2. Elementary Electronic Circuits with a BJT Transistor (continued)

2.2. *Elementary single-transistor amplifiers*

In this lecture, we build *all* the possible *practical* circuits based on a single BJT transistor and a single resistor. (We use the resistor to translate the output current of the circuit into voltage; otherwise the circuit will not be able to provide a voltage gain.) We then analyze and compare the circuits' small-signals gains to see for what applications they can be suitable.

Fig. 1 shows that only three different circuits can be based on a single transistor. This is so because the collector cannot serve as an *input*, and the base cannot serve as an *output*. The collector has a negligible effect on the base-emitter junction, which controls the injection, and, hence, the collector has a negligible effect on the current gain. The small-signal base current is much below of that in the collector and emitter, and, therefore, the voltage, current, and power gains at the base would be smaller than 1.

Depending on the input-output pair, we will distinguish among three different configurations (see Fig. 2): common emitter (CE), common collector (CC), and common base (CB). In elementary circuits, the common terminal will be grounded in the small-signal analysis

We will *always* start the analysis of a circuit from finding its static state. According to the static state, we will find the small-signal parameters of the transistor, replace the transistor by either its small-signal T or hybrid-π model, and then suppress all the static sources. We will then solve the resultant small-signal equivalent circuit in the most insightful way by applying superposition, Thévenin, Norton, and Miller theorems, and recognizing in the small-signal circuit such elementary sub-circuits as the voltage and current dividers. This will help us to better understand the circuit architecture and operation.

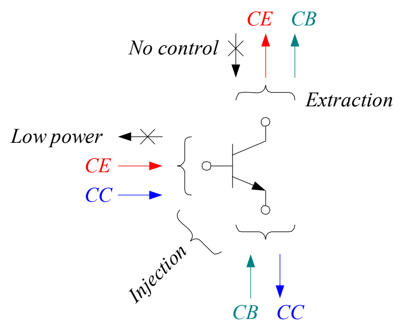

Fig. 1. Connecting practical signals to the transistor.

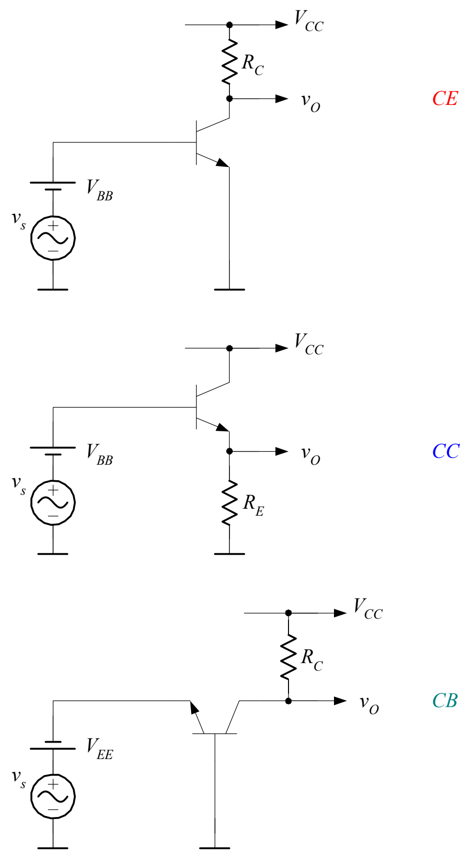

Fig. 2. Elementary single-transistor amplifiers.

CE amplifier

To define the static state of the CE amplifier (see Fig. 3), we first suppress the small-signal voltage source: v_s=0. Second, we chose the static values for the collector current, say, I_C=1 mA, and the output voltage,

$$V_O = V_{O\,\min} + 0.5(V_{CC} - V_{O\,\min}) = V_{BE} + 0.5(V_{CC} - V_{BE}) = 0.5V_{CC} + 0.5V_{BE} \quad (1)$$

The lower limit for the collector current is defined by the circuit noise level, and the upper limit is related to the maximum power, $(i_{CC}v_{CC})_{\max}$, that the transistor is able to dissipate before it reaches its maximum temperature. Choosing the output voltage in the middle of its available range, from V_{BE} to V_{CC}, provides the highest possible range for *both* the positive and negative excursions of the output signal.

