8. High-Frequency Response

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Our aim is to develop the transistor small-signal model for high frequencies and to find the high-frequency response of elementary transistor circuits. We will do this for the BJT transistors only; the analysis of JFET and MOSFET transistor circuits is very similar.

8.1. *Transistor small-signal model for high frequencies*

At high-frequencies, the impedances of the parasitic capacitances related to the transistor junctions (see Fig. 1) become comparable to the values of the corresponding *h*-parameters of the transistor model, which was obtained for dc and relatively low frequencies. Thus to adjust the transistor model for high frequencies, we include in it the parasitic capacitances of the transistor emitter and collector junctions, C_π and C_μ, and also the ohmic resistance of the base, r_b. We also rename some of the transistor small-signal parameters to allow the h_{ie} parameter represent the input impedance of the new model:

$$h_{ie} \equiv \left.\frac{v_{be}(j\varpi)}{i_b(j\varpi)}\right|_{\varpi=0} = r_b + r_\pi , \qquad (1)$$

where

$$r_\pi = (1+h_{fe})r_e . \qquad (2)$$

Unity-gain frequency (GBP)

For a given transistor, the ohmic resistance r_b of the base and the capacitance C_μ of the collector junction can be found analytically. The capacitance C_π of the emitter junction is usually measured experimentally.

The experimental setup is shown in Fig. 2. The aim is to measure the frequency ω_t ("omega test"), at which the CE amplifier with the collector, which is short-circuited for small signals, has the current gain $|A_i(j\omega_t)|=1$. The unity-gain frequency ω_t is then translated into C_π.

Note from Fig. 2 that in the small-signal analysis, r_b can be neglected because it is connected in series with the current source i_s and, hence, does not affect the base current. The transistor output resistance, r_o, can also be neglected because it is short-circuited. The capacitor C_μ can be connected in parallel with C_π, provided that the current flowing through C_μ is much smaller than the current generated by the dependent source:

