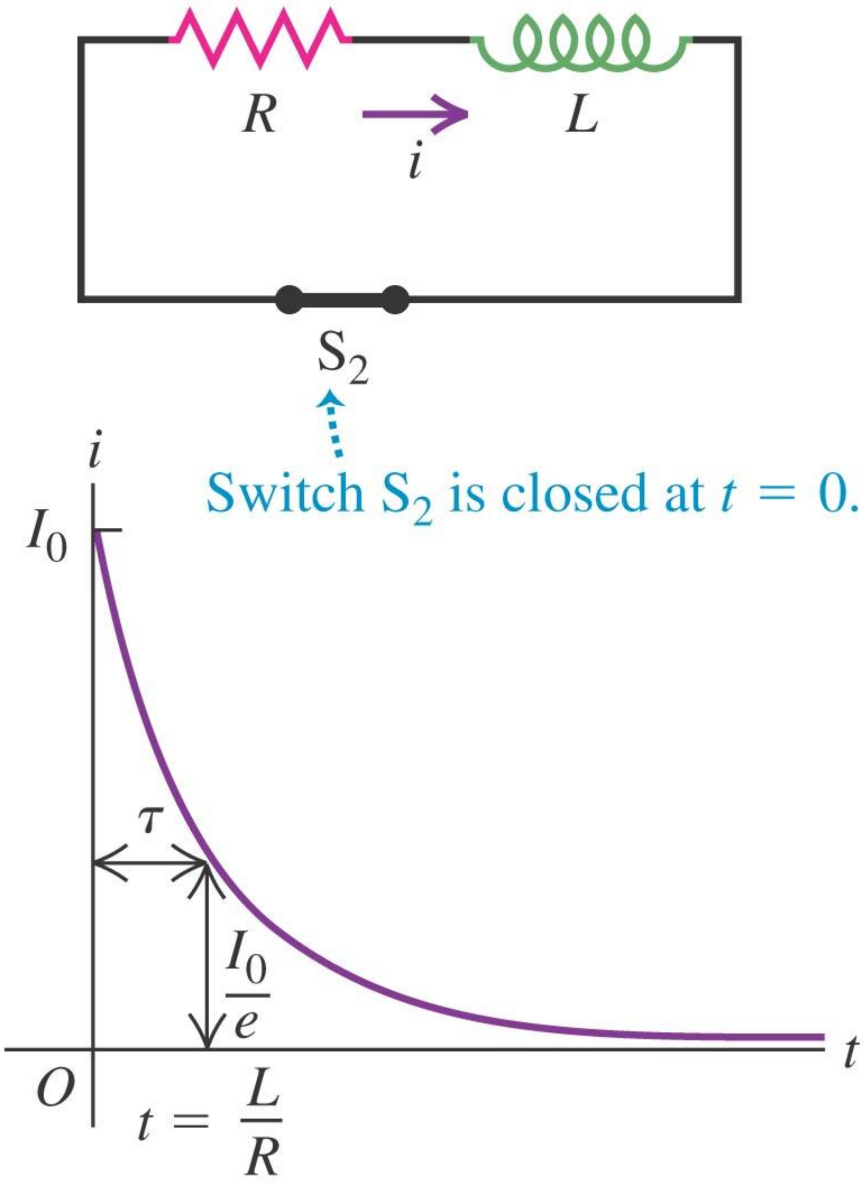
Closing switch S_1 connects the R-L combination in series with a source of emf $\mathcal{E}$.

Closing switch S_2 while opening switch S_1 disconnects the combination from the source.



