Шум в електронните схеми

Съдържание:

1. Introduction

Internal noise: noise generated in the circuits themselves.

- Noise is caused by small voltage and current fluctuations that are generated within the devices.
- Noise is due to the fact that the electric charge is not continuous and it is carried in discrete amounts – the charges of the electron.
- The noise limits the the lowest signal, which can be amplified by the circuit without significant deterioration of the quality.
- The noise limits also the upper amplification of the circuit.

External noises: noises which sources are outside of the circuit. They penetrate into the circuit through power supply; circuit inputs and outputs; by electromagnetic coupling with other circuit; etc. Examples: power supply hum; interference in the analog circuits if there is switching of electric power in the neighborhood; microphonic noise due to vibration of some circuit components, etc. These noise will be not considered here.

2. Sources of Noise

2.1. Shot Noise

Always is associated with a direct-current flow and is presenting in diodes, MOS transistors and BJT.

The current through a diode in forward biasing is composed of holes crossing from p-type semiconductor to n-type and electrons, crossing the junction in opposite direction. The passage of each carrier is a random event and is depending on the energy of the carrier, its velocity and the direction of the velocity toward the junction. Thus the current through the junction is composed of a large number of random independent current pulses. Its dependence on the time is shown in Fig. 1.

The fluctuation of the current is an ac noise signal and is termed **shot noise**.

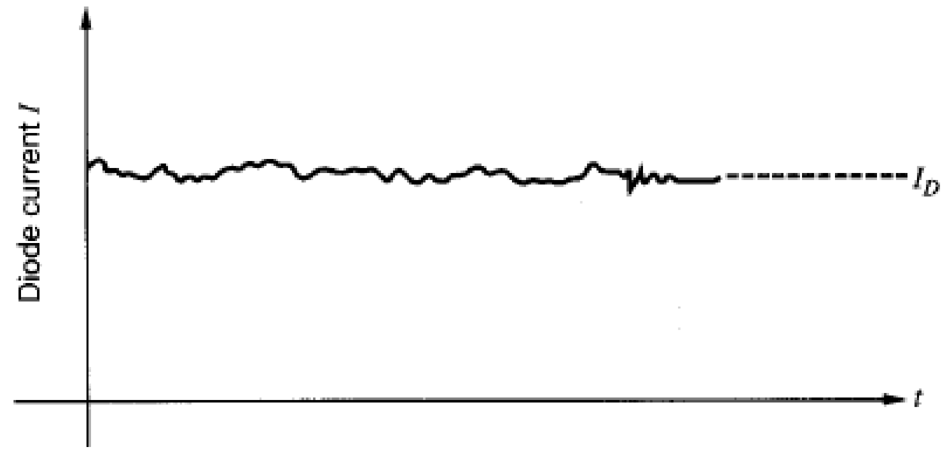

Fig. 1. Diode current as a function of time. The fluctuation is exaggerated.

The dc current I_D through the diode is the averaged value of the instantaneous current. The variable part i^2 is specified in terms of its mean-square variation about I_D

$$i^2 = \overline{(I - I_D)^2} = \lim_{T \to \infty} \frac{1}{T} \int_0^T (I - I_D)^2 \, dt \qquad (1)$$

Often instead of i^2 is used $\overline{i^2}$ in order to underline the statistical averaging.

i^2 is proportional to the frequency bandwidth Δf, in which it is measured

$$i^2 = 2qI_D\Delta f \tag{2}$$

q is the charge of the electron; $q = 1.6\times10^{-19}$C.

The spectrum density $i^2/\Delta f$ (in A^2/Hz) is constant and equal to $2qI_D$. This type of noise is called **white noise**.

Example: if I_D = 1mA and Δf = 1MHz,

$i^2 = 2\times(1.6\times10^{-19})\times(1\times10^{-3})\times(1\times10^{6})$

$= 3.2\text{x}10^{-16}A^2$;

$i = 1.8\text{x}10^{-8}$A.

The spectrum density is $3.2\text{x}10^{-22}A^2$/Hz.

The noise signal is represented in the small-signal diode model as an independent current source in parallel to the dynamic diode resistance.

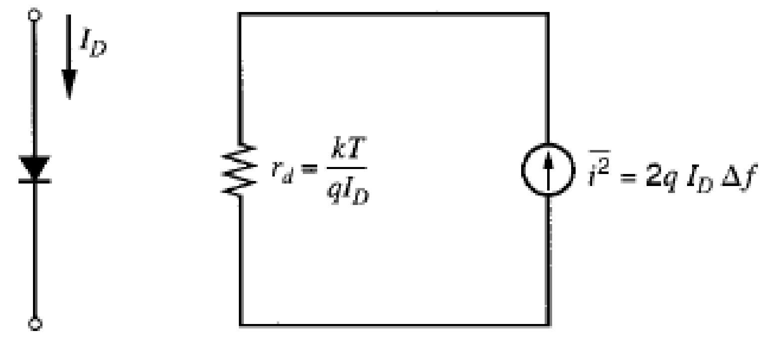

Fig. 2. Small signal equivalent circuit of a diode with noise. The direction of the current source has no significance.

Formula (2) is valid until the frequency becomes comparable to 1/τ. τ is the carrier transit time through the depletion region and is extremely small, so (2) can be used even in the GHz range.

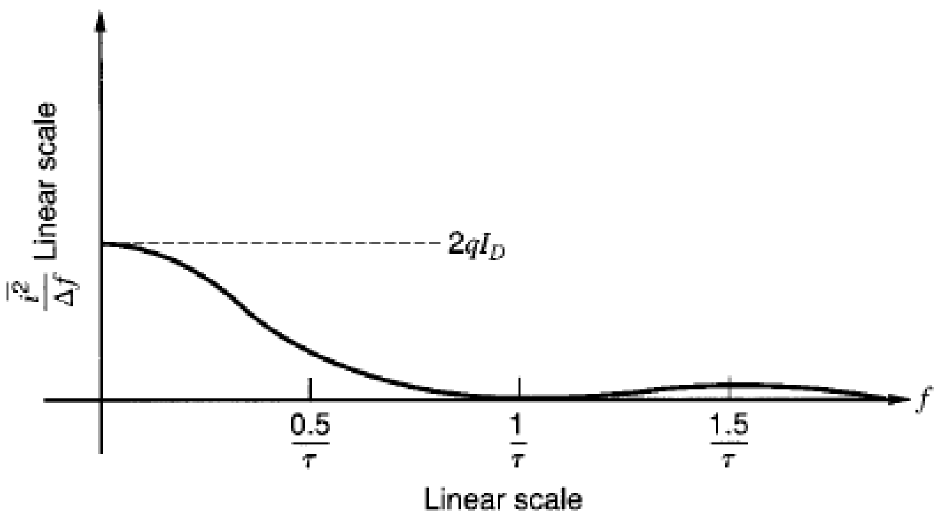

Fig. 3. Spectral density of a shot noise in a diode with transit time τ.

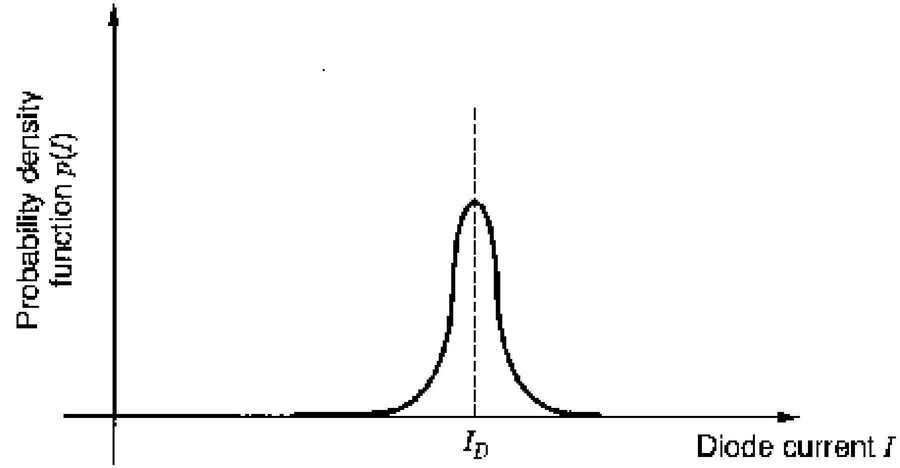

Fig. 4. Probability density function of the diode current. The current has a Gaussian distribution of its instantaneous values.

The probability density function of the instantaneous values of the diode current is shown in Fig. 4. It corresponds to the time function in Fig. 1. It gives an idea how the probability to have an instant value of the diode current, different from I_D, drops down when it is far from I_D. The **standard deviation $\sigma = i^2$** gives the practical limits for the values of the current since the current magnitude is in the limits I_D - 3σ till I_D - 3σ during 97% of the time.

2.2. Thermal Noise

- Cause: random thermal motion of the electrons.
- It presents in every resistor.
- Proportional to the absolute temperature *T* (see formula (3)).
- Doesn't depend on the existence of dc current through the resistor.
- Spectral density is constant up 10^{13}Hz – thus it is white noise.
- Gaussian amplitude distribution

The averaged noise voltage and current in a bandwidth Δf are

$$v^2 = 4kTR\Delta f; \quad i^2 = 4kT\frac{1}{R}\Delta f \qquad (3)$$

k is the Boltzmann constant; k = 1.38×10^{-23}

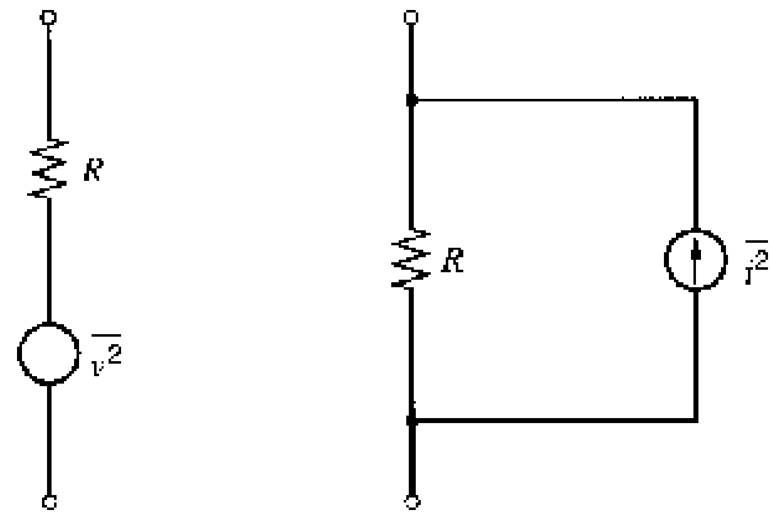

Fig. 5. Two equivalent representation of the thermal noise in the resistor *R*, corresponding to the both expressions in (3).

Since the shot noise and the thermal noise are white noises and have similar amplitude distribution, they are indistinguishable once they are introduced in the circuit.

Example: 1kΩ resistor at room temperature creates $v^2/\Delta f = 16\text{x}10^{-18}\text{V}^2/\text{Hz}$.

