# 5. Double-Transistor Circuits

Our aim is to develop and analyze basic double-transistor circuits. We will see that there are only five basic circuit topologies. A wide majority of all the analog transistor circuits, no matter how many transistors in each, can be divided in elementary blocks based on either the previously studied single-transistor topologies or on the five basic double-transistor topologies.

## 5.1. *Five basic topologies of double-transistor circuits*

The five basic double-transistor topologies (see Fig. 1) include all the possible relative positions of the transistors. It is obvious that due to a variety of interconnections between the transistors, the number of double-transistor circuits is much greater than five, but we will consider only the most generic ones.

The first topology in Fig. 1 we have already used in the current mirror (see the previous lecture).

## 5.2. *Active load*

Our aim here is to reach the highest small-signal voltage gain for a given supply voltage, $V_{CC}$, and the static output voltage of the amplifier in the nearly middle of its range: $V_O = 0.5V_{CC}$. We will define *resistive* loads as static ones since there is no difference between their static and dynamic values, and we will define loads that *do* differ in the values of their static and dynamic impedances as *active* or *dynamic* ones. To understand the difference between an amplifier with a static (resistive) and active (dynamic) load, let us first find the maximum small-signal voltage gain of the elementary CE amplifier, with a static load, (see Fig. 2), assuming that $V_O=0.5V_{CC}$,

$$A_v = -g_m(R_C \| r_o)\big|_{R_C << r_o} = -g_m R_C =$$

$$-\frac{I_C}{V_T}R_C = -\frac{V_{CE}}{V_T}\bigg|_{V_{CE}=\frac{V_{CC}}{2}} . \qquad (1)$$

$$= -\frac{V_{CC}}{2V_T}\bigg|_{300^{o}\,\mathrm{K}} = -20V_{CC}\big|_{V_{CC}=10} = -200$$

As one can see from (1), the small-signal voltage gain of the elementary CE amplifier depends solely on its voltage supply, $V_{CC}$. Since $V_{CC}$ is limited, the voltage gain is limited too.

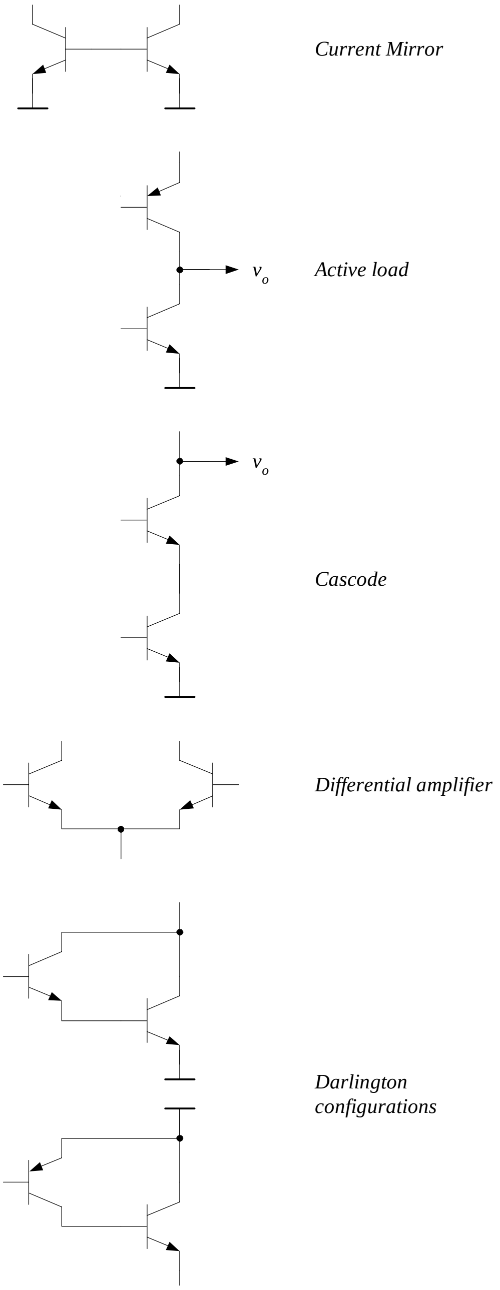

Fig. 1. Five basic double-transistor topologies.