Having, I_C we can easily find I_B=I_C/β_F, and then V_{BB}=V_{BE} by solving the following equation:

$$I_B = I_{BS}e^{V_{BE}/V_T} \Rightarrow V_{BE} = V_T \ln\frac{I_B}{I_{BS}}. \quad (2)$$

It now only remains to find the value of R_C that provides V_{CE}=0.5V_{CC}+0.5V_{BE}:

$$I_C R_C = V_{CC} - V_{CE} = V_{CC} - (0.5V_{CC} + 0.5V_{BE}) \Rightarrow R_C = \frac{0.5V_{CC} - 0.5V_{BE}}{I_C} \quad (3)$$

After finding the static state, we can calculate all the transistor small-signal parameters (see the previous lecture), replace the transistor by, for example, its hybrid-π model, suppress the V_{BB} and V_{CC} static sources in Fig. 3, and obtain in this way the equivalent small-signal circuit of the CE amplifier.

The equivalent small-signal circuit in Fig. 3 can easily be solved for the voltage, A_v, and current, A_i, gains and for the input, R_{in}, and output, R_o, impedances:

$$A_v \equiv \left.\frac{v_o}{v_s}\right|_{v_s=1\,\text{V}} = \frac{-g_m(1\,\text{V})(r_o \,\|\, R_C)}{1\,\text{V}} = -g_m(r_o \,\|\, R_C) = -\left.\frac{\alpha_f}{r_e}(r_o \,\|\, R_C)\right|_{h_{fe}>>1,\, r_o>>R_C} = -\left.\frac{R_C}{r_e}\right|_{R_C>>r_e} << -1 \quad (4)$$

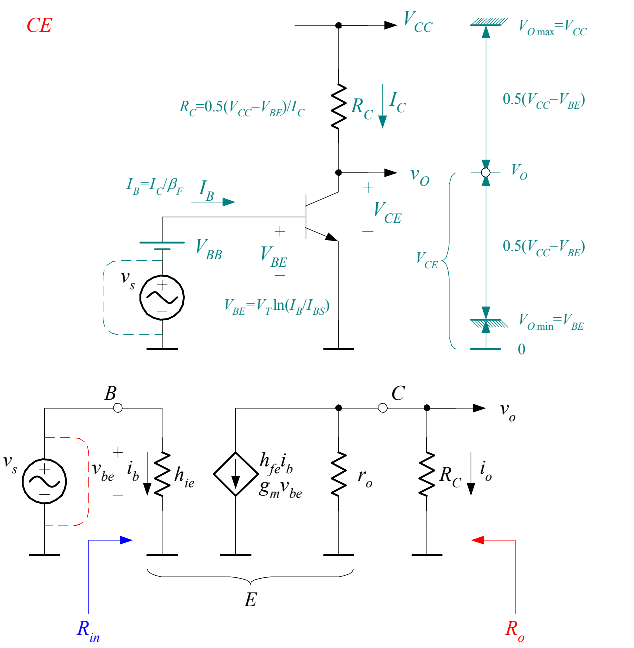

Fig. 3. Elementary CE amplifier: the static state and the equivalent small-signal circuit.

$$A_i \equiv \left.\frac{i_o}{i_s}\right|_{i_s=1\,\text{A}} = \frac{-h_{fe}(1\,\text{A})}{1\,\text{A}}\frac{r_o}{r_o + R_C} = -h_{fe}\left.\frac{r_o}{r_o + R_C}\right|_{r_o>>R_C} = -h_{fe} \quad (5)$$

$$A_p = A_v A_i\big|_{h_{fe}>>1,\, r_o>>R_C,\, R_C>>r_e} >> 1. \quad (6)$$

$$R_{in} = h_{ie} = (1 + h_{fe})r_e\big|_{300^\circ\,\text{K},\, I_C=1\,\text{mA}} \quad [2.6\,\text{k}\Omega]. \quad (7)$$

$$R_o = R_C \,\|\, r_o\big|_{R_C<<r_o} = R_C \quad [<<100\,\text{k}\Omega]$$
$$R_o(R_C) = r_o = \left.\frac{V_A + V_{CE}}{I_C}\right|_{V_A=100>>V_{CE},\, I_C=1\,\text{mA}} \quad [100\,\text{k}\Omega] \quad (8)$$

Note that the CE amplifier *reverses* the phase of the input signal. The absolute values of its voltage, current, and power gains (including dc gain values) can be greater than 1. Its input impedance has a medium value, and its output impedance, seen by R_C, is high.