2.3. Flicker Noise (1/f Noise)

- Cause: mainly due to the traps associated contamination and crystal defects. They capture and release carriers in a random fashion which is perceived as a noise.
- It presents in all active devices and carbon resistors.
- Spectral density decreases as f^{-b} (see formula (4)) – from here is the name 1/f noise.
- Non-Gaussian amplitude distribution

The averaged noise current in a bandwidth Δf is

$$i^2 = K_1 \frac{I^a}{f^b} \Delta f \qquad (4)$$

In the formula Δf is the bandwidth at frequency f; I is the dc current through the device; a is a constant in the range 0.5 to 2; b is a constant about 1. K_1 is a constant for a particular device which vary depending on device type. Even for identical devices in an integrated circuit it can be different, since it represents contaminations. Thus this noise is difficult to predict.

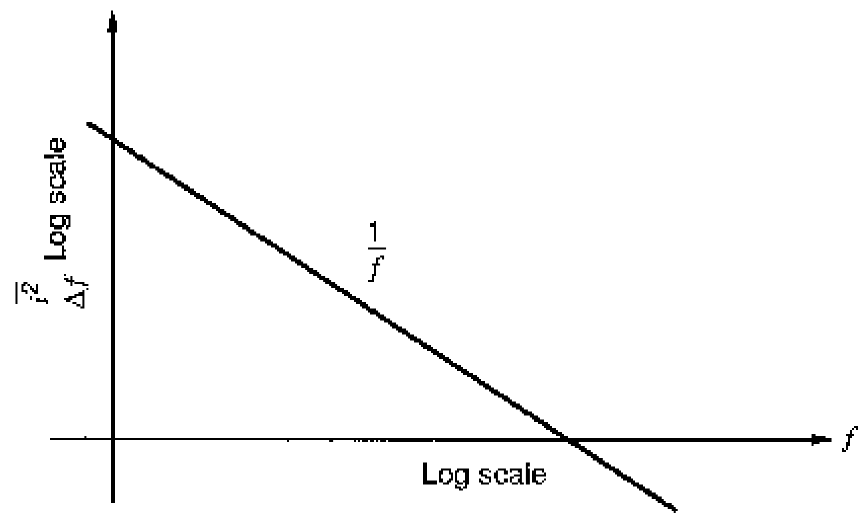

Fig. 6. Spectrum of the flicker noise. The noise is more significant at low frequencies, but for some devices it can dominate even in MHz range.

2.4. Burst Noise (Popcorn Noise)

- Cause: not fully understood. Related to the presence of heavy metal ion contamination; e.g. in gold-doped devices.
- Presents in some integrated circuits and discrete transistors.
- Appears as a bursts of noise at some levels (2 or more), which produce popping sound in loudspeakers – **popcorn noise**. See Fig. 7.
- Spectrum: low frequency noise, drops down as f^{-2} (see (5) and Fig. 8). The spectral density is

$$i^2 = K_2 \frac{I^c}{1+\left(f/f_c\right)^2}\Delta f \tag{5}$$

Δf is the bandwidth at frequency f; I is the dc current through the device; c is a constant in the range 0.5 to 2; f_c is a particular frequency for a given device. K_2 is a constant for a particular device.

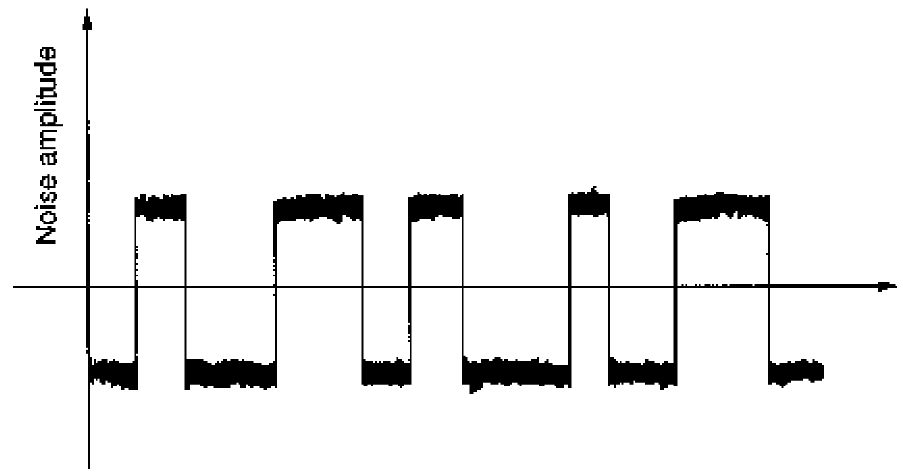

Fig. 7. Typical burst noise waveform.

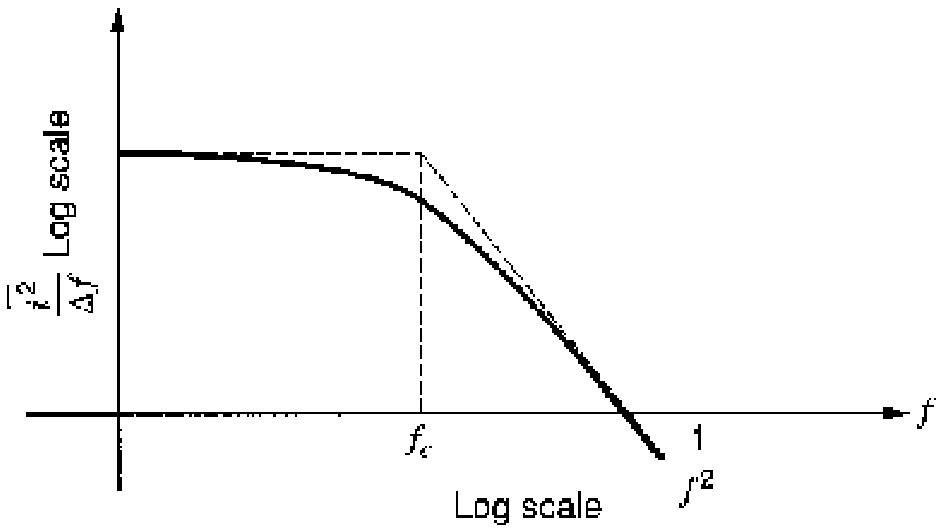

Fig. 8. Burst noise spectral density versus frequency.

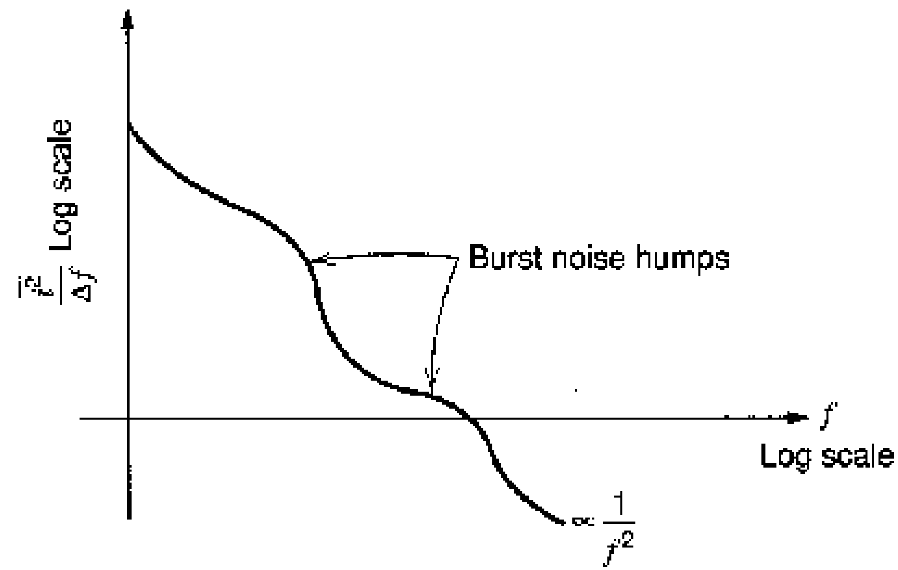

Fig. 9. Burst noise processes often occur with different time constants. Since flicker noise is invariably present, the composite low-frequency noise has multiple humps due to these time constants.

2.5. Avalanche Noise

- Cause: Zener or avalanche breakdown in a pn-junction. The holes and electrons in the depletion region of a reversed biased pn-junction acquire sufficient energy to create hole-electron pair by colliding with silicon atoms. The process is cumulative, resulting in the production of a random series of large noise spikes.
- The avalanche noise is always associated with dc current flow.
- The avalanche noise is much greater than the shot noise at the same current, since a single carrier can start an avalanching process, that results in a production of a current burst.
- Presents in Zener diodes and transistors in a breakdown.
- The magnitude of the averaged noise voltage is difficult to predict as it depends on the device structure and uniformity of the silicon crystal.
- The magnitude distribution is non-Gaussian.
- Spectrum: approximately flat.
- If present this noise is much stronger then all other noises. Due to this <u>Zener diodes are avoided in the integrated circuits</u>, especially in IC for communication applications.

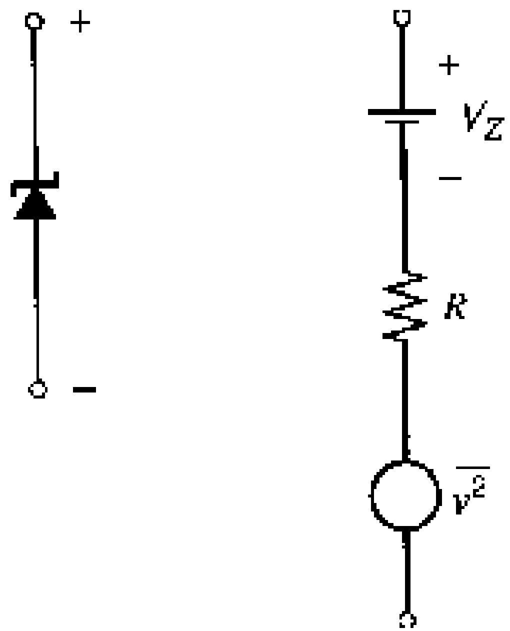

Fig. 10. The Zener diode and its small signal equivalent circuit including noise. V_Z is the breakdown voltage, R is the dynamic resistance (typically 10 to 100Ω) and v^2 is the noise voltage.

The magnitude of the noise voltage is difficult to predict, but typical measured value of the spectral density is $10^{-14}V^2/Hz$ at a Zener dc current of 0.5mA. This is equivalent to a thermal noise of a 600kΩ resistor. For comparison: if re-calculate the example for the shot noise at the same current we will receive $1.6x10^{-22}A^2/Hz$, which is equivalent to a thermal noise of a 100Ω resistor.

3. Noise Models of Electronic Components

3.1. Junction Diode

Noises, presenting in a forward biased junction diode:

- Shot noise generated in the junction;
- Flicker noise generated in the junction;
- Thermal noise generated in the silicon bulk resistance.