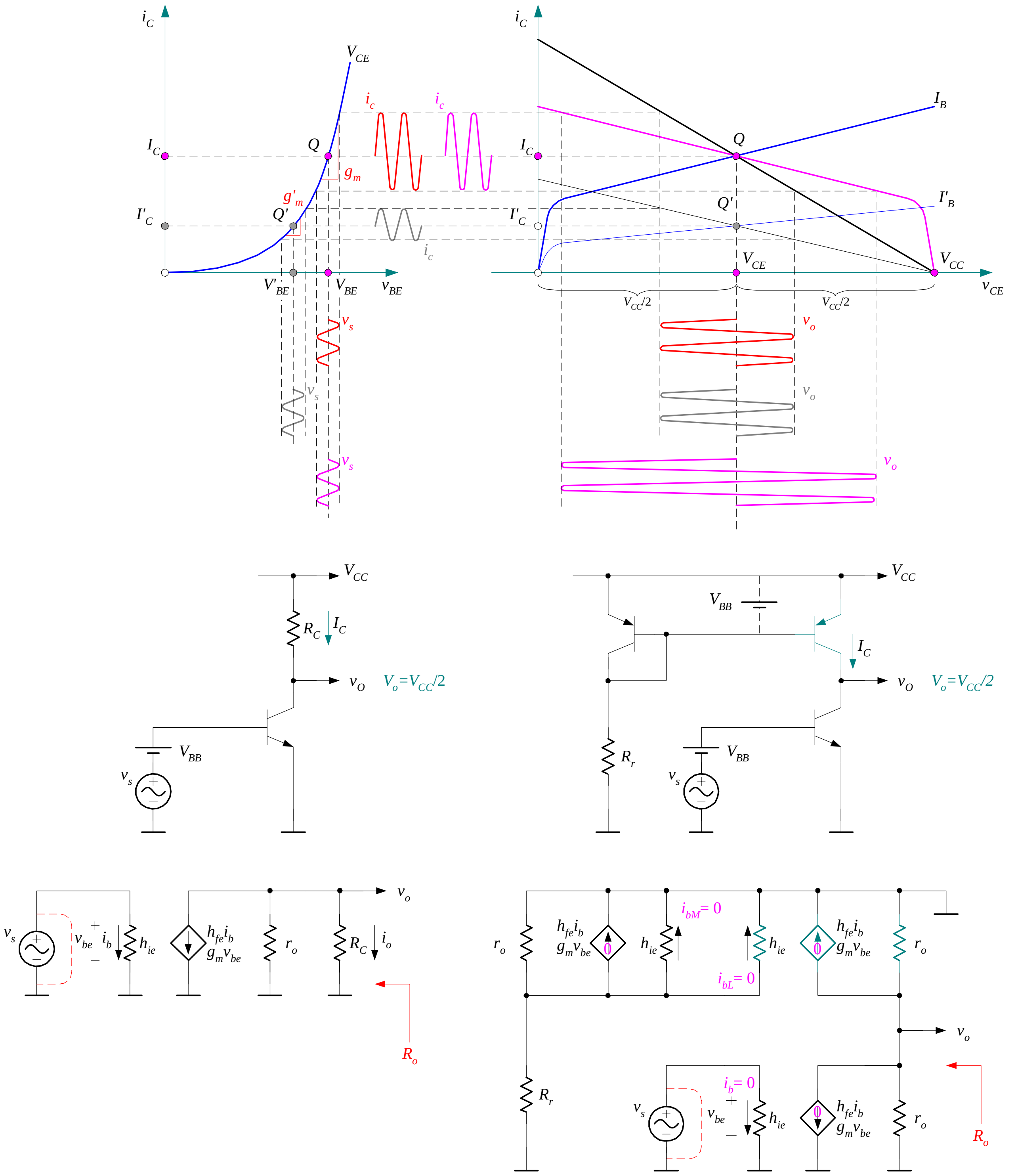

Fig. 2. CE amplifier with active load.

To understand why increasing $R_C$ does not increase the voltage gain simply note in Fig. 2 that to keep the same $V_O$=0.5$V_{CC}$, $I_C$ should be decreased with increasing $R_C$. Note from (1), that decreasing $I_C$ decreases $g_m$ and hence the voltage gain remains the same.

