6. Negative Feedback in Single-Transistor Circuits

Our aim is to study the effect of negative feedback on the small-signal gain and the small-signal input and output impedances of the single-transistor circuits. Our study will be based on generic functional models of the circuits (see Fig. 1).

6.1. *Single-transistor circuits with no feedback*

Let us start from analyzing the small-signal gain of a circuit with no feedback [see Fig. 1(a)]

$$A = G\,A_{OL}\,, \tag{1}$$

where G is the small-signal input transmission

$$G \equiv \frac{s_\varepsilon}{s_s}\,, \tag{2}$$

s_s is the signal source value, s_ε is the signal at the control port of the dependent source of the transistor model, and A_{OL} is the small-signal open-loop gain

$$A_{OL} \equiv \frac{s_o}{s_\varepsilon}\,. \tag{3}$$

Equation (1) shows that the small-signal gain A is directly proportional to the small-signal open-lop gain A_{OL}. This may be a serious disadvantage because A_{OL} depends on the transistor small-signal parameters, which are very sensitive to the transistor technology and temperature. On the other hand, this can be an advantage if the maximum gain is required and its exact value is not important. We will also see later when studying positive feedback, that the circuit with no feedback is always stable provided its G and A_{OL} gains are stable. In the next course on electronic analog circuit, you will see as well that adding negative feedback can limit the frequency range of the circuit.

6.2. *Single-transistor circuits with feedback*

Let us now find the closed-loop gain of a single-transistor circuit with feedback [see Fig. 1(b)]

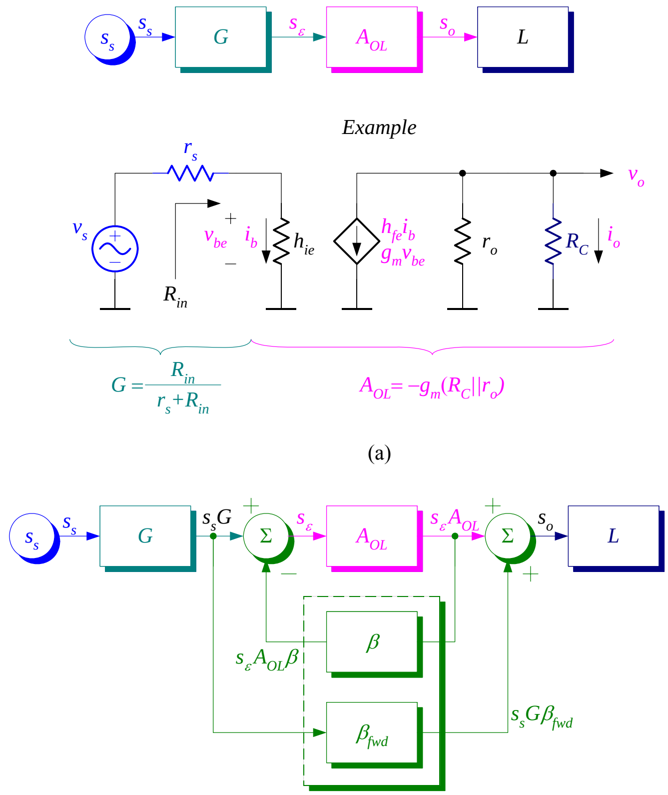

Fig. 1. Functional (block) diagrams of electronic circuits (a) without and (b) with feedback.

$$A_{CL} \equiv \frac{s_o}{s_s} = \frac{s_\varepsilon A_{OL}}{\dfrac{s_\varepsilon + s_\varepsilon A_{OL}\beta}{G}} + \frac{s_s G\beta_{fwd}}{s_s}$$

$$= G\frac{A_{OL}}{1 + A_{OL}\beta} + G\beta_{fwd} = G\frac{A_{OL}}{1+RR} + D\,, \tag{4}$$

$$= G\frac{A_{OL}}{DSF} + D$$

where β is the small-signal *feedback* transmission of the feedback network, β_{fwd} is the small-signal *feedforward* transmission of the feedback network,

$$RR \equiv A_{OL}\beta \tag{5}$$

is the return ratio,

$$D \equiv G\beta_{fwd} \tag{6}$$

is the small-signal direct transmission, and

$$DSF \equiv 1 + A_{OL}\beta \tag{7}$$

is the desensitivity factor or the amount of feedback.

