6. Negative Feedback in Single-Transistor Circuits (continued)

6.3. *Effect of negative feedback on the input and output impedances*

To determine the effect of negative feedback on the input and output impedances let us first determine the effect of feedback on the impedance at an arbitrary port x [see Fig. 1(a)]. (This impedance will be equal to the circuit input or output impedances if the port is chosen at the circuit input or at the circuit load.)

To find the impedance R_x at port x, we first connect a voltage test source v_t to the port and obtain v_t as a function of the circuit return ratio. Since the port x is short-circuited when the return ratio is calculated, we call it as RR_{SC}:

$$s_\varepsilon = s'_{\varepsilon v} + s''_{\varepsilon v} = G_v v_t - s_\varepsilon RR_{SC}, \quad \Rightarrow v_t = \frac{s_\varepsilon (1 + RR_{SC})}{G_v} \tag{1}$$

where G_v is the input transmission from the voltage test source to the control terminals of the dependent source.

We second connect a current test source $i_t = v_t/R_x$ [see Fig. 1(b)] to the port and obtain i_t as a function of the circuit return ratio. Since the port is open-circuited when this return ratio is calculated, we call it as RR_{OC}:

$$s_\varepsilon = s'_{\varepsilon i} + s''_{\varepsilon i} = G_i\, i_t - s_\varepsilon\, RR_{OC}, \quad \Rightarrow i_t = \frac{s_\varepsilon (1 + RR_{OC})}{G_i} \tag{2}$$

where G_i is the input transmission from the current test source to the control terminals of the dependent source. Note that s_ε in (1) and (2) have the same values, because i_t and v_t in Figs. 1(a) and (b) are the same.

From (1) and (2), R_x can be expressed as follows:

$$R_x = \frac{v_t}{i_t} = \frac{G_i}{G_v}\frac{1 + RR_{SC}}{1 + RR_{OC}}. \tag{3}$$

To find the ratio of the transmissions G_i and G_v in (3), we suppress in Fig. 1(c) the dependent source and connect to port x a current source $i'_t = v_t/R'_x$, where R'_x represents the open-loop port impedance for a_{OL}=0. [Note that suppressing

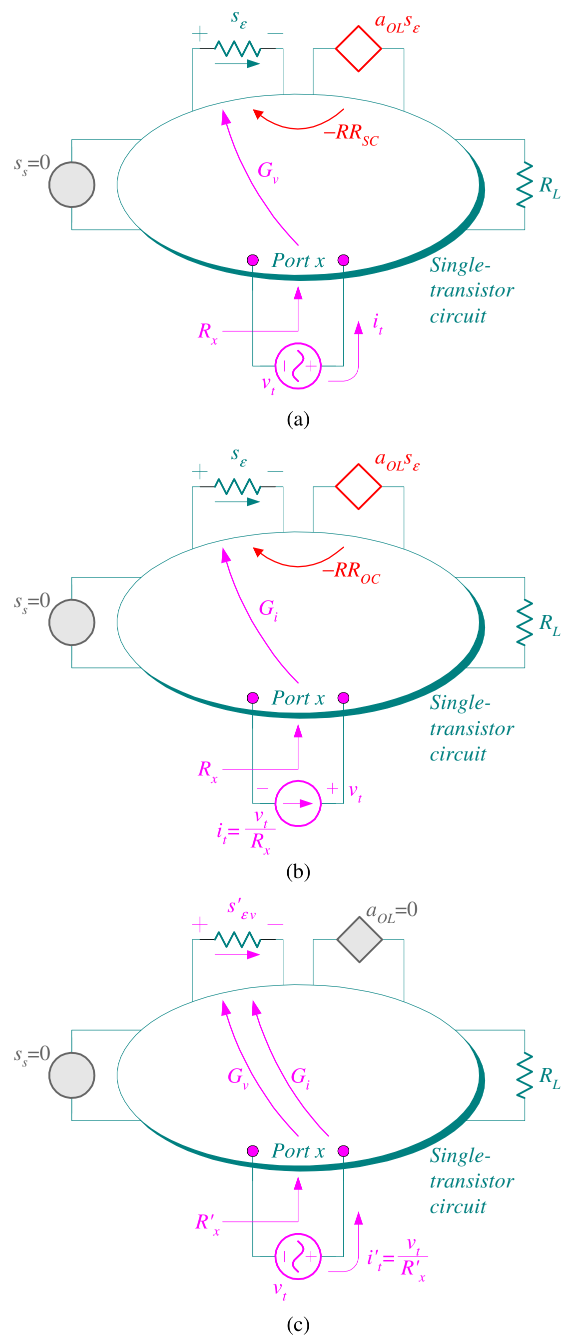

Fig. 1. Finding the impedance at an arbitrary port x.

the dependent source changes the circuit and, therefore, changes the impedance at port *x*. To keep at this port the same voltage v_t that we had in Figs. 1(a) and (b), we have to change the test current.]