Experimentally has been found that the shot noise and the flicker noise can be represented as equivalent current sources. The equivalent circuit of the diode is shown in Fig. 11. The resistor r_d represents the dynamic resistance of the junction and r_s is the equivalent resistance of the silicon at the both sides of the junction. i^2 represents noises, generated in the junction and (6) is the formula for it. The first term in (6) gives the shot noise and the second term gives the flicker noise. v_s^2 represents the thermal noise, generated in the silicon resistance r_s and its formula is (7).

$$i^2 = 2qI_D\Delta f + K\frac{I_D^a}{f}\Delta f \qquad (6)$$

$$v_s^2 = 4kTr_s\Delta f \qquad (7)$$

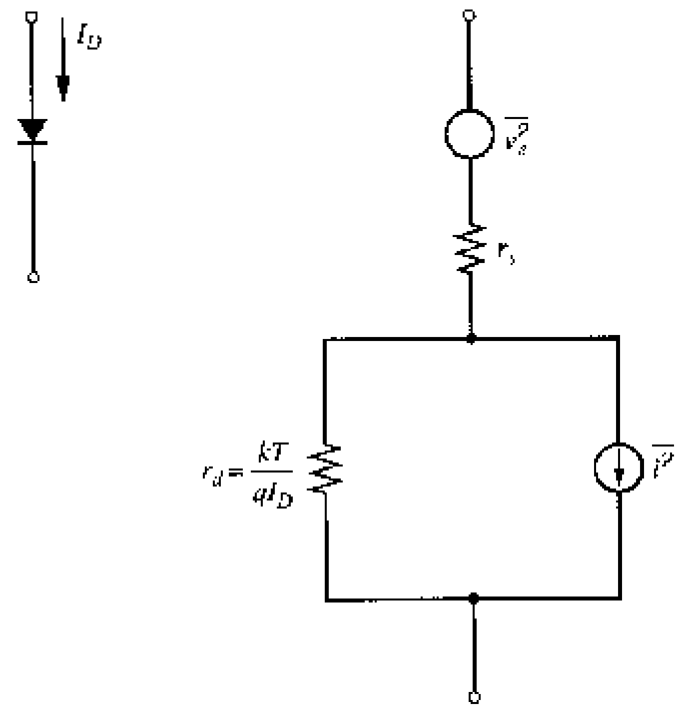

Fig. 11. Complete small-signal equivalent circuits of a diode, which includes noise sources

3.2. Bipolar Junction Transistor (BJT)

Noises, presenting in a BJT in active region:

- *Shot noise generated by the collector current*. Minority carriers, which enter from emitter in the base diffuse across the base until they reach the collector-base junction, where they are swept to the collector. The collector current I_C consists of series of random current pulses and thus it shows a full shot noise. This noise is given by formula (9).
- *Shot noise generated by the base current*. The base current is due to 1) the recombination in the base and in the base-emitter depletion region; 2) the carrier injection from the base into the emitter. The both parts are independent random processes, which is perceived as a shot noise. It is included in formula (10).
- *Thermal noise generated in the base resistor r_b*. r_b represents the ohmic resistance of the base region. It is given by formula (8).
- *Thermal noise generated in the resistor r_c* of the collector region. It is in series with high-impedance collector node and is negligible (see Fig. 12).
- *Flicker and burst noise*. Experimentally they have been found to be represented by current sources across the internal base-emitter junction. Usually they are combined with the shot noise of the base current in a single source i_b^2. They are included in formula (10).

<u>All noise sources in Fig. 12 are independent of each other</u>, since they arise from separate, physically independent mechanisms.

$$v_b^2 = 4kTr_b\Delta f \qquad (8)$$

$$i_c^2 = 2qI_C\Delta f \qquad (9)$$

$$i_b^2 = 2qI_B\Delta f + K_1\frac{I_B^a}{f}\Delta f + K_2\frac{I_B^c}{1+\left(f/f_c\right)^2}\Delta f \qquad (10)$$

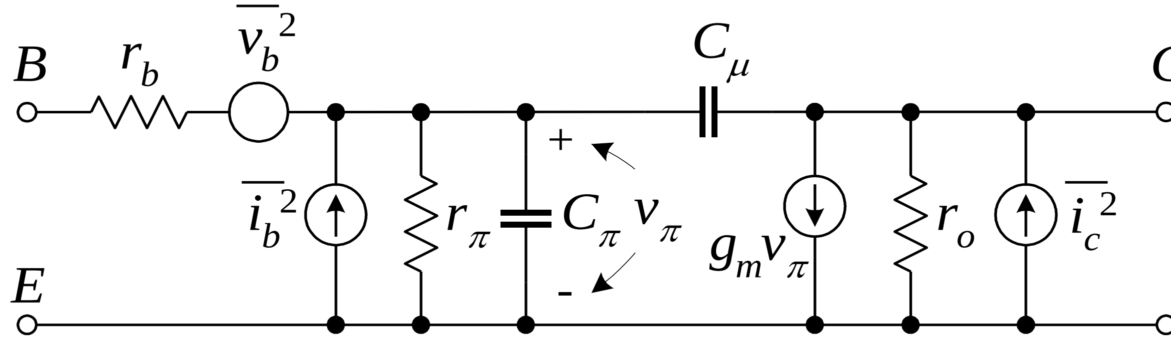

Fig. 12. Complete BJT small-signal equivalent circuits with noise sources. The resistors r_π and r_o are fictitious resistors, used for modeling purposes only and they do not exhibit a thermal noise.

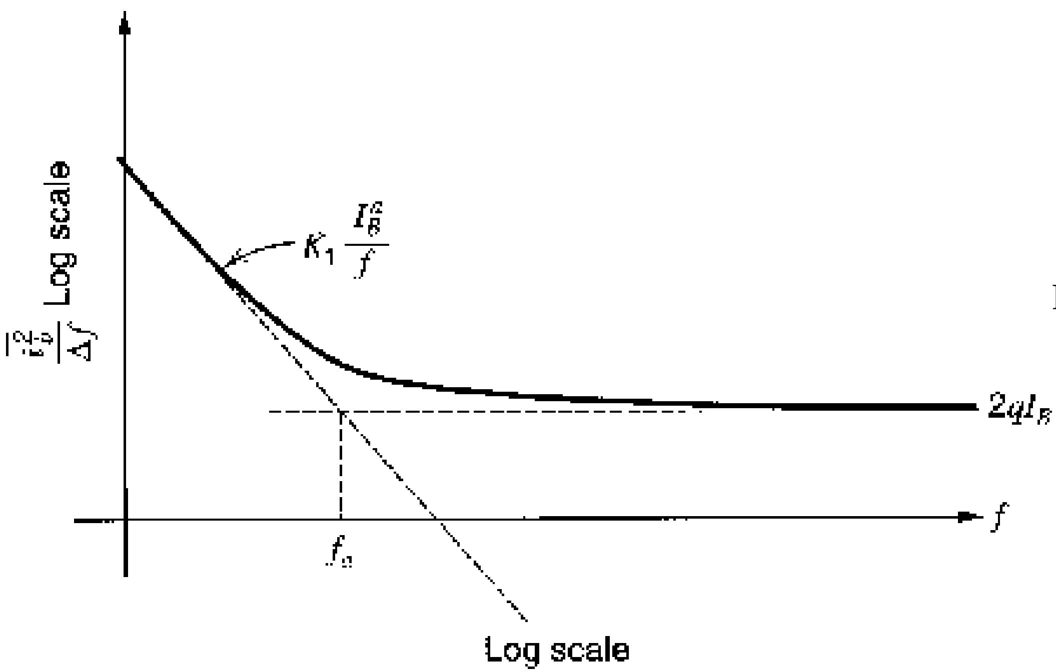

Fig. 13. Spectral density of i_b^2, corresponding to (10). The burst noise is neglected for simplicity. The frequency f_a, at which the both asymptotes meet is called *flicker noise corner frequency* and can vary from 100Hz (for careful processed BJT) till 10MHz.

3.3. MOS Transistors

Noises, presenting in a MOS transistors:

- *Thermal noise generated in the channel*. The channel is a resistor, which is controlled by the gate voltage. It exhibits thermal noise, which is a major noise source in the MOS transistor.
- *Flicker noise*. The MOS transistors conduct the current near the surface where surface acts as traps,which capture and release the current carriers. Together with the thermal noise it is represented by an equivalent drain-source current generator i_d^2, which is given by (11).
- *Shot noise generated by the gate leakage current*. This current is very small and the noise is also very small. The value of this current is given by (12).
- *HF noise component of the gate current, caused by the channel thermal noise*. In an arbitrary point of the channel, the gate-to-channel voltage has a random component due to the fluctuations along the channel caused by the thermal noise. These voltage variations create a noisy ac gate current due to capacitance between the gate and channel. The formula for the mean squared value of this current is (13).

The first three noise sources above are independent of each other. The HF noise evidently is correlated with the thermal noise in the channel.

$$i_d^2 = 4kT\left(\frac{2}{3}g_m\right)\Delta f + K\frac{I_D^a}{f}\Delta f \qquad (11)$$

$$i_g^2 = 2qI_G\Delta f \qquad (12)$$

$$i_g^2 = \frac{16}{15}kT\omega^2 C_{gs}^2\Delta f \qquad (13)$$

The thermal (first) component in (11) as well equation (13) are valid for long channel devices (>1μm). If the channel is shorter, the thermal noise is 2 to 5 times larger than it is given by the first component in (11). The HF noise also is larger.

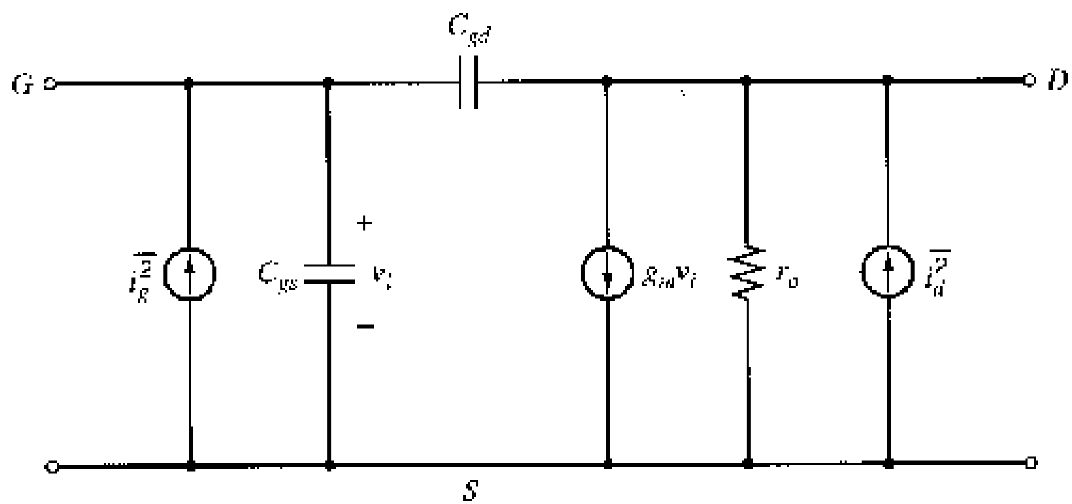

Fig. 14. MOS small-signal equivalent circuit with noise sources.

3.4. Resistors

- *Thermal noise*. Always generated in the resistors. The value of the noise current or voltage is given by (3).
- *Flicker noise*. This noise is generated only in carbon resistors.

3.5. Capacitors and Inductors

The ideal capacitors and inductors do not generate noise. But since the real components always have parasitic resistances, there is a noise generation due to these resistances.

4. Noise Analysis of the Circuits

4.1. General Remarks

Noise signals are very small, which permit to apply the small-signal linear analysis.