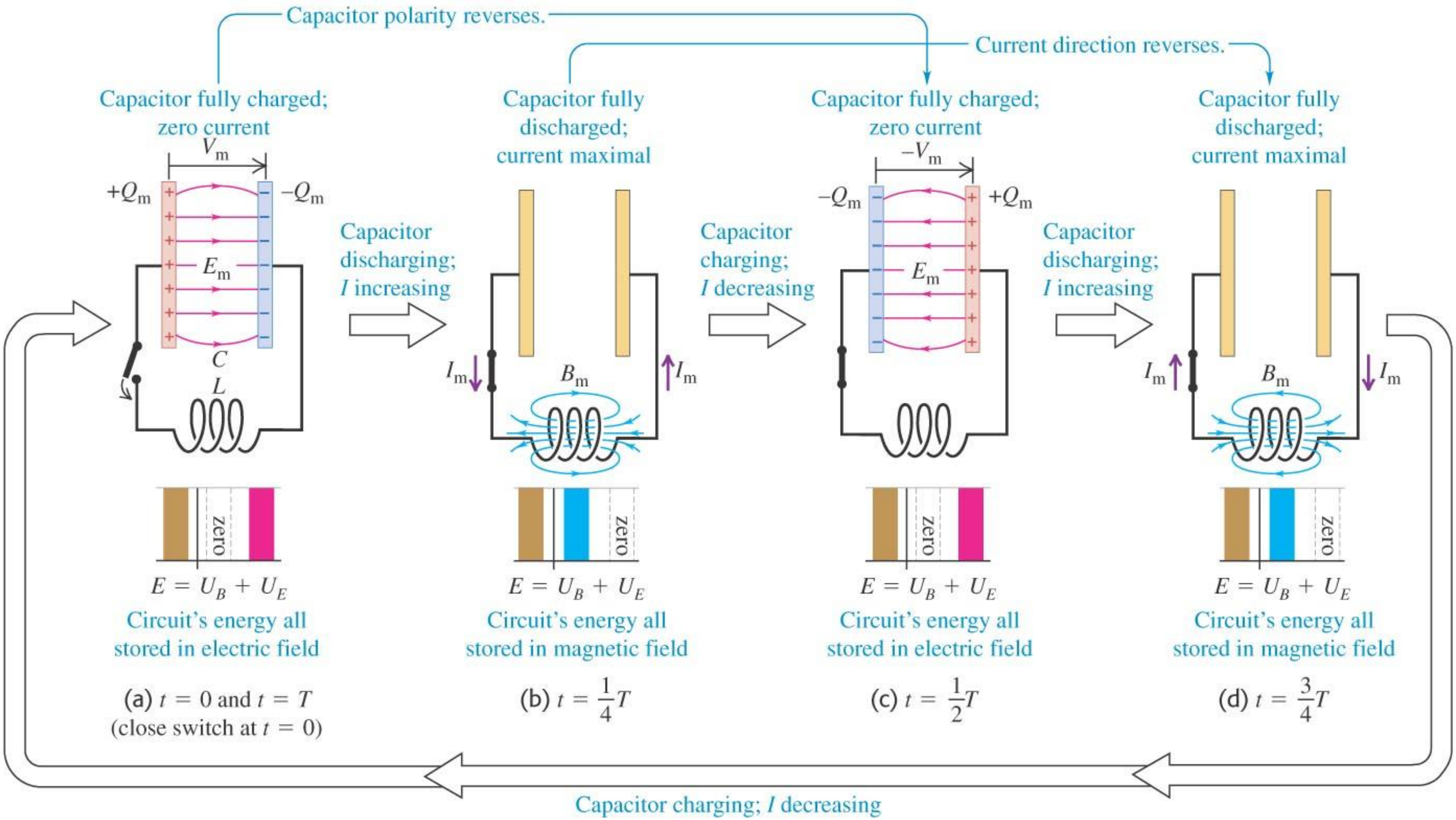
R-L circuit II

- LR and RC circuits both have a time constant. For RC τ= RC for LR τ = L/R
- Recall reactance for a capacitor and inductor are:
- **$Z_C = 1/ i \omega C$**
- **$Z_L = i \omega L$**



The L-C circuit

- **Electric and magnetic field energy transfer**



(a) $t = 0$ and $t = T$ (close switch at $t = 0$)

(b) $t = \frac{1}{4}T$

(c) $t = \frac{1}{2}T$

(d) $t = \frac{3}{4}T$

Applications and comparisons

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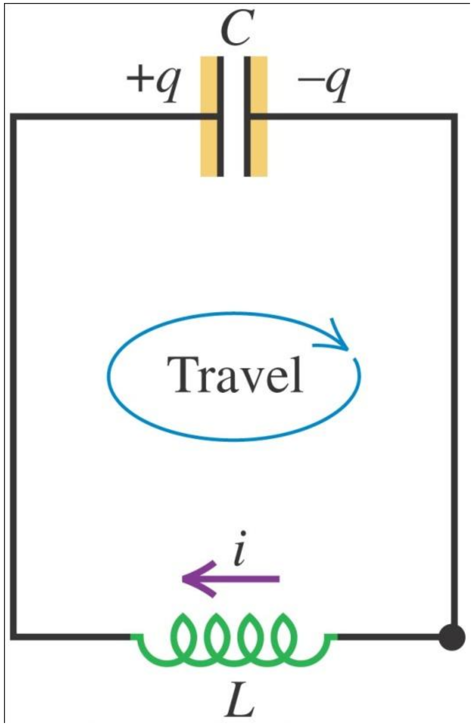