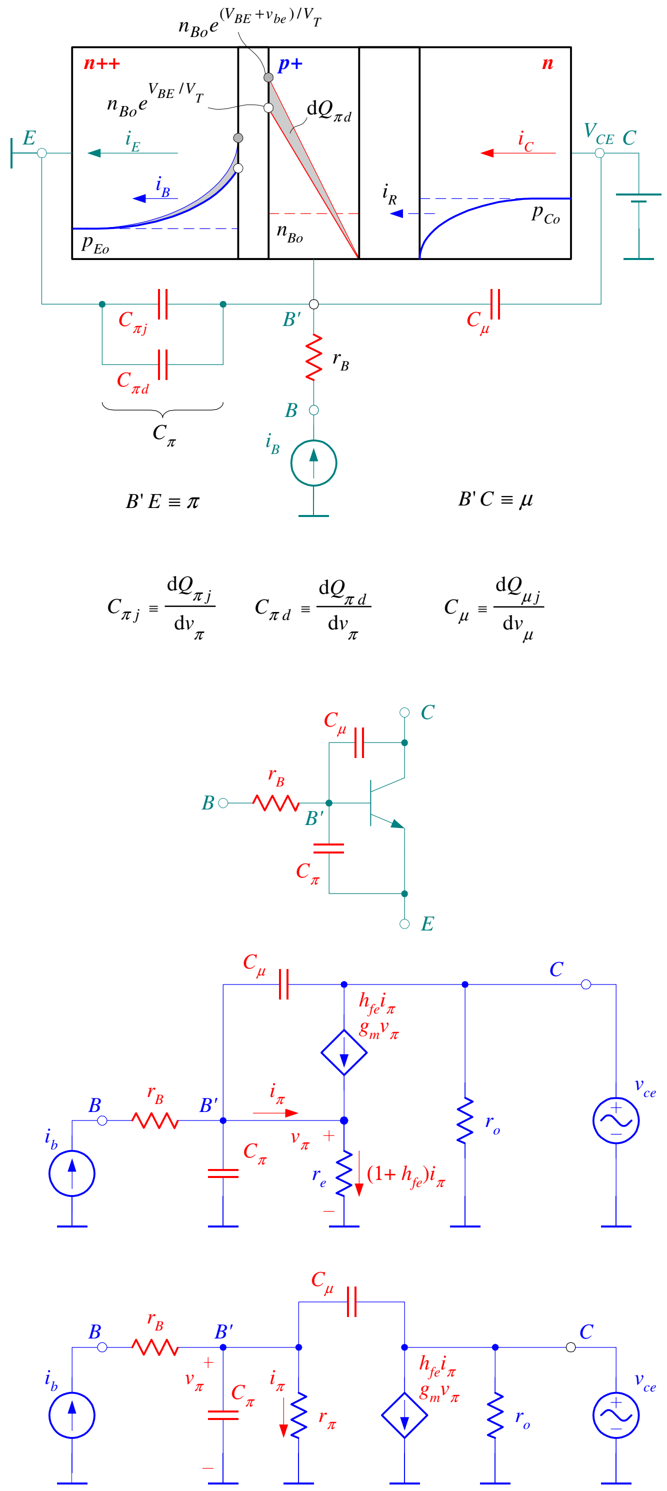

Fig. 1. Parasitic capacitances of the BJT transistor and its high-frequency small-signal models: *T* and hybrid-π.

$$\frac{v_\pi}{\dfrac{1}{\varpi C_\mu}} << g_m v_\pi \Rightarrow \varpi << \frac{g_m}{C_\mu}. \tag{3}$$

As a result, the circuit current gain

$$A_i(j\varpi) \equiv \frac{i_c(j\varpi)}{i_s(j\varpi)}\Bigg|_{i_s=1} = i_c = \frac{\dfrac{1}{j\varpi(C_\pi + C_\mu)}}{\dfrac{1}{j\varpi(C_\pi + C_\mu)} + r_\pi} h_{fe}$$

$$= \frac{h_{fe}}{1 + j\varpi(C_\pi + C_\mu) r_\pi} = \frac{h_{fe}}{1 + j\varpi/\varpi_c}, \tag{4}$$

where $\omega_c = 1/[(C_\pi + C_\mu) r_\pi]$ is the circuit corner frequency.

Assuming $h_{fe}>>1$,

$$|A_i(j\varpi_t)| = \left|\frac{h_{fe}}{1 + j\varpi_t/\varpi_c}\right| = 1$$

$$\Rightarrow |1 + j\varpi_t/\varpi_c| = h_{fe} >> 1$$

$$\Rightarrow \varpi_t/\varpi_c = h_{fe} \tag{5}$$

$$\Rightarrow \varpi_t = h_{fe}\cdot\varpi_c$$

According to (5), ω_t has an additional meaning: "gain-bandwidth product", where h_{fe} is the maximum, dc gain of the circuit, and ω_c is the circuit bandwidth.

Since in the vicinity of ω_t the circuit is not lamped and the current gain does not meet (5) (see Fig. 2), ω_t is measured indirectly, namely, a current gain $|A_i'(j\omega')|$ is measured at a frequency $\omega_t >> \omega' >> \omega_c$.

$$|A_i'(j\varpi')| = \left|\frac{h_{fe}}{1 + j\varpi'/\varpi_c}\right|_{\varpi'>>\varpi_c}$$