Noise analysis is a frequency analysis. Consider a noise current source with mean-square value

$$i^2 = S(f)\Delta f \qquad (14)$$

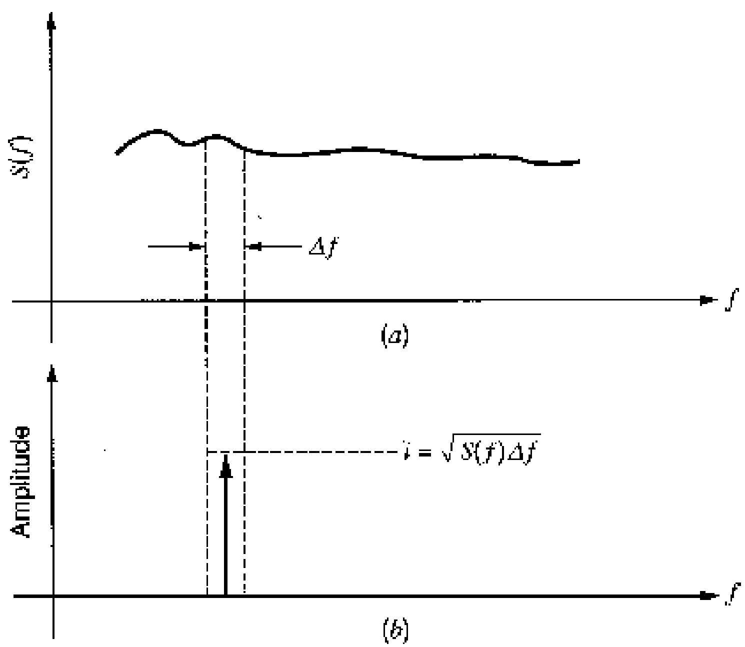

Fig. 15. Representing of a noise with a bandwidth Δf by an equivalent sinusoid, having corresponding rms value.

S(*f*) is a spectral density, given by above formulas. If Δf is very small, the current source can be represented at the frequency *f* by a sinusoidal current source, having a rms value

$$i = \sqrt{S(f)\Delta f} \qquad (15)$$

This representation reduces the noise analysis to familiar sinusoidal circuit analysis. Each circuit contains many noise sources. If they are not correlated, as it is usually, the circuit can be analyzed separately for each of them, assuming that other sources are 0. The total output noise in the bandwidth Δf is calculated as mean square value by adding the individual mean-square contributions from each noise source.

Example: Equivalent noise of a two resistors in series. For the circuit in Fig. 15 we have

$$\overline{v_1^2} = 4kTR_1\Delta f\,; \qquad \overline{v_2^2} = 4kTR_2\Delta f \tag{16}$$

The instantaneous value $v_T(t)$ of the total noise voltage produced by the both resistors is

$$v_T(t) = v_1(t) + v_2(t) \tag{17}$$

The averaging gives

$$\begin{aligned} \overline{v_T^2(t)} &= \overline{[v_1(t) + v_2(t)]^2} \\ &= \overline{v_1^2(t)} + \overline{v_2^2(t)} + \overline{2v_1(t)v_2(t)} \end{aligned} \tag{18}$$

Since there is no cross-correlation between $v_1(t)$ and $v_2(t)$ the last term is 0 and

$$\overline{v_T^2(t)} = \overline{v_1^2(t)} + \overline{v_2^2(t)} = 4kT(R_1 + R_2)\Delta f \tag{19}$$

Thus **the total noise of the both series resistor is equal to the noise, created by their equivalent resistor R_1+R_2.** The same is true for parallel connected resistors.

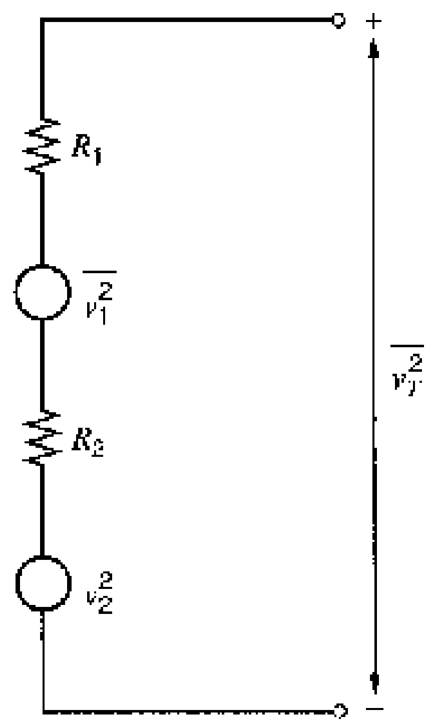

Fig. 16. Two resistors with their equivalent noise sources.

4.2. BJT Noise Performance

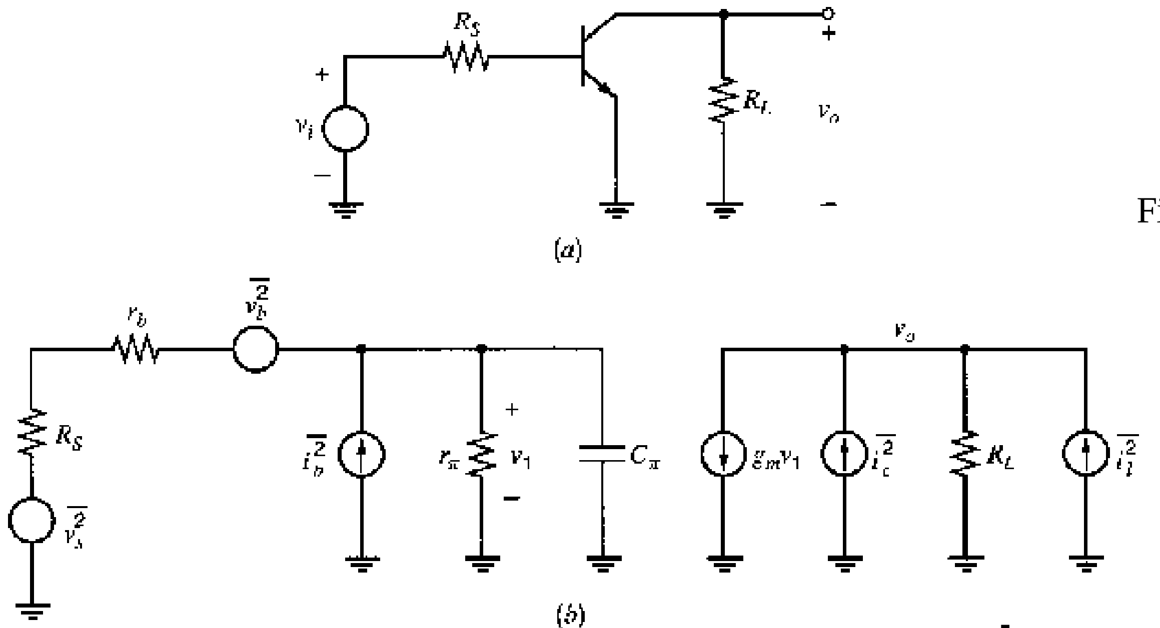

Fig. 17. (a) BJT common-emitter amplifier – small signal equivalent circuit. (b) BJT is replaced by its model and the noise sources are added.

The analysis will be preformed for every noise source separately, assuming other noise generators to be 0. Zero values of noise voltage sources means that they are replaced by a short circuit; zero values for the noise current sources means that they are disconnected from the circuit.

The source v_s produces output noise voltage v_{o1}, which can be determined by the following analysis:

$$v_1 = \frac{Z}{Z + r_b + R_s} v_s \quad \text{where } Z = r_\pi \,||\, \frac{1}{j\omega C_\pi} \tag{20}$$

$$v_{o1} = -g_m R_L v_1 = -g_m R_L \frac{Z}{Z + r_b + R_s} v_s \tag{21}$$

The mean-square value is:

$$v_{o1}^2 = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_s|^2} v_s^2 \tag{22}$$

The output noise due to the other input noise sources are:

$$v_{o2}^2 = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_s|^2} v_b^2 \quad (23)$$

$$v_{o3}^2 = g_m^2 R_L^2 \frac{(R_s + r_b)^2 |Z|^2}{|Z + r_b + R_s|^2} i_b^2 \quad (24)$$

The currents of the both output noise current sources flow through R_L only and produce voltages:

$$v_{o4}^2 = i_l^2 R_L^2 \quad (25)$$

$$v_{o5}^2 = i_c^2 R_L^2 \quad (26)$$

The total mean square output noise voltage is a sum of above five components, since all noise sources are independent:

$$v_o^2 = \sum_{n=1}^{5} v_{on}^2 \quad (27)$$

$$= g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_s|^2} \left[v_s^2 + v_b^2 + (R_s + r_b)^2 i_b^2 \right] + R_L^2 \left(i_l^2 + i_c^2 \right)$$

After replacing the expressions for the corresponding noises and for Z and after neglecting the flicker noise as small we have:

$$\frac{v_o^2}{\Delta f} = g_m^2 R_L^2 \frac{r_\pi^2}{(r_\pi + r_b + R_s)^2} \frac{1}{1 + \left(\frac{f}{f_1} \right)^2} \times$$
$$\times \left[4kT(R_s + r_b) + (R_s + r_b)^2 2qI_B \right] + \quad (28)$$
$$+ R_L^2 \left(4kT \frac{1}{R_L} + 2qI_C \right)$$

where:

$$f_1 = \frac{1}{2\pi \left[r_\pi \parallel (R_s + r_b) \right] C_\pi} \quad (29)$$

Example: Let:

$I_C = 100\mu A \quad \beta = 100 \quad r_b = 200\Omega$

$R_S = 500\Omega \quad C_\pi = 10pF$

$R_L = 5k\Omega$

Then $f_1 = 23.3MHz$ and

$$\frac{v_o^2}{\Delta f} = \left[\frac{4.13\times10^{-15}}{1+\left(\frac{f}{f_1}\right)^2} + 0.88\times10^{-15} \right] V^2/Hz \qquad (30)$$

The frequency dependence of the spectrum, given by (30), is plotted in Fig. 18.

Let suppose further that the next stages limit the spectrum of the amplifier to 1MHz. The noise spectrum till 1MHz is approximately constant and equal to $5x10^{-15}V^2/Hz$ therefore total noise voltage in 1MHz bandwidth at the output is

$$v_{oT}^2 = 5\times10^{-15}\times10^6 V^2 = 5\times10^{-15} V^2 \text{ or } v_{oT} = 71\mu V$$

If the spectrum is not constant, it is necessary to integrate across the bandwidth.