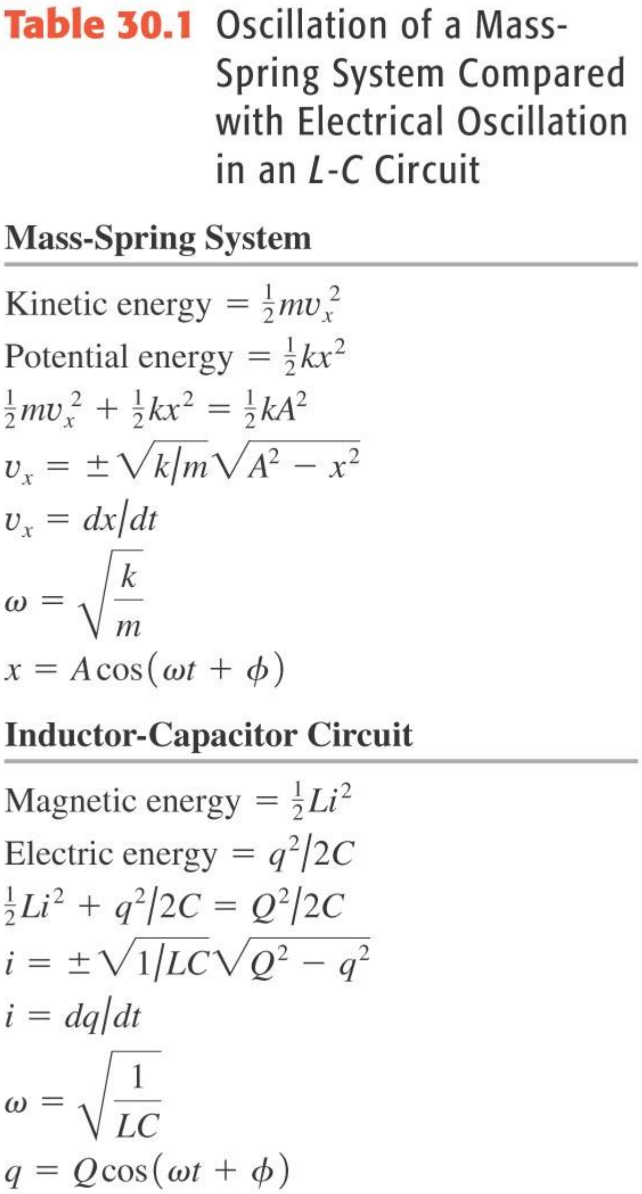


Table 30.1 Oscillation of a Mass-Spring System Compared with Electrical Oscillation in an *L*-*C* Circuit

Mass-Spring System

Kinetic energy $= \frac{1}{2}mv_x^2$

Potential energy $= \frac{1}{2}kx^2$

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$

$v_x = dx/dt$

$\omega = \sqrt{\frac{k}{m}}$

$x = A\cos(\omega t + \phi)$

Inductor-Capacitor Circuit

Magnetic energy $= \frac{1}{2}Li^2$

Electric energy $= q^2/2C$

$\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$

$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$

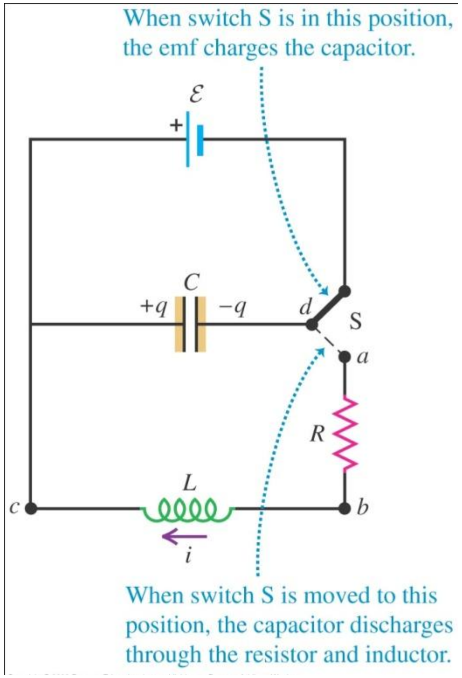
$i = dq/dt$

$\omega = \sqrt{\frac{1}{LC}}$

$q = Q\cos(\omega t + \phi)$

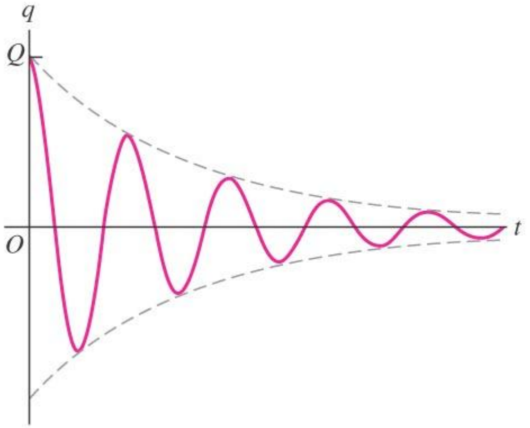
The L-R-C circuit - Dissipation

- In an ideal L-R-C circuit the only dissipation is through the resistor. The L and C have no dissipation and are lossless. The resistor converts the electrical energy into heat. Thus decay of voltage and current.

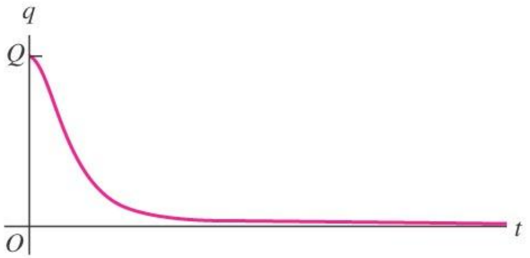

When switch S is in this position, the emf charges the capacitor.

When switch S is moved to this position, the capacitor discharges through the resistor and inductor.

(a) Underdamped circuit (small resistance R)



(b) Critically damped circuit (larger resistance R)



(c) Overdamped circuit (very large resistance R)