It is obvious now that if we like to increase the small-signal voltage gain, we have to keep the same $g_m$ and not to decrease $I_C$. This is possible if a *nonlinear* load, with the nonlinear red load line in Fig. 2, is used instead of the linear resistive load. Note that the new load line has a lower slope (higher dynamic impedance) at the operating point. Note also that we have to draw this new load line through the ($V_{CC}$, 0) point because at this point the voltage across the load is zero and, hence, a zero current should be expected for a practical non-reactive load.

Recalling that reversing (flipping horizontally) the load line and shifting it back to the origin of the coordinate system reproduces the load characteristic in its own coordinate system, we can see that the load characteristic is identical to the characteristic of the transistor in the given static state. Therefore, replacing the static load with a transistor identical to the transistor of the elementary CE amplifier and providing to the new transistor the same static state allows us to increase the dynamic load value without decreasing $I_C$ and $g_m$ and, therefore, to increase $A_v$:

$$A_v = -g_m \frac{r_o}{2}\bigg|_{V_{CE}<<V_A} = -\frac{I_C}{V_T}\frac{V_A}{2I_C} \quad . \qquad (2)$$

$$= -\frac{V_A}{2V_T}\bigg|_{\substack{V_A=100\\ 300^\circ\mathrm{K}}} = -\frac{100}{2\cdot 26\mathrm{mV}} = -2000$$

Note that, for $V_{CC}$ = 10 V, the small-signal gain $A_v$ is increased by an order of magnitude compared to the previous case. Note also that any further increase in $A_v$ is not possible without increasing either $V_A$ or $r_o$ of the transistors.

### 5.3. *Cascode*

The Early voltage in (2) is a technological parameter, which is difficult to increase. Therefore, we will look for a circuit, based on the transistors with the same $V_A$ as in the above example, but having a much greater *equivalent* $r_o$ to replace the transistors with not especially high $r_o$. Of course, all the other small-signal parameters of the new circuit should match the corresponding small-signal parameters of the transistors to be replaced.

Recalling that the output impedance seen by the load of the elementary CE amplifier $R_o(R_C)=r_o>>h_{ie}$, and the output impedance seen by the load of the elementary CB amplifier is by a two orders of magnitude higher than $r_o$ when the input source impedance $r_s>>h_{ie}$, we simply use the CE amplifier as a signal source for the CB amplifier (see Fig. 3). Such a configuration is called cascode (cascade-cathode, since this idea was first implemented for vacuum tubes; many transistor topologies and their names are derived from vacuum-tube circuits). Connecting the elementary CE and CB amplifiers in series allows us to use the high current gain of the CE amplifier, high voltage gain of the CB amplifier, and its very high output impedance. The resulting circuit will have current and voltage gains as high as those of the elementary CE amplifier but a much higher output impedance.

Note that both the transistors in the cascode configuration have almost the same static state in terms of the collector currents.