To illustrate the phase reversal of the CE amplifier, we give in the Appendix a graphical solution for the small-signal voltage gain A_v=−$g_m R_C$.

CC amplifier

To define the static state of the CC amplifier (see Fig. 4), we first choose I_E, say, I_E=1 mA, and V_O=0.5(V_{CC}–V_{BE}), V_{CE}= V_{BE}+0.5(V_{CC}–V_{BE})= 0.5V_{CC}+0.5V_{BE}. Note that the lower limit for V_O is ground and the upper limit is V_{CC}–V_{BE}; for greater values of V_O the base-collector junction will be forward biased.

This gives us

$$R_E = \frac{V_O}{I_E} = \frac{0.5(V_{CC} - V_{BE})}{I_E} \qquad (9)$$

$$I_C = \alpha_F I_E;\; I_B = \frac{I_C}{\beta_F}, \qquad (10)$$

and

$$V_{BB}=V_O+V_{BE}=0.5V_{CC}+0.5V_{BE}, \qquad (11)$$

where V_{BE} can be found from (2). Note that an approximate value of V_{BB} can be found by simply adding a 0.7 V voltage to V_O: $V_{BE}\approx 0.5(V_{CC}-0.7)+0.7=0.5V_{CC}+0.35$.

After finding the static state, we can obtain equivalent small-signal circuit of the CC amplifier (see Fig. 4). Note that this time we are using the T small-signal model for the transistor. The equivalent circuit in Fig. 4 can easily be solved for

$$A_v \equiv \frac{v_o}{v_s} = \underbrace{\frac{R_E \| r_o}{R_E \| r_o + r_e}}_{\text{voltage divider}}\Bigg|_{r_o>>R_E>>r_e} = 1. \qquad (12)$$

$$A_i \equiv \frac{i_o}{i_s}\Bigg|_{i_s=1\,\mathrm{A}} = \frac{1\,\mathrm{A} + h_{fe}(1\,\mathrm{A})}{1\,\mathrm{A}}\underbrace{\frac{r_o}{r_o + R_E}}_{\text{current divider}}\Bigg|_{R_E<<r_o} = 1 + h_{fe}\Big|_{h_{fe}>>1} >> 1 \,. \qquad (13)$$

$$A_p = A_v A_i\Big|_{h_{fe}>>1,\, r_o>>R_C>>r_e,\, R_E<<r_e} >> 1\,. \qquad (14)$$

$$R_{in} = \underbrace{(1+h_{fe})(r_e + R_E \| r_o)}_{\text{Miller's theorem}} \quad [>> 2.6\,\mathrm{k}\Omega]. \qquad (15)$$

$$R_o = r_e \| R_E \| r_o\Big|_{r_e<<R_E<<r_o} = r_e \quad [26\,\Omega]$$
$$R_o(R_E) = r_e \| r_o\Big|_{r_e<<r_o} = r_e \quad [26\,\Omega] \qquad (16)$$