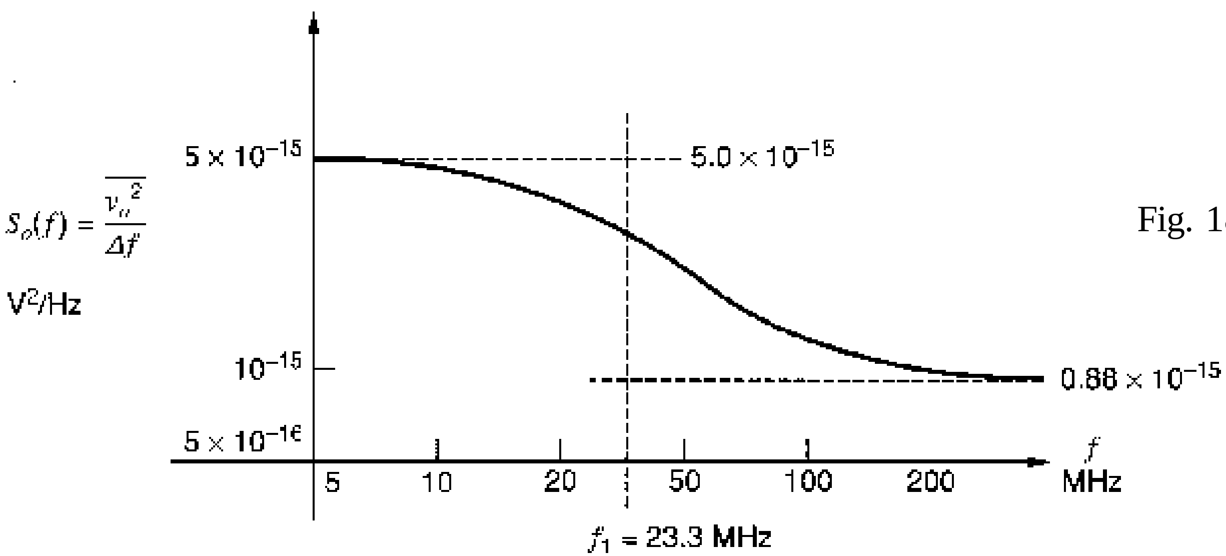

Fig. 18. Noise voltage spectrum of the circuit in the example

4.3. Equivalent Input Noise and the Minimum Detectable Signal

Usually the output noise is considered as an equivalent input noise, which is amplified together with the signal and after the amplification gives the same output noise. In this model all components in the circuit are assumed to be noiseless and the output noise is due to the equivalent input noise source only. This permits to compare the equivalent input noise with the input signal and to determine the **minimum detectable signal**. This is the minimum input signal, which can be effectively amplified without significant corruption by the noise.

The output noise of the circuit in Fig. 19 is

$$v_o^2 = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2} v_{iN}^2 \tag{31}$$

The comparison with (28) gives

$$\frac{v_{iN}^2}{\Delta f} = 4kT(R_s + r_b) + (R_s + r_b)^2 2qI_B + \\ + \frac{1}{g_m^2} \frac{|Z + r_b + R_s|^2}{|Z|^2} \left(4kT \frac{1}{R_L} + 2qI_C \right) \tag{32}$$

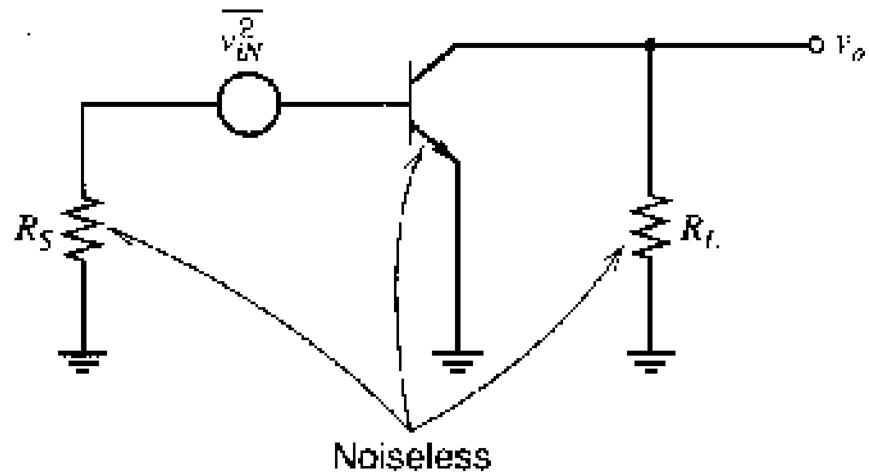

Fig. 19. Representation of circuit noise performance by an equivalent input noise voltage.

For the previous example we have:

$$A_v = 18.7$$

$$v_{iNT}^2 = \frac{v_{oT}^2}{A_v^2} = \frac{5\times10^{-9}}{18.7^2} = 14.3\times10^{-12}\,\mathrm{V}^2$$

$$v_{iNT} = 3.78\mu\mathrm{V}$$

The minimum detectable signal depends on the application, but often it is determined to be equal to the input noise voltage in the passband of the amplifier – 3.78μV in our case.

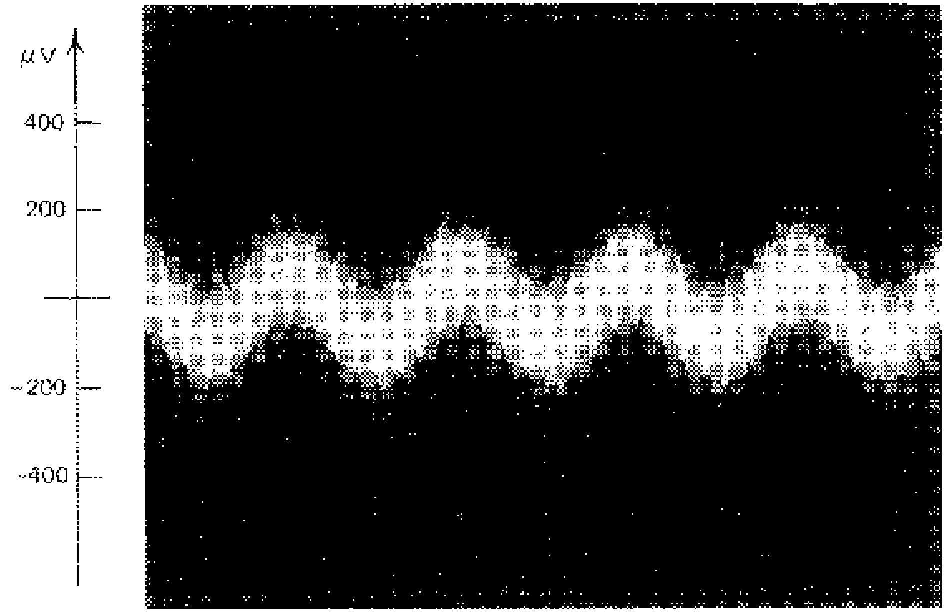

Fig. 20. Output voltage waveform of the circuit, considered in the example.

5. Equivalent Input Noise Generators. Noise Performances of the Basic Amplifiers.

5.1. Basic Principle

Basic idea: To replace all noises of a noisy two-port by two equivalent noise voltage and current generators at the input as it is shown in Fig. 21. The two-port further is considered as noiseless and all output noises are assumed to be due to the both equivalent input generators.

The approach is well working if the equivalent sources are not correlated, which generally is not the case, because they depend on the same set of original noise generators. In the following cases is possible to apply the method:

- When the cross-correlation is small and it is possible to neglect it, which is satisfied for a large number of practical circuit;
- When either voltage source; either current source dominates the cross-correlation can be neglected also.

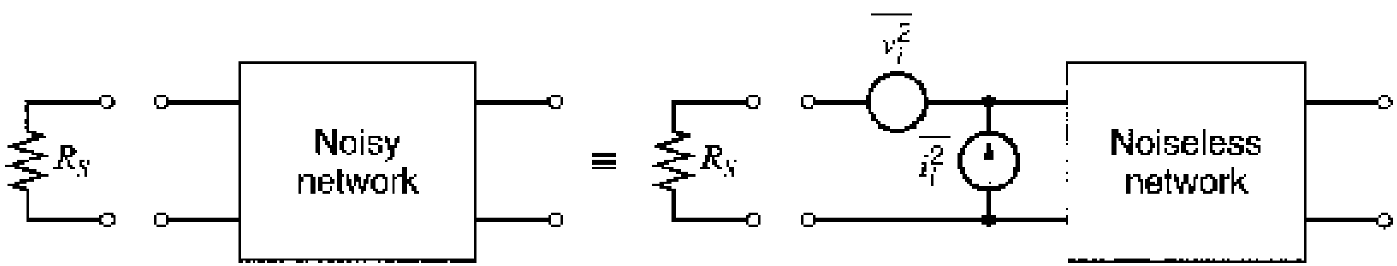

Fig. 21. Representation of noise in a two-port network by equivalent input voltage and current generators.

The both generators are necessary. If the voltage generator absents, it is impossible to reflect the case, when the input signal is from a source with zero or very small resistance R_S; when the current source absents it is impossible to consider the case when $R_S \to \infty$. In the first case the remaining current noise generator is short connected through the signal voltage source; in the second case the remaining voltage generator is disconnected from the input.

How to determine the equivalent generators:

- Consider the case when the input is short circuited, determine the output noise voltages of the both circuits and equalize them. In this case only v_i^2 has effect in the right hand side circuit in Fig. 21.
- Next consider the case when the input is open, determine the output noise voltages of the both circuits and equalize them. In this case only i_i^2 in Fig. 21 has effect in the right hand side circuit.

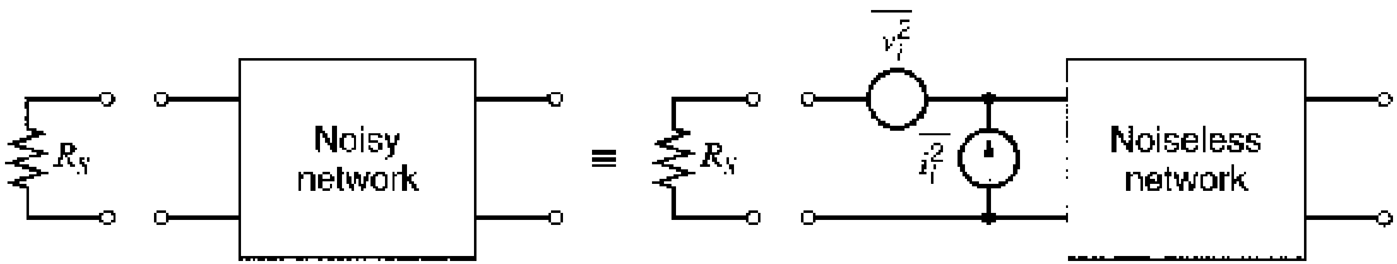

Fig. 21(repeating). Representation of noise in a two-port network by equivalent input voltage and current generators.