$$= \frac{h_{fe}}{\varpi'/\varpi_c} \tag{6}$$

$$\Rightarrow |A_i'(j\varpi')|\varpi' = h_{fe}\varpi_c = \varpi_t$$

$$\Rightarrow \varpi_t = |A_i'(j\varpi')|\varpi'$$

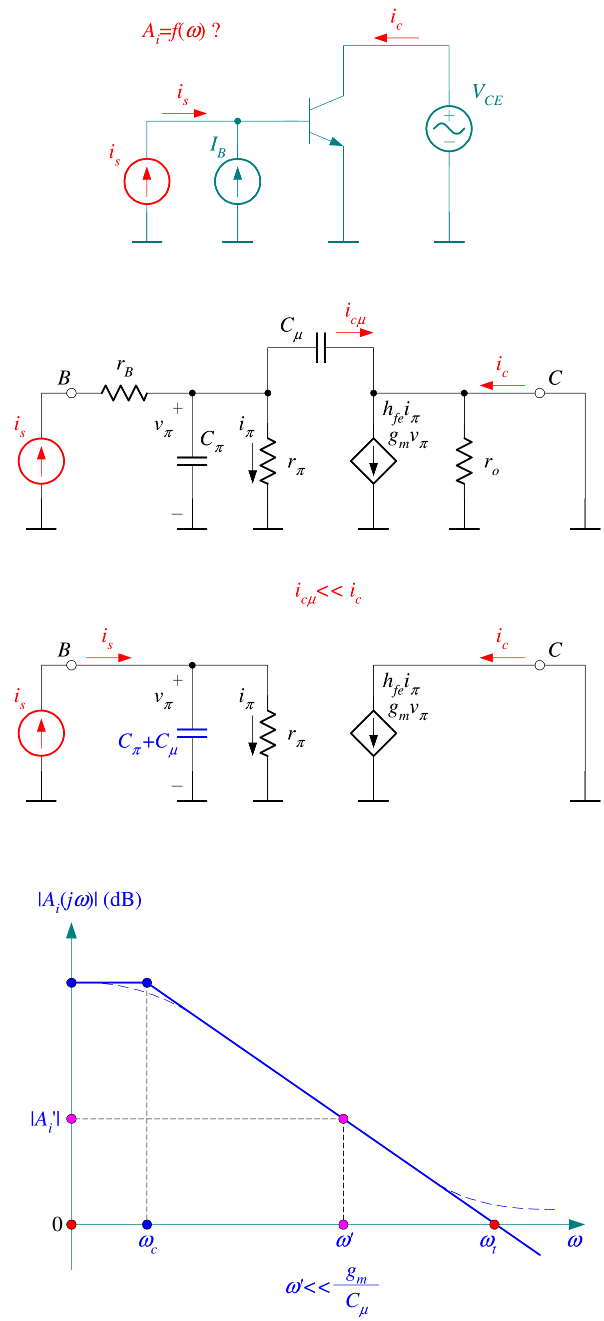

Fig. 2. Experimental setup for measuring ω_t and C_π.

Having ω_t, we can eventually find C_π as follows

$$\varpi_t = h_{fe} \cdot \varpi_c = \frac{h_{fe}}{(C_\pi + C_\mu) r_\pi} = \frac{h_{fe}}{(C_\pi + C_\mu) r_e (1 + h_{fe})}$$

$$= \frac{\alpha_f I_E}{(C_\pi + C_\mu) V_T} = \frac{I_C}{(C_\pi + C_\mu) V_T} = \frac{g_m}{C_\pi + C_\mu} < \frac{g_m}{C_\mu}. \quad (7)$$

$$\Rightarrow C_\pi = \frac{I_C}{V_T} \frac{1}{\varpi_t} - C_\mu$$

This experimental value of C_π is given in the data sheets of the transistor for a particular I_C, which was set in the experimental setup.

For a different I_C, the value of C_π should be adjusted. Since the diffusion capacitance of the emitter junction is usually much greater than the junction capacitance, and the diffusion capacitance is directly proportional to the emitter and collector currents, the C_π value, which given in the date sheets, can simply be multiplied by the ratio of the collector current that is set in the developed circuit and that is given in the data sheets.

8.2. *Miller's theorem for voltages*

The Miller theorem for voltages allows splitting (see Fig. 2) the impedance connected to an arbitrary port of a circuit into two impedances connected to ground, provided the ratio μ (Miller gain) is known of the port terminal voltages relative to ground. As the Miller theorem for current, this new theorem will help us to divide a circuit into two separate parts and to solve them independently, thus, simplifying the analysis.