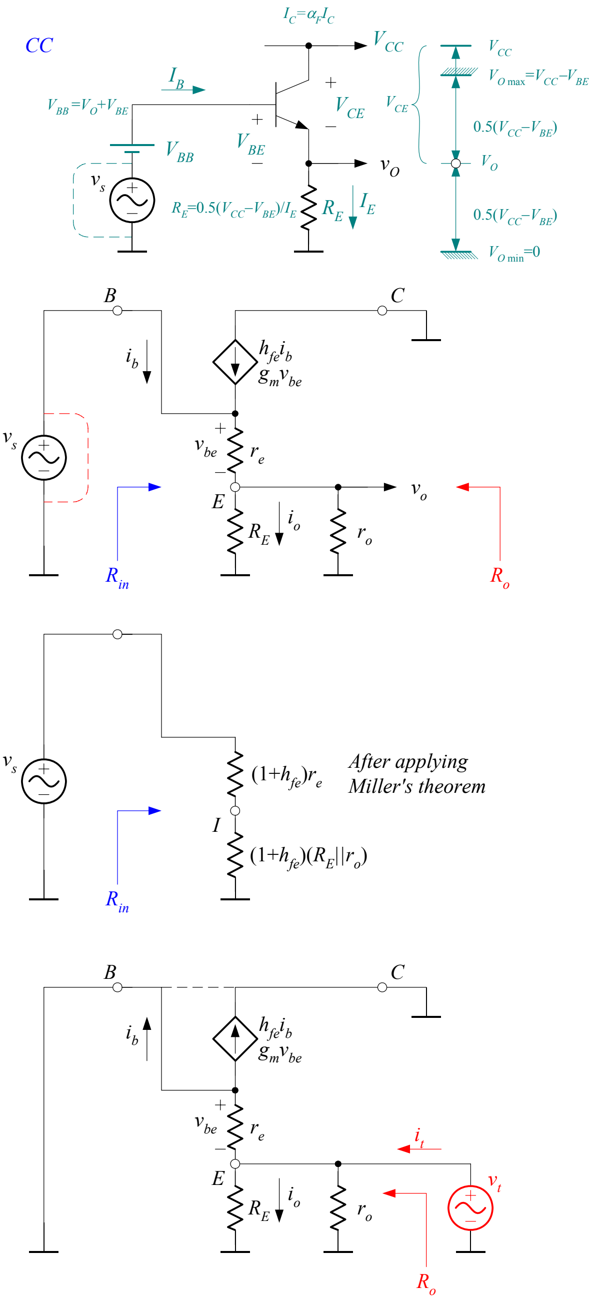

Fig. 4. Elementary CC amplifier: the static state and the equivalent small-signal circuit.

To obtain (16), we observe that the dependent source in Fig. 4 is short-circuited.

Note that the CC amplifier does not reverse the phase of the input signal. The absolute values of its current (including dc) and power gains can be greater than 1, and its voltage gain can be almost unity. Its input impedance can be very high, and its output impedance, seen by R_E, is very low.

CB amplifier

To define the static state of CB amplifier (see Fig. 5), we first choose I_C, say, I_C=1 mA. We then note that for V_B=0, the collector voltage range can be as great as from ground to V_{CC}, therefore, we can chose V_O=0.5V_{CC}. Since V_E=–V_{BE}, the collector-emitter voltage V_{CE} = V_C–V_E=V_O–V_E=0.5V_{CC}+V_{BE}. V_{BE} can be found from (2), where I_B=I_C/β_F. R_C=(V_{CC}–V_O)/I_C=0.5V_{CC}/I_C.

To obtain the equivalent small-signal circuit for the CB amplifier, we use the hybrid-π model for the transistor. To solve the equivalent circuit in the easiest way, we replace the $h_{fe}i_b$ current source and the r_o resistor by their Thévenin equivalent. The resultant circuit can very easily be solved for

$$A_v \equiv \left.\frac{v_o}{v_s}\right|_{v_s=1\,\mathrm{V}} = \frac{1\,\mathrm{V}+g_m(1\,\mathrm{V})r_o}{1\,\mathrm{V}}\,\frac{R_C}{r_o+R_C}$$

$$= \left.\frac{R_C}{r_o+R_C}+g_m\frac{r_oR_C}{r_o+R_C}\right|_{r_o>>R_C} \qquad (17)$$

$$= \left.\alpha_f\frac{R_C}{r_e}\right|_{\alpha_f=1,\,R_C>>r_e} >>1$$

$$A_i \equiv \left.\frac{i_o}{i_s}\right|_{v_s=1\,\mathrm{V}} = \frac{v_o/R_C}{(1\,\mathrm{V})/h_{ie}+v_o/R_C}$$

$$= \left.\frac{\dfrac{1}{r_o+R_C}+\underbrace{g_m}_{\alpha_f/r_e}\dfrac{r_o}{r_o+R_C}}{\dfrac{1}{h_{ie}}+\dfrac{1}{r_o+R_C}+\underbrace{g_m}_{\alpha_f/r_e}\dfrac{r_o}{r_o+R_C}}\right|_{r_o>>R_C>>1} . \qquad (18)$$