We will define a feedback as negative or positive if it decreases or increases, correspondingly, the closed loop gain A_{CL} relative to GA_{OL}. It is obvious from (4) that $|DSF|>1$ corresponds to a negative feedback, $|DSF|<1$ corresponds to a positive feedback, and $DSF=1$ corresponds to no feedback.

Advantages of negative feedback

From mathematical point of view, both the advantages and disadvantages of negative feedback are related to the denominator, $1+A_{OL}\beta$ or DSF, in (4).

For $A_{OL}\beta >> 1$, the closed loop gain A_{CL} becomes insensitive to the open-loop gain A_{OL}:

$$A_{CL} = G\frac{1}{\beta} + G\beta_{fwd}\ . \tag{8}$$

A_{CL} mainly depends on the small-signal input transmission G and the transmissions β and β_{fwd} of the feedback circuit.

For an arbitrary return ratio $A_{OL}\beta$, we can find the sensitivity of A_{CL} to A_{OL} as follows:

$$\frac{\mathrm{d}A_{CL}}{\mathrm{d}A_{OL}} = G\left[\frac{1}{1+A_{OL}\beta} - \frac{A_{OL}\beta}{(1+A_{OL}\beta)^2}\right]$$

$$= G\left[\frac{1+A_{OL}\beta}{(1+A_{OL}\beta)^2} - \frac{A_{OL}\beta}{(1+A_{OL}\beta)^2}\right]$$

$$= G\frac{1}{(1+A_{OL}\beta)^2} = \underbrace{G\frac{A_{OL}}{(1+A_{OL}\beta)}}_{A_{CL}\ \text{if}\ A_{CL}>>D}\frac{1}{(1+A_{OL}\beta)}\frac{1}{A_{OL}}\ . \tag{9}$$

$$= A_{CL}\frac{1}{1+A_{OL}\beta}\frac{1}{A_{OL}}$$

$$\Rightarrow \frac{\mathrm{d}A_{CL}}{A_{CL}} = \frac{\mathrm{d}A_{OL}}{A_{OL}}\frac{1}{1+A_{OL}\beta},\ \text{or}\ \delta A_{CL} = \frac{1}{DSF}\delta A_{OL}$$

This means that the relative change in A_{CL} is by a factor of DSF lower than the relative change in A_{OL}. Note that with no feedback, like the case of Fig. 1(a), $\delta A_{CL} = \delta A_{OL}$. Due to the negative feedback, A_{CL} becomes by a factor of DSF less sensitive to A_{OL}.

The main disadvantage of the negative feedback is related to the frequency dependence of the return ratio $A_{OL}\beta$. If there is a frequency where $A_{OL}\beta = -1$, then in (4), $A_{CL}=\infty$; the feedback turns out to be positive, and the circuit (amplifier) becomes unstable: it can produce a sustained output, for example, sustained oscillations, with no input. We will study such a behavior of transistor circuits in the lectures dedicated to positive feedback oscillators.

Finding partial gains

To define the partial gains in (4)–(7) in terms of the signals at the circuit input, s_s, output, s_o, and the control terminals of the dependent source, s_ε, we first assume that in a generic single-transistor circuit (see Fig. 2) there is only a single dependent source and, then, we will solve this circuit by applying superposition. Yes, we will apply superposition despite the fact that one in the sources in Fig. 2 depends on the other. However, we will do it carefully.

We have no difficulty to find the contribution of the s_s source to all the other signals in Fig. 2(a). To do this, we simply suppress the dependent source as shown in Fig. 2(b).

However, when we are finding the contribution of the dependent source $a_{OL}s_\varepsilon$, we cannot simply suppress the independent source s_s. This is so because zeroing the s_s source also zeroes the $a_{OL}s_\varepsilon$ source and it contributes nothing. To let the dependent source $a_{OL}s_\varepsilon$ to contribute the same way it does in Fig. 2(a), before applying superposition, we have to "remind" the $a_{OL}s_\varepsilon$ source in Fig. 2(c) its *original* value $a_{OL}s_\varepsilon$, which it had in Fig. 2(a). [If you do not like the word to "remind" something to a dependent source, you can replace it in Fig. 2(d) with an equivalent independent source having the value $a_{OL}s_\varepsilon$. Or you can assume that the dependent source in Fig. 2(c) *still depends* on the *original* value of s_ε in Fig. 2(a).]