5.2. Equivalent Noise Generators of a BJT

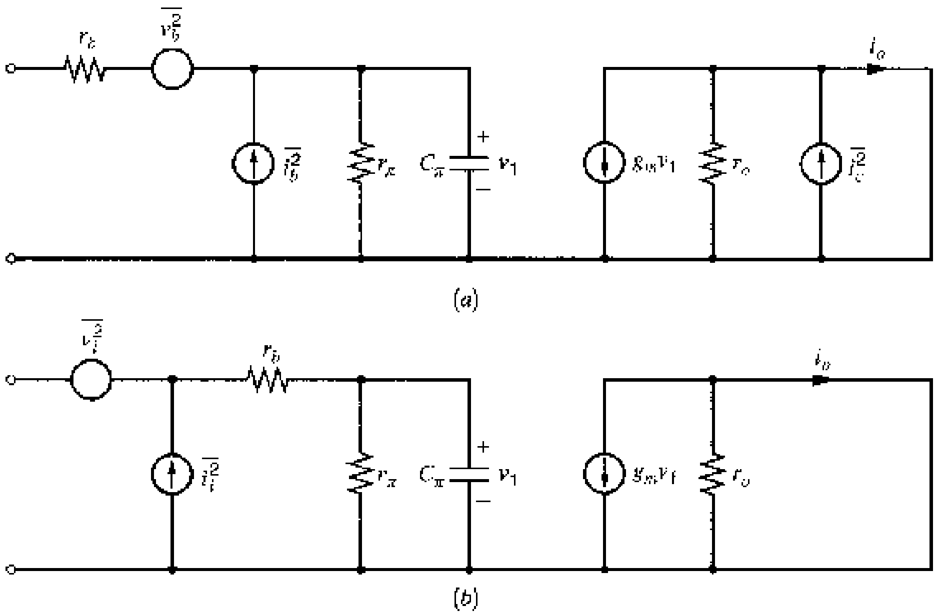

Fig. 22. (a) BJT small signal equivalent circuit with noise generators. (b) Representation of the noise performance by equivalent input generators. In the both circuits is used the r_b as a name of the resistance r_x from the BJT equivalent circuit.

Assumptions in the analysis:

- Short circuit at the output;
- C_μ is neglected;
- r_b is neglected since $r_b << r_\pi$.

When short-circuit the both inputs, i_b^2 in Fig 22(a) is short-circuited and has no effect. From equating i_o in the both circuit we receive

$$g_m v_b + i_c = g_m v_i \quad \text{or} \quad v_i = v_b + \frac{1}{g_m} i_c \tag{33}$$

Since v_b and i_c are not correlated

$$v_i^2 = v_b^2 + \frac{1}{g_m^2} i_c^2 \tag{34}$$

The replacement of v_b and i_c from (8) and (9) gives

$$\frac{v_i^2}{\Delta f} = 4kT\left(r_b + \frac{1}{2g_m}\right) = 4kTR_{eq} \tag{35}$$

The equivalent input noise voltage generator v_i can be considered as produced by an **equivalent input noise resistance** R_{eq}, equal to

$$R_{eq} = r_b + \frac{1}{2g_m} \tag{36}$$

The first term in (36) is approximately constant, but the second term decreases with increasing the collector current in the quiescent point. In this way is possible to minimize R_{eq} and v_i^2.

The equivalent input noise current is determined by considering the case when the in both inputs in Fig. 22 are open.

$$g_m Z i_i = i_c + g_m Z i_b; \qquad Z = r_\pi \parallel C_\pi \tag{37}$$

or

$$\beta(jf)i_i = i_c + \beta(jf)i_b \tag{38}$$

where

$$\beta(jf) = g_m Z = \frac{\beta_0}{1 + j\dfrac{f}{f_\beta}} \tag{39}$$

β_0 is the low-frequency value of β ($\beta_0 = g_m r_\pi$). Since i_b and i_c are independent

$$i_i^2 = i_b^2 + \frac{i_c^2}{\left|\beta(jf)\right|^2} \tag{40}$$

Next we substitute i_b and i_c from (9) and (10) and β from (39)

$$\frac{i_i^2}{\Delta f} = 2q\left[I_B + K_1' \frac{I_B^a}{f} + \frac{I_C}{\beta_0^2}\left(1 + \frac{f^2}{f_\beta^2}\right)\right] \tag{41}$$

$$K_1' = \frac{K_1}{2q}$$

The popcorn noise component in i_b is neglected. The equivalent input current generator can be considered as a shot noise created by an **equivalent shot noise current**

$$I_{eq} = I_B + K_1' \frac{I_B^a}{f} + \frac{I_C}{\beta_0^2}\left(1 + \frac{f^2}{f_\beta^2}\right) \tag{42}$$

From (41) and (42) follows that the input noise current generator can be minimized by choosing the current in the quiescent point as small as possible. This is opposite to the requirement for minimizing of v_i^2.

On Fig. 23 is plotted i_i^2 vs. frequency based on (41). The frequency f_b is given by

$$f_b = \frac{f_T}{\sqrt{\beta_F}} \tag{43}$$

where f_T is the transit frequency, at which the common emitter current gain β is equal to 1 ($f_T = \beta_0 f_\beta$) and β_F is dc current gain.

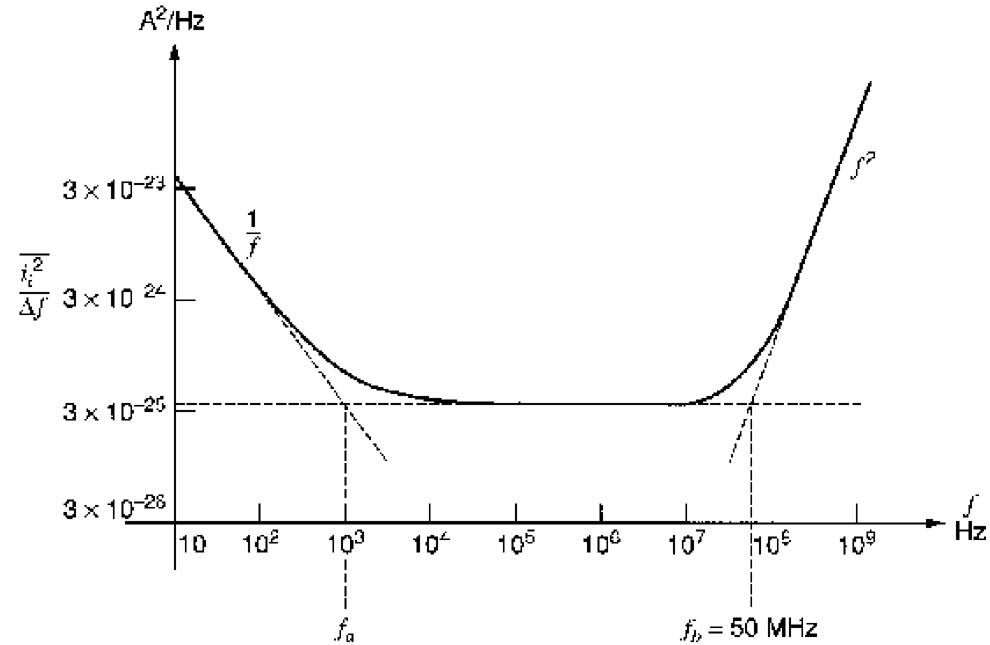

Fig. 23. Equivalent input noise current spectral density of a BJT with I_C = 100μA, $\beta_0 = \beta_F$ = 100, f_T = 100MHz.

5.3. Equivalent Noise Generators of MOS Transistor

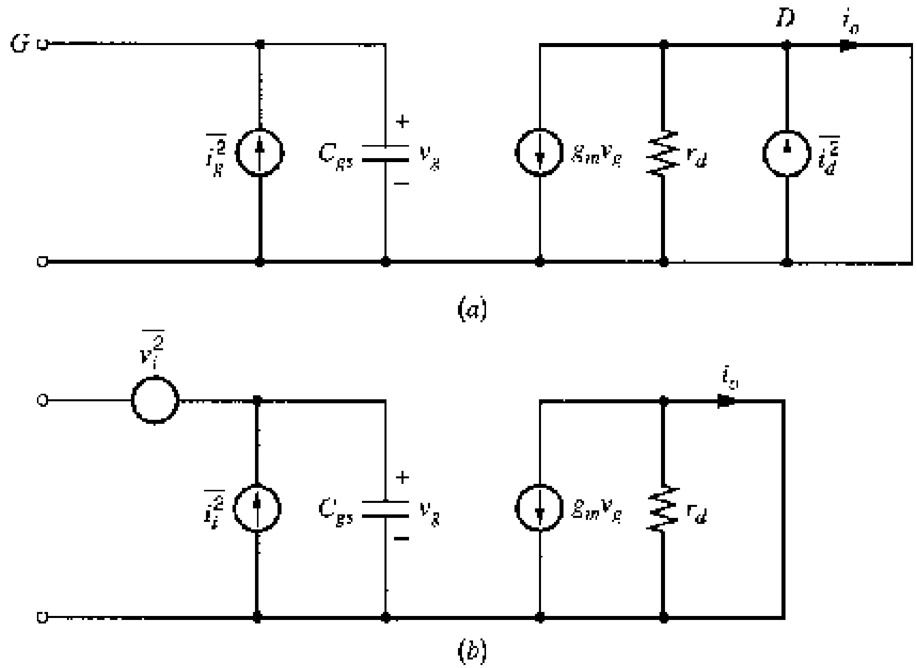

Fig. 24. (a) MOSFET small signal equivalent circuit with noise generators. (b) Representation of the noise performance by equivalent input generators.

When the both inputs in Fig. 24 are short circuited we receive

$$i_d = g_m v_i \tag{44}$$

After substituting i_d from (11)

$$\frac{v_i^2}{\Delta f} = 4kT\frac{2}{3}\frac{1}{g_m} + K\frac{I_D^a}{g_m^2 f} = 4kTR_{eq} \tag{45}$$

The equivalent input noise resistance is

$$R_{eq} = \frac{2}{3}\frac{1}{g_m} + K'\frac{I_D^a}{g_m^2 f}; \quad K' = \frac{K}{4kT} \tag{46}$$

To determine i_i^2 we assume the both inputs in Fig. 24 opened

$$i_i \frac{g_m}{j\omega C_{gs}} = i_g \frac{g_m}{j\omega C_{gs}} + i_d \tag{47}$$

i_g and i_d are not correlated and therefore

$$i_i^2 = i_g^2 + \frac{\omega^2 C_{gs}^2}{g_m^2} i_d^2 \tag{48}$$

The substitution for i_g and i_d from (11) and (13) gives

$$\frac{i_i^2}{\Delta f} = 2qI_G + \frac{\omega^2 C_{gs}^2}{g_m^2}\left(4kT\frac{2}{3}g_m + K\frac{I_D^a}{f}\right) \tag{49}$$

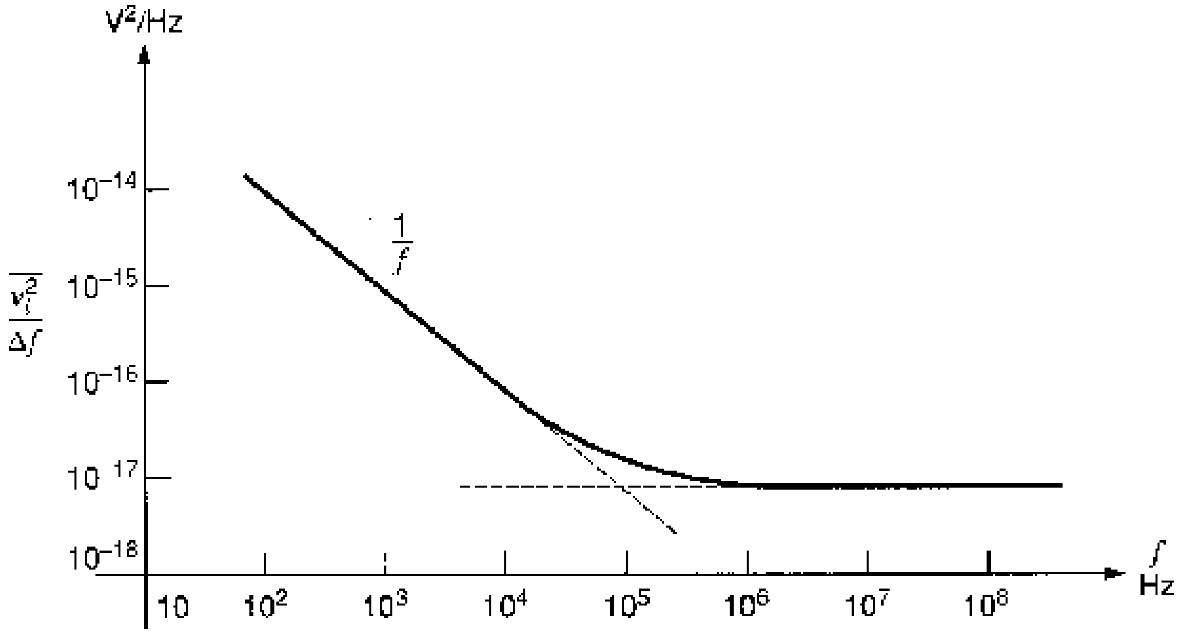

Fig. 25. Spectral density of the equivalent input noise voltage of a MOSFET.