$$= \frac{\dfrac{h_{fe}}{\{1+h_{fe}\}}\dfrac{1}{\{r_e\}}}{\dfrac{1}{\{(1+h_{fe})r_e\}}+\dfrac{h_{fe}}{\{1+h_{fe}\}}\dfrac{1}{\{r_e\}}} = \frac{h_{fe}}{1+h_{fe}} = \alpha_f$$

$$A_p = A_vA_i\Big|_{h_{fe}>>1,\,r_o>>R_C>>1} >>1 . \qquad (19)$$

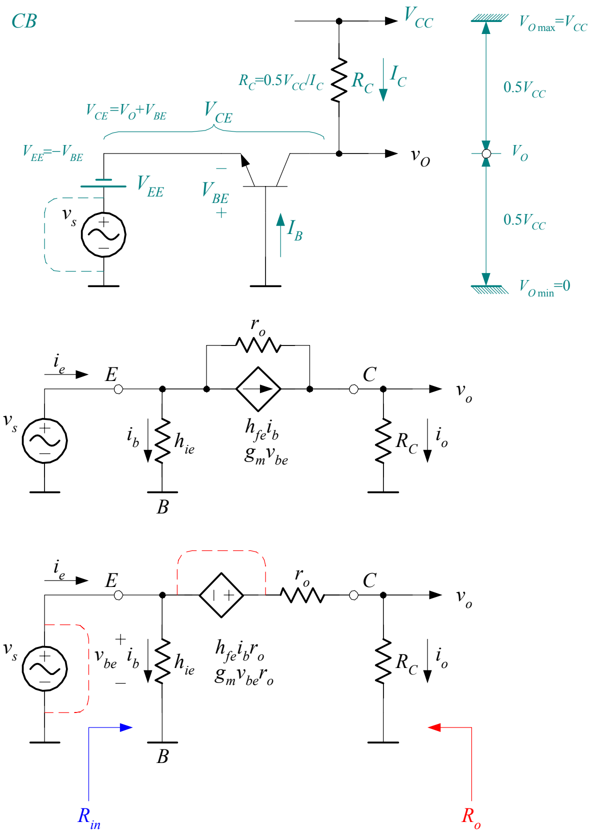

Fig. 5. Elementary CB amplifier: the static state and the equivalent small-signal circuit.

$$R_{in} = \left.\frac{v_s}{i_s}\right|_{v_s=1} = \left.\frac{1\,\mathrm{V}}{\dfrac{1}{h_{ie}}+\dfrac{1}{r_o+R_C}+g_m\dfrac{r_o}{r_o+R_C}}\right|_{r_o>>R_C>>1}$$

$$= \frac{1}{\dfrac{1}{(1+h_{fe})r_e}+\dfrac{h_{fe}}{1+h_{fe}}\dfrac{1}{r_e}} = \frac{1}{\dfrac{1}{(1+h_{fe})r_e}(1+h_{fe})} . \qquad (20)$$

$$= r_e \quad [26\,\Omega]$$

$$R_o = R_C \,\|\, r_o\Big|_{R_C<<r_o} = R_C$$
$$. \qquad (21)$$
$$R_o(R_C) = r_o \quad [100\,\mathrm{k\Omega}]$$

Note that the CB amplifier does not reverse the phase of the input signal. The absolute values of its voltage (including dc) and power gains can be greater than 1, and its current gain can be almost unity. Its input impedance is very small, and its output impedance, seen by R_C, is high. You will see from the coming class exercise that adding a great enough impedance, r_s>>h_{ie}, to the signal source *significantly*, by two