We now can define

$$G \equiv \frac{s'_\varepsilon}{s_s}, \tag{10}$$

$$\beta_{fwd} \equiv \frac{s'_o}{s'_\varepsilon}, \tag{11}$$

$$A_{OL} \equiv \frac{s''_o}{s_\varepsilon}, \tag{12}$$

$$\beta \equiv -\frac{s''_\varepsilon}{s''_o}, \tag{13}$$

$$RR \equiv A_{OL}\beta \equiv -\frac{s''_\varepsilon}{s_\varepsilon}. \tag{14}$$

In a general case, the number of independent signal sources in a single-transistor circuit can be greater than one, but by applying superposition, the circuit can always be solved for each of them separately. Each independent source s_i will have different G_i, β_{fwd}, and D_i gains, where i is the dependent source number. Individual closed-loop gains can be found in this case as

$$A_{CLi} \equiv G_i \frac{A_{OL}}{DSF} + D_i \,. \tag{15}$$

It important to note that the solution for the *dependent* source remains the same, and A_{OL} and *DSF* in (13) should be found only once.

The very important advantages of the new generic approach are that it allows for:

- recognizing the feedback in a single-transistor circuit,
- finding the partial transmissions and gains of the circuit,
- distinguishing between negative an positive feedback,

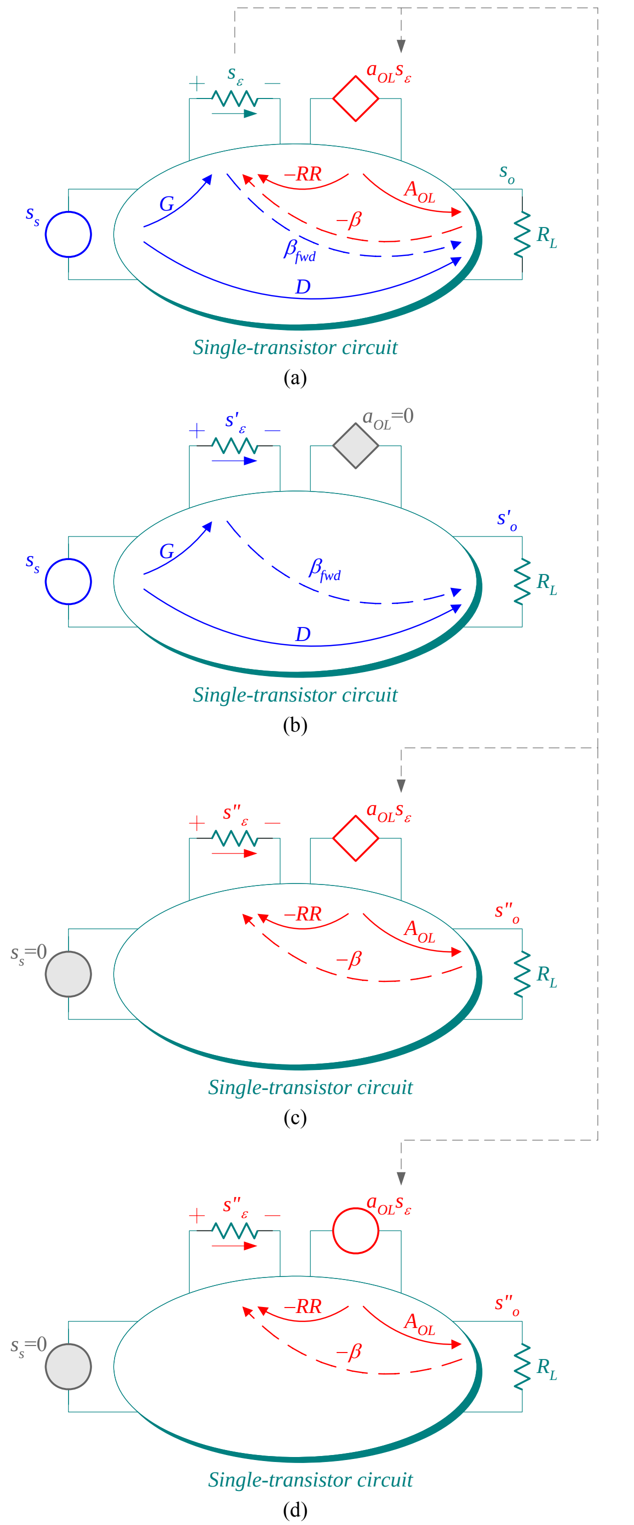

Fig. 2. Finding partial small-signal transitions and gains of a single-transistor circuit.