$$G_v = \frac{s'_{\varepsilon v}}{v_t}; \quad G_i = \frac{s'_{\varepsilon v}}{i'_t}$$
$$\Rightarrow \frac{G_i}{G_v} = \frac{v_t}{s'_{\varepsilon v}} \frac{s'_{\varepsilon v}}{i'_t} = \frac{v_t}{i'_t} = R'_x \quad (4)$$

Considering (3) and (4), we obtain Blackman's formula for the closed-loop impedance:

$$R_x = R'_x \frac{1 + RR_{SC}}{1 + RR_{OC}} = R'_x \frac{DSF_{SC}}{DSF_{OC}} . \quad (5)$$

Equation (5) reveals the effect of feedback on the closed-loop impedance at an arbitrary port of a single-transistor circuit: the closed-loop impedance R_x can be higher, equal, or lower than the open-loop impedance R'_x depending on the ratio of the amounts of feedback, one, DSF_{SC}, obtained when the port is short-circuited and the other, DSF_{OC}, obtained when the port is open-circuited.

According to (5), the effect of feedback is most pronounced when either RR_{SC} or RR_{OC} equals zero. Let us see it in the following examples.

Since we are interested in the effect of the feedback, or *DSF*, that is present in the original circuit, we will express in this course, wherever possible, — *and in the exam as well* — any closed-loop impedance through either the input or output open-loop impedances and the *DSF* of the entire circuit and *not* of a part of it.

Finding the closed-loop input and output impedances: example 1

To analyze the effect of feedback on the closed-loop input and output impedances of the CC amplifier (see Fig. 2), we first find the open-loop input and output impedances:

$$\begin{aligned} R'_{in} &= r_s + h_{ie} + R_E \parallel r_o \\ R'_o &= R_E \parallel r_o \parallel (r_s + h_{ie}) \end{aligned} \quad . \qquad (1)$$

We second find the return ratios RR_{SC} and RR_{OC} for the input

$$\begin{aligned} RR_{SC(in)} &\equiv \left. \frac{-s''_\varepsilon}{s_\varepsilon} \right|_{SC(in)} = \left. \frac{-i''_b}{i_b} \right|_{\substack{i_b=1 \\ SC(in)}} \\ &= h_{fe} \frac{R_E \parallel r_o}{R_E \parallel r_o + r_s + h_{ie} + 0} = RR \\ RR_{OC(in)} &\equiv \left. \frac{-s''_\varepsilon}{s_\varepsilon} \right|_{OC(in)} = \left. \frac{-i''_b}{i_b} \right|_{\substack{i_b=1 \\ OC(in)}} \\ &= h_{fe} \frac{R_E \parallel r_o}{R_E \parallel r_o + r_s + h_{ie} + \infty} = 0 \end{aligned} \quad . \qquad (2)$$

and output ports

$$\begin{aligned} RR_{SC(o)} &\equiv \left. \frac{-s''_\varepsilon}{s_\varepsilon} \right|_{SC(o)} = \left. \frac{-i''_b}{i_b} \right|_{\substack{i_b=1 \\ SC(o)}} \\ &= h_{fe} \frac{R_E \parallel r_o \parallel 0}{R_E \parallel r_o \parallel 0 + r_s + h_{ie}} = 0 \\ RR_{OC(o)} &\equiv \left. \frac{-s''_\varepsilon}{s_\varepsilon} \right|_{OC(o)} = \left. \frac{-i''_b}{i_b} \right|_{\substack{i_b=1 \\ OC(o)}} \\ &= h_{fe} \frac{R_E \parallel r_o \parallel \infty}{R_E \parallel r_o \parallel \infty + r_s + h_{ie}} = RR \end{aligned} \quad . \qquad (3)$$

Considering (1)-(3), we finally obtain the closed-loop input and output impedances according to Blackman's formula (note that the negative feedback increases by the circuits *DSF* the closed-loop input impedance relative to its open-loop value and decreases to the same extent the closed-loop output impedance relative to its open-loop value):

