APPENDIX E

VECTOR OPERATORS

The gradient of a scalar ($\nabla\Phi$), the divergence of a vector ($\nabla \cdot \mathbf{F}$), and the curl of a vector ($\nabla \times \mathbf{F}$) are listed below for rectangular, cylindrical, and spherical coordinates.

RECTANGULAR COORDINATES

$$\nabla\Phi = \mathbf{a}_x \frac{\partial\Phi}{\partial x} + \mathbf{a}_y \frac{\partial\Phi}{\partial y} + \mathbf{a}_z \frac{\partial\Phi}{\partial z}$$

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \mathbf{F} = \mathbf{a}_x \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) + \mathbf{a}_y \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) + \mathbf{a}_z \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)$$

CYLINDRICAL COORDINATES

$$\nabla\Phi = \mathbf{a}_\rho \frac{\partial\Phi}{\partial\rho} + \mathbf{a}_\phi \frac{1}{\rho}\frac{\partial\Phi}{\partial\phi} + \mathbf{a}_z \frac{\partial\Phi}{\partial z}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \mathbf{F} = \mathbf{a}_\rho \left(\frac{1}{\rho}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z} \right) + \mathbf{a}_\phi \left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial\rho} \right) + \mathbf{a}_z \left(\frac{1}{\rho}\frac{\partial(\rho F_\phi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial F_\rho}{\partial\phi} \right)$$

SPHERICAL COORDINATES

$$\nabla\Phi = \mathbf{a}_r \frac{\partial \Phi}{\partial r} + \mathbf{a}_\theta \frac{1}{r}\frac{\partial \Phi}{\partial \theta} + \mathbf{a}_\phi \frac{1}{r\sin\theta}\frac{\partial \Phi}{\partial \phi}$$

$$\nabla \cdot F = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(F_\theta \sin\theta)$$

$$+\frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

$$\nabla \times F = \frac{\mathbf{a}_r}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(F_\phi \sin\theta) - \frac{\partial F_\theta}{\partial\phi}\right) + \frac{\mathbf{a}_\theta}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial}{\partial r}(rF_\phi)\right)$$

$$+\frac{\mathbf{a}_\phi}{r}\left(\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial\theta}\right)$$