- solving the circuit in the simplest way: each time it is solved for a single source only (this approach is most powerful and insightful in noise analysis, when quite a few independent equivalent noise sources are connected to the circuit and contributions of all of them should be found separately to identify the dominant noise source).

Finding closed-loop gain: example circuit 1

Let us now to solve the elementary CC amplifier (see Fig. 3) for the closed-loop gain:

$$G \equiv \frac{s'_\varepsilon}{s_s} = \frac{i'_b}{v_s}\bigg|_{v_s=1} = \frac{1}{h_{ie} + R_E \| r_o}, \qquad (1)$$

$$\beta_{fwd} \equiv \frac{s'_o}{s'_\varepsilon} = \frac{v'_o}{i'_b}\bigg|_{i'_b=1} = R_E \| r_o, \qquad (2)$$

$$A_{OL} \equiv \frac{s''_o}{s_\varepsilon} = \frac{v''_o}{i_b}\bigg|_{i_b=1} = h_{fe}(h_{ie} \| R_E \| r_o), \qquad (3)$$

$$RR \equiv A_{OL}\beta \equiv -\frac{s''_\varepsilon}{s_\varepsilon} = -\frac{i''_b}{i_b}\bigg|_{i_b=1} = h_{fe}\frac{R_E \| r_o}{R_E \| r_o + h_{ie}}. \qquad (4)$$

$$A_{CL} \equiv \frac{s_o}{s_s} = \frac{v_o}{v_s} = \frac{1}{h_{ie} + R_E \| r_o}\,\frac{h_{fe}(h_{ie} \| R_E \| r_o)}{1 + h_{fe}\dfrac{R_E \| r_o}{R_E \| r_o + h_{ie}}} + \frac{1}{h_{ie} + R_E \| r_o}(R_E \| r_o). \qquad (5)$$

In the exam, you do *not* have to simplify any solution for A_{CL}, but we will do it here to be sure that the new solution is identical to that obtained in Lecture 3.

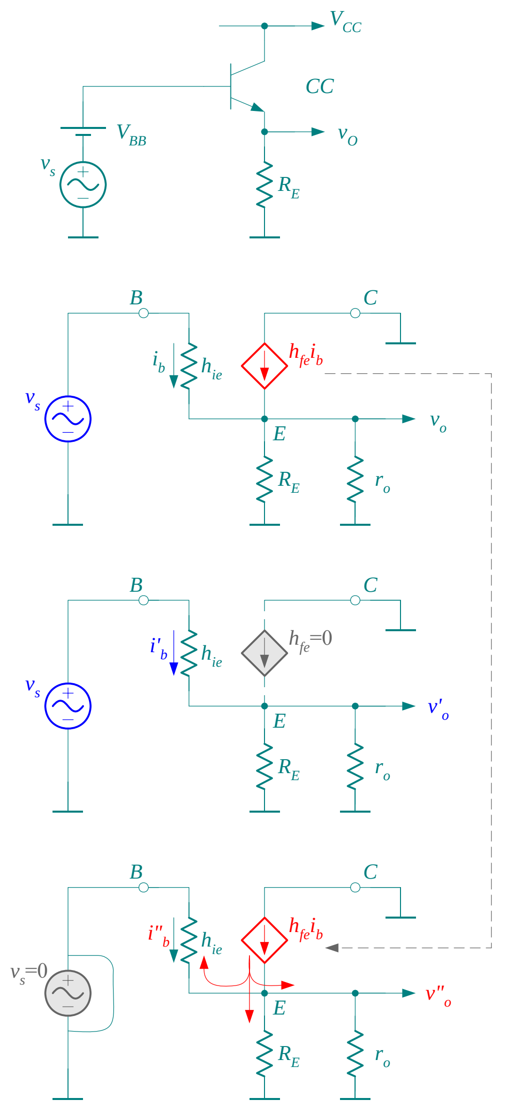

Fig. 3. Example circuit 1: the elementary CC amplifier, solved with the π model of the transistor.