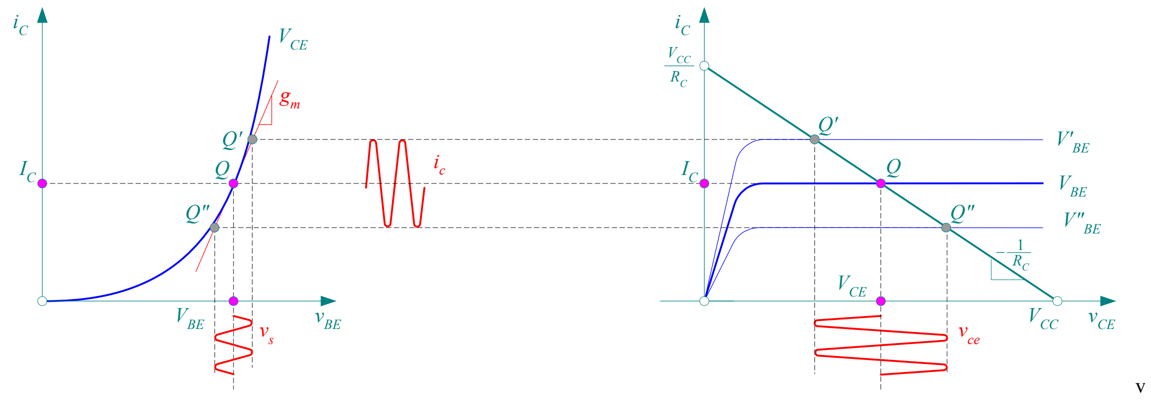

Fig. A1. A graphical solution for the elementary CE amplifier, assuming V_A=∞.

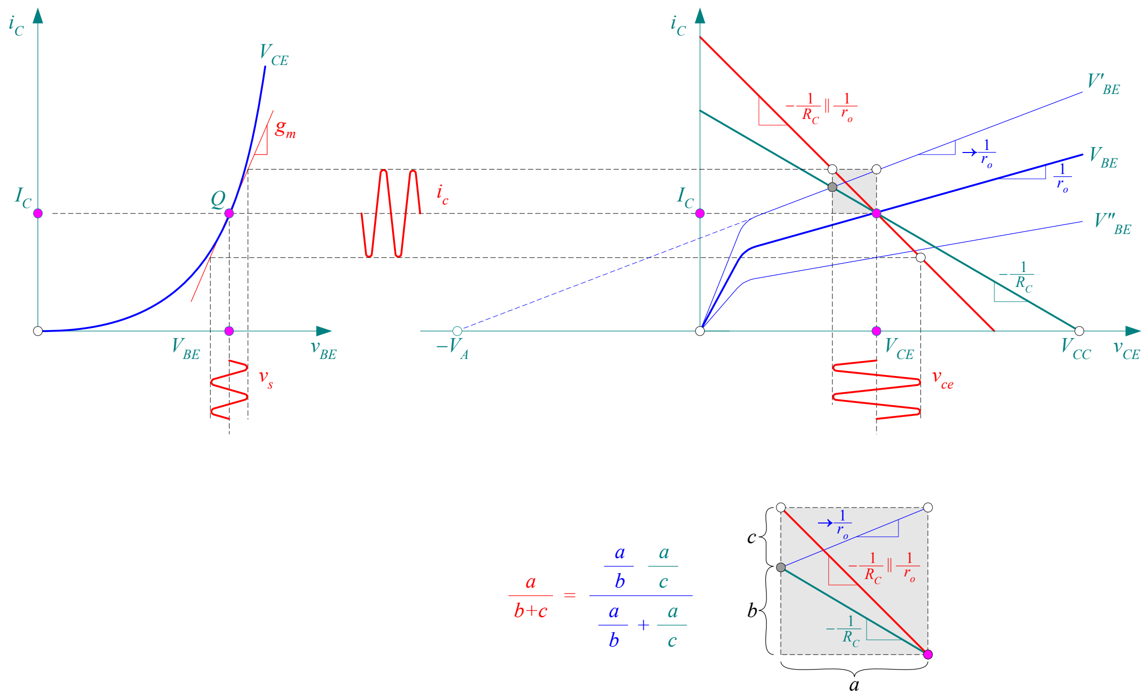

Fig. A2. A graphical solution for the elementary CE amplifier, assuming V_A<∞.

orders of magnitude, increases the output impedance of the CB amplifier.

Appendix

Fig. A1 shows a graphical solution for the small-signal voltage gain, $A_v = -g_m R_C$ of the CE amplifier. It is assumed that $V_A = \infty$. Note that the input signal v_s is first amplified by g_m (the translation by the i_c-v_{be} characteristic) and then by $-R_C$ (the translation by the v_{ce}-i_c characteristic).

Fig. A2 shows a graphical solution for $V_A < \infty$.

References

[1] J. Millman and C. C. Halkias, Integrated electronics, McGraw-Hill.
[2] A. S. Sedra and K. C.Smith, Microelectronic circuits.