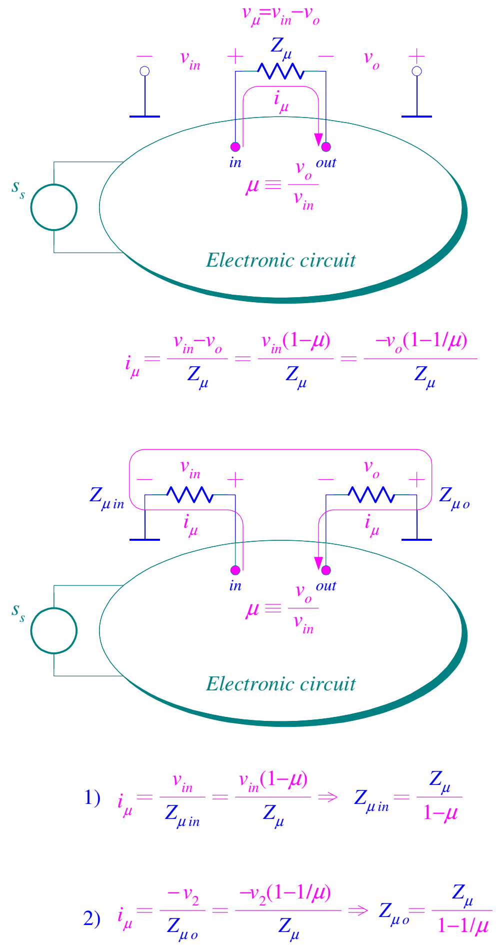

Fig. 3. Miller's theorem for voltages.

Example circuit: CE amplifier at high frequencies

Our aim is to find the upper limit, ω_H, of the amplifier small-signal frequency band. To simplify the analysis, we split C_μ into $C_{\mu\,in}$ and $C_{\mu\,o}$ (see Fig. 4) by applying the Miller theorem. This allows us to separate the circuit input and output loops.

To find $C_{\mu\,in}$ approximately, we suppose that ω_H depends on the corner frequency of the circuit input part only, and that the $C_{\mu\,o}$ capacitor impedance is high enough to be neglected at ω_H in the circuit output part. Thus, we can find $C_{\mu\,in}=C_\mu(1-\mu_{dc})$, where μ_{dc} does not depend on $C_{\mu\,o}$. To find $C_{\mu\,o}$, we suppose that $|\mu|>>1$, which gives us $C_{\mu\,o}=C_\mu\,(1-1/\mu)\approx C_\mu$.

The corner frequencies ω_{in} and ω_o of the separate input and output parts of the circuit can very easily be found (see Fig. 5). If $\omega_o>4\omega_{in}$ (see Fig. 4), then $\omega_H\approx\omega_{in}$, and the impedance of the $C_{\mu\,o}$ capacitor is indeed high enough, and its effect on the Miller gain μ can be neglected. If $\omega_o<4\omega_{in}$, then our above assumption is not correct, and we cannot solve this circuit by hand.

Note that for negative $|\mu|>>1$, the $C_{\mu\,in}=C_\mu(1-\mu)$ capacitance in the input part of the CE amplifier can be much greater than C_π. Such an increase of the equivalent input capacitance and the corresponding reduction of the circuit frequency band is called Miller effect.