5.4. Comparison of the Noise Properties of BJT and MOSFET

1) *Equivalent input noise voltage generator*.

For BJT the equivalent noise resistance is given by (36):

$$R_{eq} = r_b + \frac{1}{2g_m} \qquad (36)$$

For MOSFET the equivalent noise resistance is given by (46):

$$R_{eq} = \frac{2}{3}\frac{1}{g_m} + K'\frac{I_D^a}{g_m^2 f}; \qquad K' = \frac{K}{4kT} \qquad (46)$$

For BJT: if I_C = 1μA $1/2g_m$ = 13kΩ; $r_b \approx$ 100Ω and R_{eq} = 13kΩ - determined from $1/2g_m$.

If I_C = 1mA $1/2g_m$ = 13Ω; $r_b \approx$ 100Ω and R_{eq} = 113Ω - determined basically from r_b.

For MOSFET: R_{eq} has proportional frequency response to v_i^2, shown in Fig. 25. At higher frequencies R_{eq} is determined only from the the first term in (46). If g_m is 1mA/V, high frequency value of R_{eq} = 667Ω - much higher then for the BJT.

Conclusion: Input noise voltage generator of MOSFET has typically higher value than the same generator in BJT. The voltage noise generator in MOSFET exhibits flicker noise at low frequencies which is absent in BJT.

2) *Equivalent input noise current generator*.

For BJT the equivalent input current spectrum density is given by (41):

$$\frac{i_i^2}{\Delta f} = 2q\left[I_B + K_1{}'\frac{I_B^a}{f} + \frac{I_C}{\beta_0^2}\left(1 + \frac{f^2}{f_\beta^2}\right)\right] \qquad (41)$$

$$\frac{i_i^2}{\Delta f} = 2qI_G + \frac{\omega^2 C_{gs}^2}{g_m^2}\left(4kT\frac{2}{3}g_m + K\frac{I_D^a}{f}\right) \qquad (49)$$

The multiplier before the parenthesis in (49) is $1/A_i^2$, where A_i is the current gain of the circuit. The first term in (49) is also very small since I_G is extremely small. Therefore the equivalent input noise current generator for MOSFET is smaller then the same generator for BJT.

v_i^2 is more important when the signal source is with low impedance; i_i^2 affects the circuit when the signal source is with high impedance. Therefore:

- *BJT has superior noise performance when the source impedance is low;*
- *MOSFET has superior noise performance when the source impedance is high.*

5.5. Noise Performance of Common Base Circuit

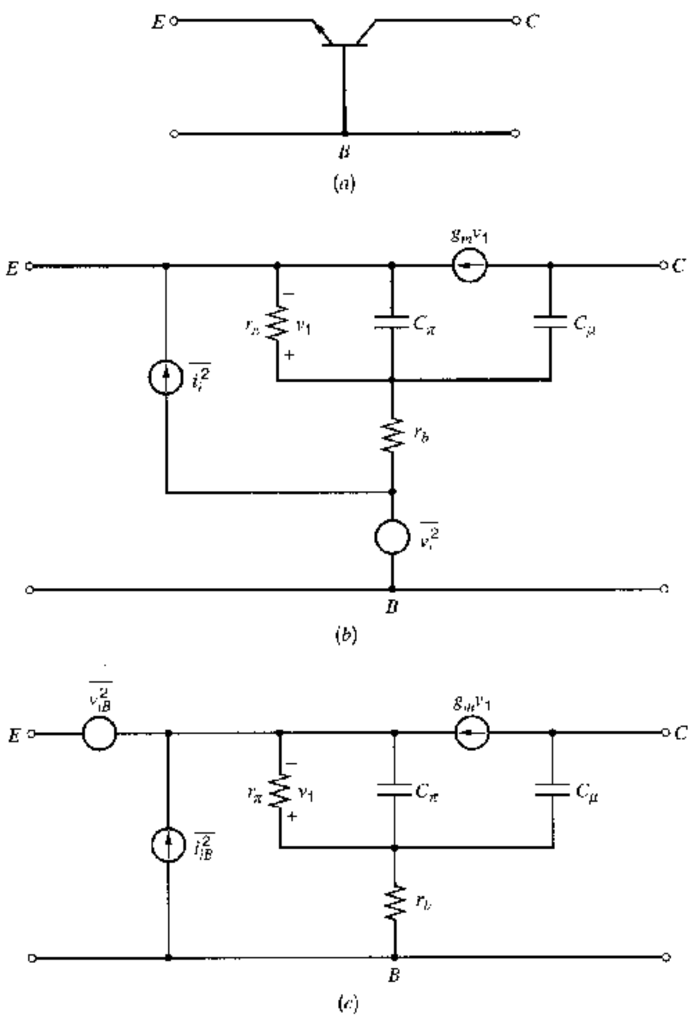

Fig. 26. (a) Common-base circuit. (b) Small-signal equivalent circuit with equivalent input noise generators for CE circuit. (c) Small-signal equivalent circuit with equivalent input noise generators for CB circuit.

In Fig. 26 (b) the BJT is replaced by its hybrid small-signal equivalent circuit and the equivalent input voltage and current generators v_i^2 and i_i^2 for common-emitter connection are introduced. In Fig. 26 (c) is the same circuit, but with the equivalent input voltage and current generators v_{iB}^2 and i_{iB}^2 for common-base circuit.

When the inputs in Fig. 26 (b) and Fig. 26 (c) are short connected between base and emitter of the BJT in Fig. 26 (b) appears v_i^2 and in Fig. 26 (c) - v_{iB}^2. Thus they must be equal. In the same way when the inputs are opened in Fig. 26 (b) acts i_i^2 and in Fig. 26 (c) acts i_{iB}^2 therefore they are equal.

$$v_{iB}^2 = v_i^2; \quad i_{iB}^2 = i_i^2 \tag{50}$$

It seems that CB has the same noise performance as CE circuit. This is not fully true. CB circuit has a current gain < 1 and therefore the *output noise current is referred back without any reduction* and can corrupt the input signal significantly.

5.6. Noise Performance of the Differential Pair

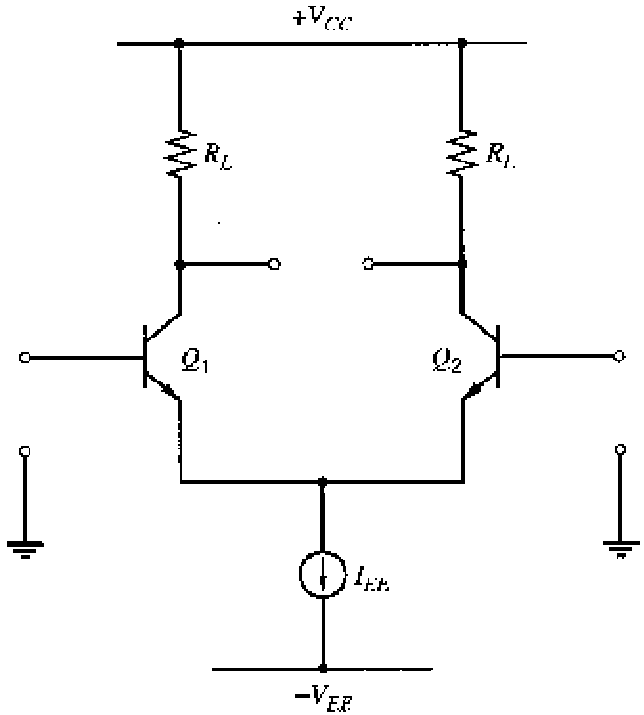

Fig. 27. The basic differential amplifier.

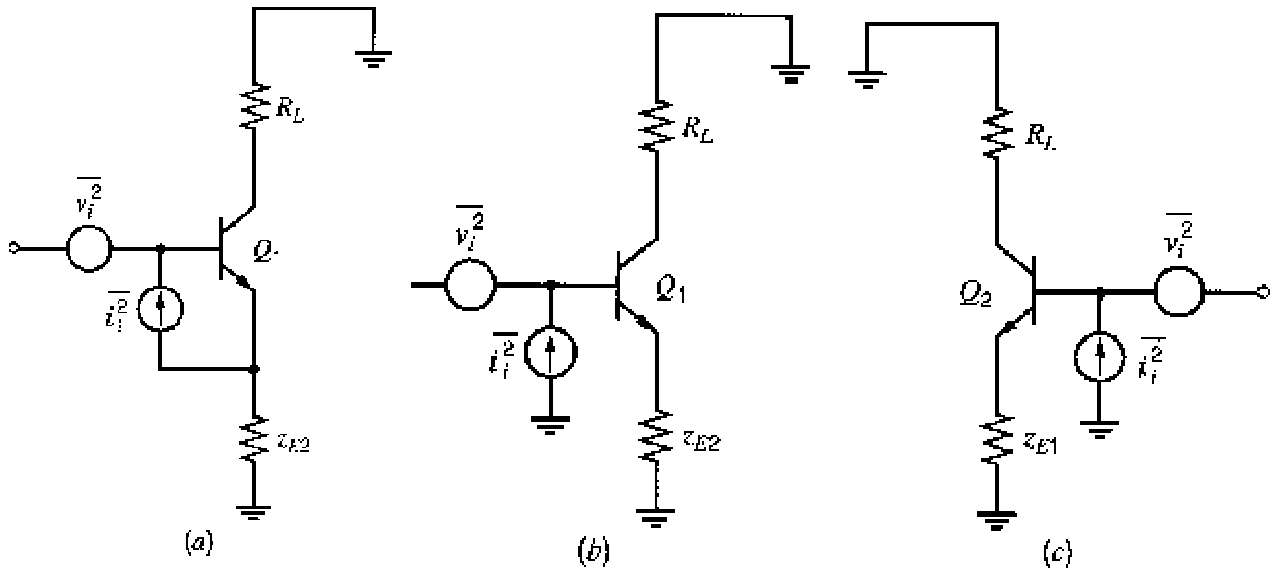

Fig. 28. Transformation of the noise sources of Q_1 and Q_2 in the differential pair.