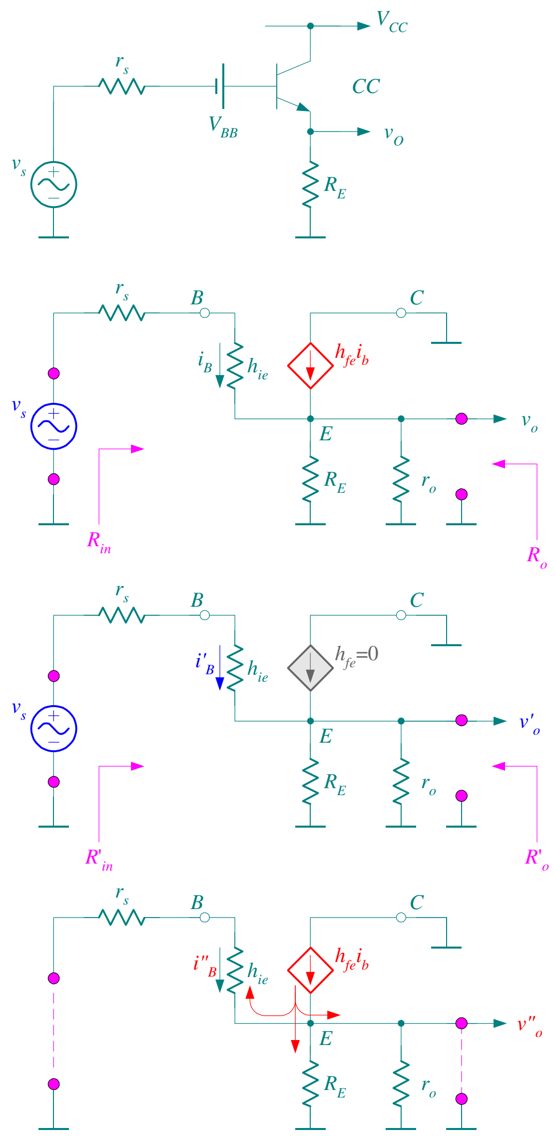

Fig. 2. Example 1: finding closed-loop input and output impedances for a CC amplifier.

$$\begin{aligned} R_{in} &= (r_s + h_{ie} + R_E \parallel r_o)\left(1 + h_{fe} \frac{R_E \parallel r_o}{R_E \parallel r_o + r_s + h_{ie}}\right) \\ &= (r_s + h_{ie} + R_E \parallel r_o)(1 + RR) = R'_{in}\ DSF \\ R_o &= [R_E \parallel r_o \parallel (r_s + h_{ie})] \frac{1}{1 + h_{fe} \dfrac{R_E \parallel r_o}{R_E \parallel r_o + r_s + h_{ie}}} \\ &= [R_E \parallel r_o \parallel (r_s + h_{ie})] \frac{1}{1 + RR} = R'_o \frac{1}{DSF} \end{aligned} \quad . \qquad (4)$$

Finding the closed-loop input and output impedances: example 2

To find the closed-loop impedance $R_{in}(r_s)$ seen by r_s in Fig. 3, we simply note that

$$R_{in} = R_{in}(r_s) + r_s \quad , \quad \Rightarrow R_{in}(r_s) = R_{in} - r_s \qquad (1)$$

or, considering (4) in the previous subsection,

$$R_{in}(r_s) = R'_{in}\ DSF - r_s\ . \qquad (2)$$

To find the closed-loop impedance $R_o(R_E)$ seen by R_E in Fig. 3, we simply note as well that

$$R_o = R_o(R_E) \parallel R_E = \frac{R_E\ R_o(R_E)}{R_E + R_o(R_E)}$$

$$\Rightarrow R_o R_E + R_o R_o(R_E) = R_E\ R_o(R_E)\ . \qquad (3)$$

$$\Rightarrow R_o(R_E) = \frac{R_o R_E}{R_E - R_o}$$

or, considering (4) in the previous subsection,

$$R_{in}(R_E) = \frac{R'_o \dfrac{1}{DSF} R_E}{R_E - R'_o \dfrac{1}{DSF}}\ . \qquad (4)$$

Thus, we have expressed the $R_{in}(r_s)$ and $R_o(R_E)$ closed-loop impedances through either the input or output open-loop impedances and *DSF* of the entire circuit.

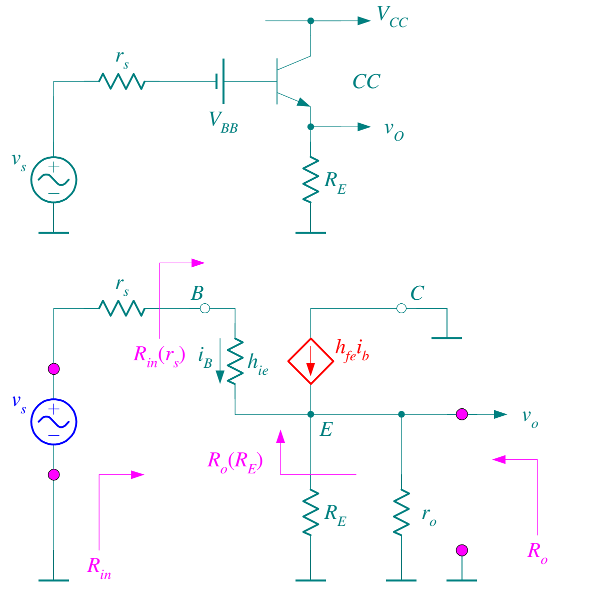

Fig. 3. Example 2: finding the impedances $R_{in}(r_s)$ seen by r_s and $R_{in}(R_E)$ seen by R_E.

References

[1] P. R. Gray, P. J. Hurst, S. H. Lewis, and R. G. Meyer., *Analysis and Design of Analog Integrated Circuits.* (4th Edition.)

Important!

Read the next lecture notes (Lecture 10: Positive-Feedback Oscillators) *before* attending the class! Reserve enough time for this work! Refer [2] while reading.

Those students who will attend the class without reading the lecture 10 notes in advance will not be allowed to ask any questions.