Note also that the Miller gain and ω_o can be found with no approximation:

$$\mu=-g_m\frac{(R_C\parallel r_o)\dfrac{1}{j\varpi C_\mu(1-1/\mu)}}{(R_C\parallel r_o)+\dfrac{1}{j\varpi C_\mu(1-1/\mu)}}$$

$$=-g_m\frac{R_C\parallel r_o}{1+j\varpi(R_C\parallel r_o)C_\mu(1-1/\mu)}$$

$$\Rightarrow\ \mu=-[1+g_m(R_C\parallel r_o)]\frac{1}{1+j\varpi(R_C\parallel r_o)C_\mu}+1$$

$$\varpi_o=\left.\frac{1}{C_\mu(R_C\parallel r_o)\sqrt{1-\left(\dfrac{\sqrt{2}}{g_m(R_C\parallel r_o)}\right)^2}}\right|_{R_C<<r_o}$$

$$=\frac{1}{C_\mu(R_C\parallel r_o)\sqrt{1-\left(\dfrac{\sqrt{2}}{\alpha_f}\dfrac{r_e}{R_C}\right)^2}}\qquad .\qquad(8)$$

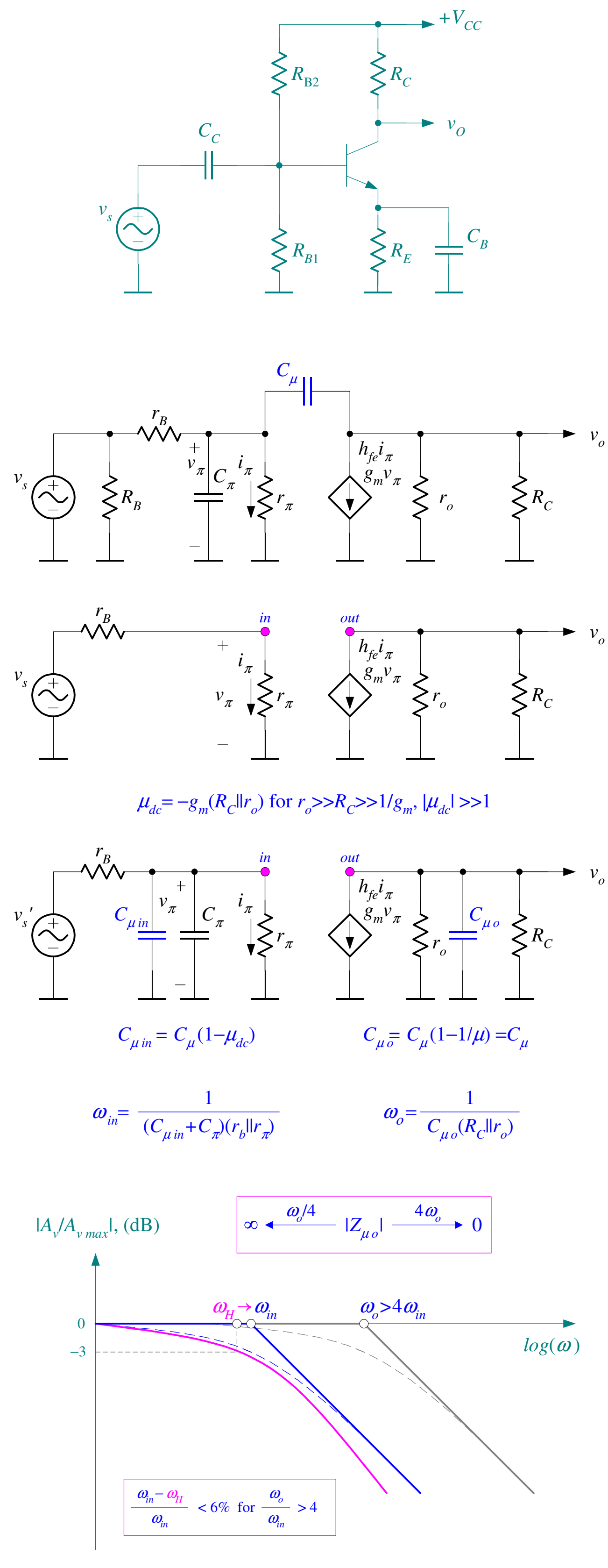

Fig. 4. Example circuit: CE amplifier at high frequencies.