Since the noise sources in Q_1 and Q_2 are independent, their contribution can be considered separately. In Fig. 28 (a) is shown the equivalent circuit when Q_1 is noisy and Q_2 is noiseless. z_{E2} is the equivalent impedance of Q_2 seen at its emitter, it is a function of the impedance between the the base of Q_2 and the ground. Since we are considering the mean-square values, the circuits in Fig. 28(a) and Fig. 28(b) are equivalent. In Fig. 28(c) is shown the equivalent presentation when Q_2 is considered as noisy.

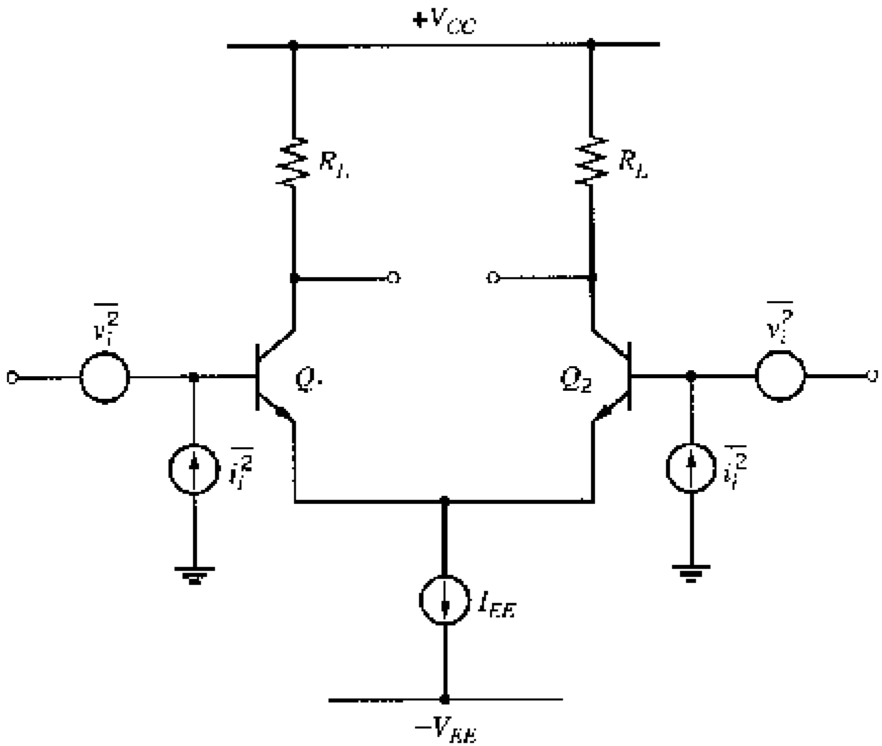

Fig. 29. The circuit of the differential pair with added noise generators for both transistors according Fig. 28.

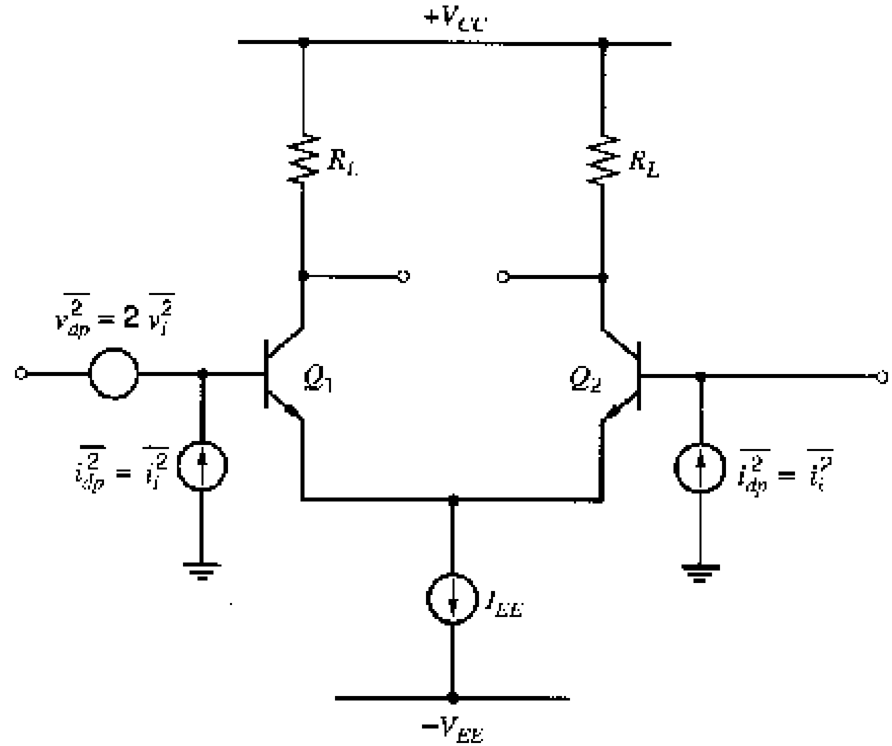

Fig.30. The circuit of the differential pair with both input voltage noise generators united.

The equivalent voltage noise sources appear in series to the differential source. Since they are independent, they can be combined as it is shown in Fig. 30.

The differential voltage gain is the same as the voltage gain of a unique CE stage. In the same time the equivalent input voltage noise generator has $\sqrt{2}$ times higher magnitude (3dB).

If the noise due to R_L is significant, it must be referred back symmetrically to the inputs.

The current mirror I_{EE} also produce a noise. But this noise is a common mode signal and if the circuit is well balanced, it can't produce a significant noise at the output.

5.7. Effect of the Feedback on the Noise Performance. Noise Performance of the Emitter Follower.

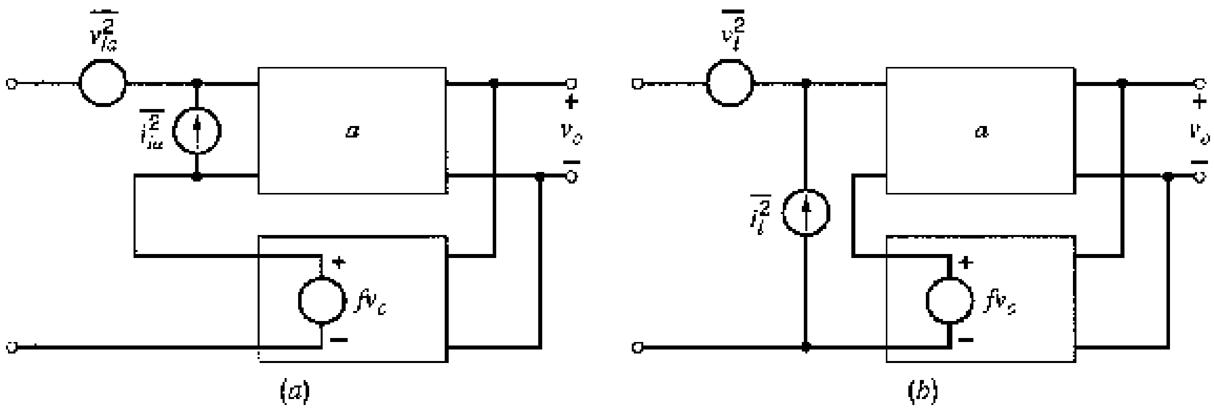

Fig. 31. (a) An amplifier *a* with noise generators, having noiseless series-voltage feedback. (b) Equivalent representation of (a) with two input noise generators.

First will be considered an *ideal noiseless feedback*, shown in Fig. 31.

1) We short circuit the both inputs, the outputs are opened. The noise current generators in the both circuits are short circuited and the noise voltage generators in the both circuits are connected in identical way – thus they are equal:

$$v_i^2 = v_{ia}^2 \quad (51)$$

2) The both inputs and outputs are open. The noise voltage generators in the both circuits have free hanging terminal and have no effect in the circuit. The noise current generators are connected to the input of the amplifier: in (a) directly; in (b) through the controlled voltage source fv_o, which is short connection for the noise signal. Therefore again the both sources are equal

$$i_i^2 = i_{ia}^2 \quad (52)$$

In the same way can be considered other feedback types. For all of them we conclude that the **ideal noiseless feedback doesn't improve the noise performance of the amplifier**..

The real feedback circuit also produces noise, since usually it includes resistors. This noise is independent on the noise sources in the amplifier and the noise produced from the feedback circuit adds to the noise from the amplifier. Therefore the **real feedback circuits worsen the noise performance of the amplifiers**.

Emitter follower.

1) The emitter is a circuit with series-voltage feedback as considered here. Therefore its equivalent noise generators are the same as for the CE circuit.

2) The emitter follower has a unity gain, therefore the equivalent input noise voltage generator of the next stage transforms into equivalent voltage noise generator at the input of the emitter follower without reduction.

6. Definition of Some Noise Terms

6.1. Signal-to-Noise Ratio (SNR)

In a specified bandwidth the signal-to-noise ratio is defined as a ratio of signal power to the noise power at a port:

$$\mathrm{SNR} = \frac{P_s}{P_n} \tag{53}$$

P_s is proportional to the current or voltage rms value in square and P_n – to the mean-square value of the voltage or the current at the port:

$$\mathrm{SNR} = \frac{v_s^2}{v_n^2} = \frac{i_s^2}{i_n^2} \tag{54}$$

Often SNR is measured in dB

$$\mathrm{SNR(dB)} = 10\log_{10}\frac{P_s}{P_n} \tag{55}$$

The larger SNR is, the less signal is corrupted by the noise.

6.2. Noise Figure

The noise figure F is a commonly used method for specifying of the noise performance of two-ports. It is defined as

$$F = \frac{\text{input SNR}}{\text{output SNR}} \tag{56}$$

The noise figure is expressed often in dB, thus

$$F(\text{dB}) = 10\log_{10}\frac{\text{input SNR}}{\text{output SNR}} \tag{57}$$
$$= \text{input SNR(dB)} - \text{output SNR(dB)}$$

Let N_i is the noise power at the input; N_o – the noise power at the output; S_i - the signal power at the input; S_o - the signal power at the output. Then

$$F = \frac{S_i}{N_i}\frac{N_o}{S_o} \tag{58}$$

Let G is the power gain of the two-port, then

$$S_o = GS_i \tag{59}$$

For an **ideal noiseless two-port** $N_o = GN_i$ and from (58) follows that ***F* = 1 or 0dB.**

Alternative definition of noise figure. (58) can be transformed to

$$F = \frac{S_i}{N_i}\frac{N_o}{S_o} = \frac{N_o}{GN_i} \tag{60}$$

Therefore the noise figure is a ratio between the total output noise N_o to that part of output noise due to the amplified input noise (the noise due to the source resistance).

If N_c is the noise produced by the circuit, $N_o = GN_i + N_c$ and

$$F = 1 + \frac{N_c}{GN_i} \tag{61}$$

Often the noise figure is specified for a small bandwidth Δf at a frequency $f >> \Delta f$. In this case it is called **spot noise figure**.

If the noise figure is utilized over a wide bandwidth, it is called **average noise figure**.

6.3. Noise Temperature

The noise temperature is defined as the temperature at which the source resistance R_S must be held so that the noise output of the circuit due to R_S equals to noise output of the circuit itself.

7. References

P. R. Gray, P. J. Hurst, S. H. Lewis, R. G. Mayer, Analysis and design of analog integrated circuits, 4th ed., J.Wiley&Sons, 2001.