The small-signal solution for the cascode output impedance,

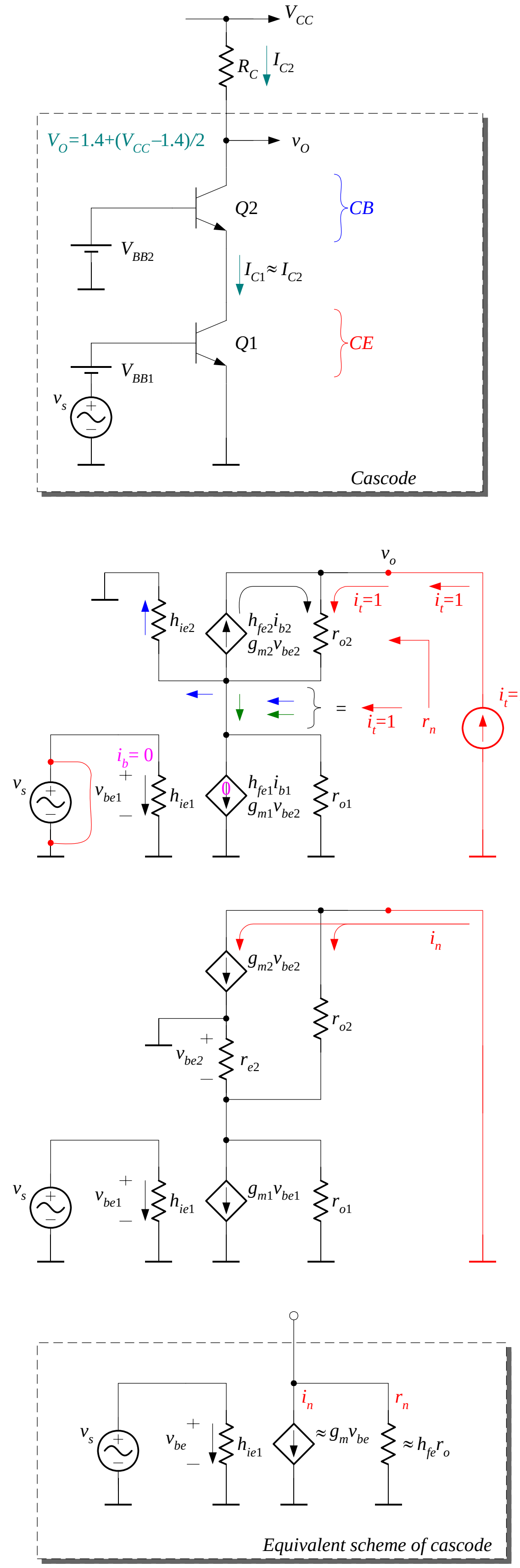

Fig. 3. Cascode configuration.

seen by its load, can easily be found for a current test source supplying the current *into* the circuit (note that when the current is supplied in the different direction then the negative value of $v_t/i_t$ corresponds to the positive value of the output impedance).

$$r_n = \left.\frac{v_t}{i_t}\right|_{i_t=1} = \frac{v_t}{1\text{A}} =$$

$$= (h_{ie2} \,\|\, r_{o1})\, 1\text{A} + \left.\left(1 + h_{fe2}\frac{r_{o1}}{\underbrace{r_{o1} + h_{ie2}}_{ib2}}\right) r_{o2}\right|_{h_{ie2} << r_{o1}} . \qquad (3)$$

$$= h_{fe2} \cdot r_{o2}$$

As one can see from (3), the small-signal impedance of the cascode is by two orders higher than that of a single transistor.

Having the output impedance, $r_n$, and the short-circuit current of the cascode configuration (see Fig. 3)

$$i_n = \underbrace{\underbrace{\underbrace{g_{m1} v_{be1} \frac{r_{o1} \,\|\, r_{o1}}{\underbrace{r_{o1} \,\|\, r_{o1} + r_{e2}}_{\to 1}}}_{i_{re2}} \cdot r_{e2}}_{v_{be2}} \cdot g_{m2}}_{g_{m2} v_{be2}}$$

$$\left. + \underbrace{g_{m1} v_{be1} \frac{r_{e2} \,\|\, r_{o1}}{\underbrace{r_{e2} \,\|\, r_{o1} + r_{o2}}_{\to 0}}}_{i_{ro2}} \right|_{r_o >> r_e} \quad , \qquad (4)$$

$$= g_{m1} v_{be1} \cdot r_{e2} \cdot g_{m2} = g_{m1} v_{be1} \cdot r_{e2} \cdot \frac{\alpha_{f2}}{r_{e2}}$$

$$= g_{m1} v_{be1} \alpha_{f2} \approx g_m v_{be}$$

we can obtain small-signal equivalent circuit for the cascode: $i_n$, $r_n$. Note that this circuit is almost identical to that of a single transistor in terms of the dependent source value $g_m v_{gs}$ but have a two orders of magnitude higher dynamic output impedance.

Replacing both the transistors in the amplifier with active load in Fig. 2 by the cascode configurations allows us to increase the amplifier gain in (2) by two orders of magnitude and to reach a 2 $10^5$ small-signal voltage gain in a single four-transistor stage with no load resistors.

Note in Fig. 3 that employing cascode configurations requires a higher $V_{CC}$ for the same range of the output.

## REFERENCES

[1] A. S. Sedra and K. C.Smith, *Microelectronic circuits*.