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Example circuit: CB amplifier at high frequencies

The aim is to find the upper limit, ω_H, of the amplifier small-signal frequency band. To find ω_H easily, we neglect r_o and also the voltage drop across r_b (because both i_b and r_b are small). The latter allows us to ground r_b and the controlled source (see Fig. 5) and to separate the circuit input and output parts.

The corner frequencies ω_{in} and ω_o of the separate input and output parts can very easily be found (see Fig. 5). When one of these frequencies is greater than the other by at least of a factor of 4, then ω_H is nearly equal to the lower corner frequency. When $\omega_{in}=\omega_o$, then $\omega_H=(2^{0.5}-1)^{0.5}\omega_{in/o}$ (*prove this!*). In all the other cases, ω_H cannot by found in a simple way.

Note that the CB amplifier does not suffer from the Miller effect, and, thus, has a much wider frequency band than the CE amplifier.

REFERENCES

[1] S. Sedra, K. C. Smith, Microelectronic Circuits, 4th ed. New York: Oxford University Press, 1998.

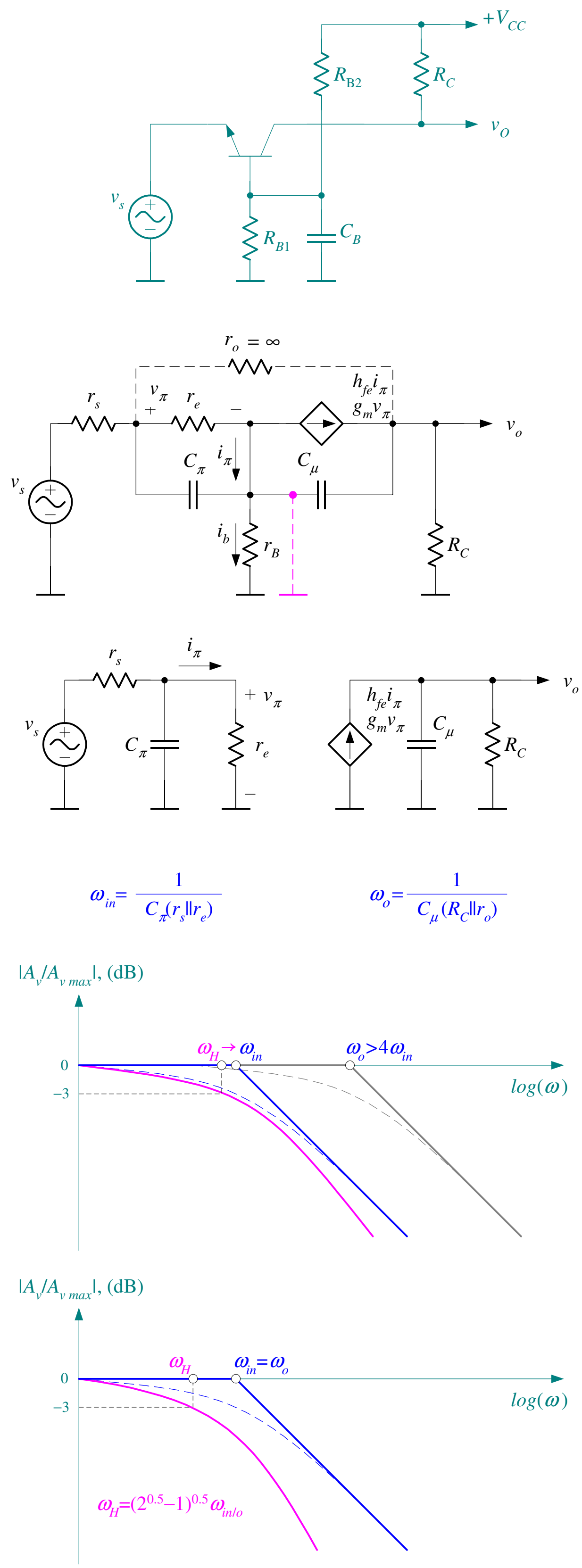

Fig. 5. Example circuit: CB amplifier at high frequencies.