

APPENDIX D

MATH IDENTITIES

TRIGONOMETRIC

$$\varepsilon^{jx} = \cos x + j \sin x$$

$$\varepsilon^{-jx} = \cos x - j \sin x$$

$$\sin x = \frac{\varepsilon^{jx} - \varepsilon^{-jx}}{2j}$$

$$\cos x = \frac{\varepsilon^{jx} + \varepsilon^{-jx}}{2}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x \cos^2 x = \frac{\sin^2 2x}{4}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

HYPERBOLIC

$$\varepsilon^x = \cosh x + \sinh x$$

$$\varepsilon^{-x} = \cosh x - \sinh x$$

$$\sinh x = \frac{\varepsilon^x - \varepsilon^{-x}}{2}$$

$$\cosh x = \frac{\varepsilon^x + \varepsilon^{-x}}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

TRIGONOMETRIC-HYPERBOLIC RELATIONSHIPS

$$\sinh jx = j \sin x \quad \sin jx = j \sinh x$$

$$\cosh jx = \cos x \quad \cos jx = \cosh x$$

$$\tanh jx = j \tan x \quad \tan jx = j \tanh x$$

$$\sinh(x \pm jy) = \sinh x \cos y \pm j \cosh x \sin y$$

$$\cosh(x \pm jy) = \cosh x \cos y \pm j \sinh x \sin y$$