$$A_{CL} = \frac{1}{h_{ie} + R_E \| r_o} \frac{h_{fe} \dfrac{h_{ie}(R_E \| r_o)}{h_{ie} + (R_E \| r_o)}}{1 + h_{fe} \dfrac{R_E \| r_o}{R_E \| r_o + h_{ie}}} + \frac{R_E \| r_o}{h_{ie} + R_E \| r_o}$$

$$= \frac{R_E \| r_o}{h_{ie} + R_E \| r_o} \times \left[\frac{h_{fe} h_{ie}}{h_{ie} + R_E \| r_o + h_{fe}(R_E \| r_o)} + 1 \right]$$

$$= \frac{R_E \| r_o}{h_{ie} + R_E \| r_o} \times \left[\frac{h_{fe}(1 + h_{fe}) r_e}{(1 + h_{fe}) r_e + (1 + h_{fe})(R_E \| r_o)} + 1 \right]. \quad (6)$$

$$= \frac{R_E \| r_o}{h_{ie} + R_E \| r_o} \frac{\overbrace{h_{fe} r_e + r_e}^{h_{ie}} + (R_E \| r_o)}{r_e + (R_E \| r_o)}$$

$$= \frac{R_E \| r_o}{r_e + R_E \| r_o}$$

Finding closed-loop gain: example circuit 2

Let us now find the closed-loop gain for the same circuit but by using the T model for the transistor.

$$G \equiv \frac{s'_\varepsilon}{s_s} = \frac{i'_b}{v_s}\bigg|_{v_s = 1} = \frac{1}{r_e + R_E \| r_o}, \quad (1)$$

$$\beta_{fwd} \equiv \frac{s'_o}{s'_\varepsilon} = \frac{v'_o}{i'_b}\bigg|_{i'_b = 1} = R_E \| r_o, \quad (2)$$

$$A_{OL} \equiv \frac{s''_o}{s_\varepsilon} = \frac{v''_o}{i_b}\bigg|_{i_b = 1} = 0, \quad (3)$$

$$RR \equiv A_{OL}\beta \equiv -\frac{s''_\varepsilon}{s_\varepsilon} = -\frac{i''_b}{i_b}\bigg|_{i_b = 1} = h_{fe}. \quad (4)$$

$$A_{CL} \equiv \frac{s_o}{s_s} = \frac{1}{r_e + R_E \| r_o} \frac{0}{1 + h_{fe}} + \frac{1}{r_e + R_E \| r_o}(R_E \| r_o) \quad . \quad (5)$$

$$= \frac{R_E \| r_o}{r_e + R_E \| r_o}$$

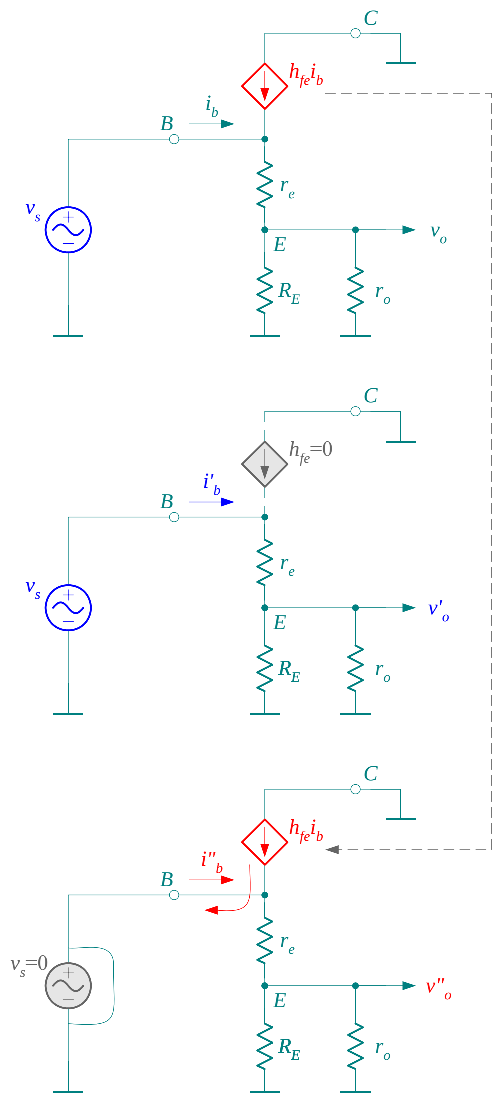

Fig. 4. Example circuit 2: the elementary CE amplifier, solved with the T model of the transistor.

REFERENCES

[1] P. R. Gray, P. J. Hurst, S. H. Lewis, and R. G. Meyer., *Analysis and Design of Analog Integrated Circuits.* (4th Edition.)